Improved Lightpath (Wavelength) Routing in Large WDM Networks

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0. Outline of the Talk

1. Introduction

The emerging optical network offers the possibility of interconnecting hundreds of thousands of users, covering local to wide area, and providing capacities exceeding substantially those conventional networks. The network promises data transmission rates several orders of magnitudes higher than current electronic networks.

The key to high speed in the network is to maintain the signal in optical form rather than traditional electronic form.

1.1 Optical Communication vs Electronic Communication

The high bandwidth of fiber-optic links is utilized through *Wavelength-Division Multiplexing* **(WDM) technology which supports the propagation of multiple laser beams through a single fiber-optic link provided that each laser beam uses a distinct optical wavelength.**

The major applications of the network are video conferencing, scientific visualization, real-time medical imaging, supercomputing, and distributed computing.

The WDM Optical Model

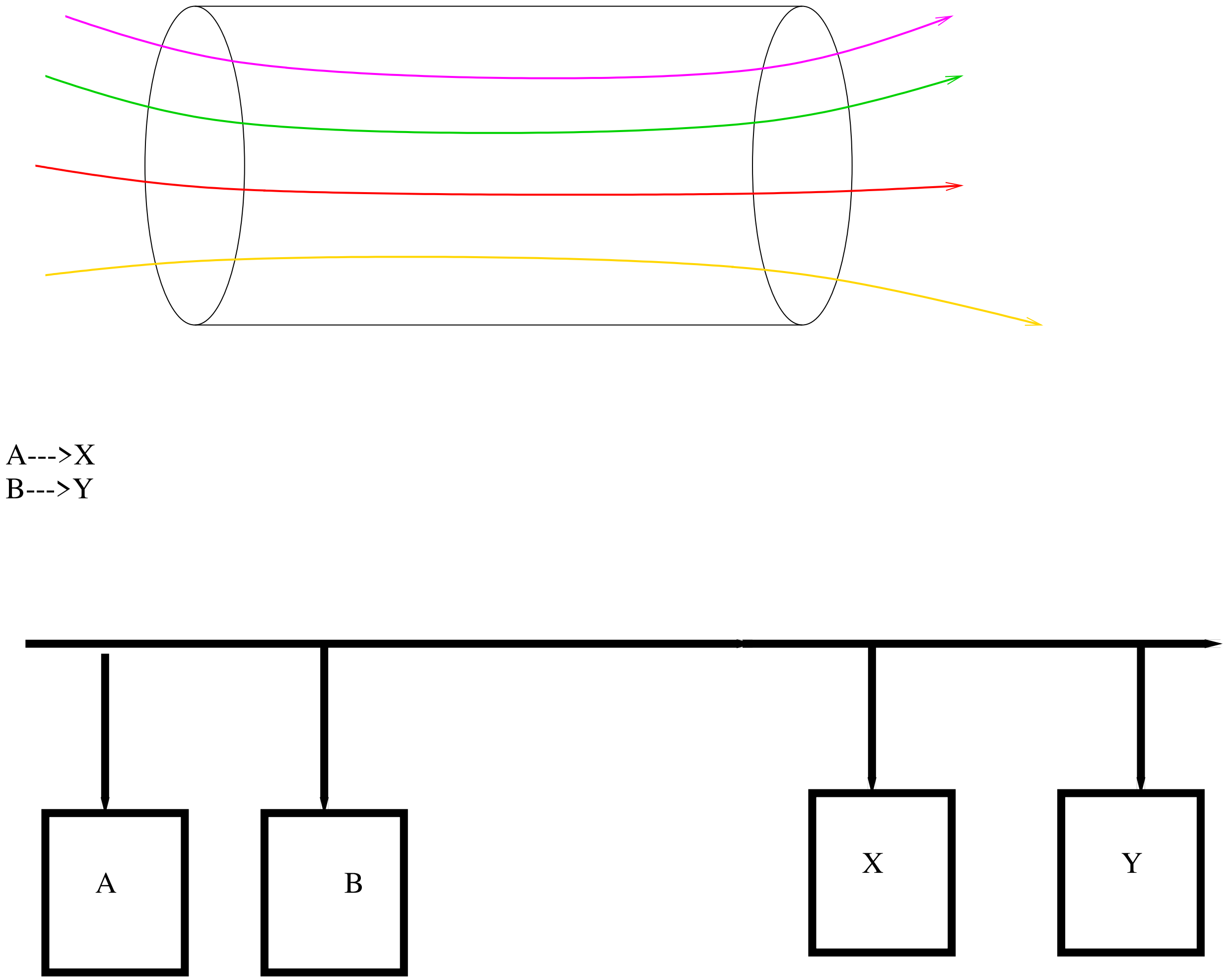

Figure 1: The WDM optical model

1.1 Optical Communication vs Electronic Communication (cont.)

One major topic related to WDM networks is the routing issue. The message transfer in the networks is through first establishing the ***lightpath*** **and then proceeding the transfer. Lightpaths thus provide a powerful approach to utilize the vast available bandwidth in WDM networks.**

A lightpath **is an all-optical transmission path between two nodes in the network, implemented by assigning a unique wavelength throughout the path.**

A semilightpath **is a transmission path by chaining several lightpaths together. Thus, the wavelength conversions on a semilightpath are required at some intermediate nodes, but generally not at every node.**

1.2 Objective

The objective of this paper is to present algorithms for finding a routing semilightpath between a given source and destination such that the cost sum of links and nodes on the path is minimum in terms of the following cost measurements:
1) the cost for traversing a link on some wavelength; and
2) the cost for wavelength conversion when the path has to switch to a different wavelength at some intermediate nodes.

1.3 Our Contributions

- **An efficient algorithm for the problem is proposed, which requires $O(k^2n + km + kn\log(kn))$ time.**
- **A distributed implementation of the proposed algorithm is also given. The communication and time complexities of the distributed version are $O(km)$ and $O(kn)$, respectively.**
- **A special version of the problem is studied, the proposed algorithm takes $O(d^2nk_0^2 + mk_0\log n)$ time, where d is the maximum in-degree or out-degree and k_0 is the number of wavelengths in a single link (load).**

1.3 Our Contributions (cont.)

Compared with a previous $O(k^2n + kn^2)$ **time algorithm for the problem, our algorithm has the following advantages.**

(i) We take into account the physical topology of the network which makes our algorithm outperform the previous one. In particular, when k **is small (e.g.** $k = O(\log n)$**) and** $m = O(n)$**, our algorithm runs in time** $O(n \log^2 n)$**, while the previous algorithm runs in time** $O(n^2 \log n)$**.**

(ii) Since our algorithm has high locality, it can be efficiently implemented on the network distributively.

2. Preliminaries

2.1 The problem statement

The optical network is modeled by a *directed graph* $G = (V, E)$**, where** V **and** E **are a set of nodes (vertices) and a set of directed links (edges) of the network.**

Let $n = |V|$ **and** $m = |E|$**. Let** $d_{in}(G, v)$ **and** $d_{out}(G, v)$ **be the in-degree and out-degree of** v **in** G**.**

Define $d_{in} = \max\{d_{in}(G, v) \mid v \in V\}$**,** $d_{out} = \max\{d_{out}(G, v) \mid v \in V\}$**, and** $d = \max\{d_{in}, d_{out}\}$ **the** *maximum degree* **of** G**. It is well known that** $\Sigma_{v\in V}\, d_{in}(G, v) = \Sigma_{v\in V}\, d_{out}(G, v) = m$**.**

Assume that $\Lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_k\}$ is the set of wavelengths in G. Associated with each link e, there is a set of wavelengths $\Lambda(e)$. By the definition of Chlamtac et al, the *cost structure* of using the resources in G is defined as follows.

- **For each link e and wavelength λ_i a nonnegative weight $w(e, \lambda_i)$ is associated, representing the "cost" of using λ_i on link e. If λ_i is not available on the link, then the weight is infinite.**

- **The "cost" of wavelength conversion is modeled via cost factors of the form $c_v(\lambda_p, \lambda_q)$, which is the cost of wavelength conversion at node v from λ_p to λ_q. For some given v, p and q, if the conversion at v is not available, then $c_v(\lambda_p, \lambda_q)$ is infinite. If the two wavelengths are equal (i.e, $p = q$), then $c_v(\lambda_p, \lambda_p) = 0$.**

A *semilightpath* $\mathcal{P}$ **is a sequence** $e_1, e_2, \ldots, e_l$ **of links such that the tail of** e_{i+1} **coincides with the head of** e_i**,** $i = 1, \ldots, l$**. Further, a wavelength** $\lambda_{j_i} \in \Lambda(e_i)$ **is associated with each** e_i**, this is the wavelength used by the path on link** e_i**.**

Denote by $head(e)$ **and** $tail(e)$ **the head and the tail of a directed link** e**, which are the two endpoints of** e**. Then, the** *cost* $C(\mathcal{P})$ **of the semilightpath** $\mathcal{P}$ **is**

$$C(\mathcal{P}) = \sum_{i=1}^{l} w(e_i, \lambda_{j_i}) + \sum_{i=1}^{l-1} c_{head(e_i)}(\lambda_{j_i}, \lambda_{j_{i+1}}), \tag{1}$$

The *optimal semilightpath problem* **is thus to find a semilightpath from** s **to** t **with miniming the path cost.**

For this problem, not only do we need to find an optimal semilightpath, but we also need to assign every link e **on the path a specific, valid wavelength** $\lambda(e) \in \Lambda(e)$ **and to set the wavelength conversion switch at every intermediate node on the path if necessary.**

3. The Improved Algorithm

The basic idea behind our algorithm is to transform the problem to a single-source shortest path problem in an auxiliary directed graph $G_{s,t}$ which will be defined later. Then, a solution of $G_{s,t}$ corresponds to a solution for the optimal semilightpath problem on G exactly. Therefore, the key here is how to construct $G_{s,t}$.

3.1 The Steps of the Improved Algorithm

1. **Construct a multigraph** $G_M(V_M, E_M)$ **using** G **and** $\Lambda(e)$ **for every link** $e \in E$**.**
2. **Construct a weighted bipartite graph** $G_v(X_v, Y_v, E_v)$ **using** G_M **and the wavelength conversion function at node** v **for every** $v \in V_M$**.**
3. **Construct a weighted directed graph** $G_{s,t}$ **using** $G_v(X_v, Y_v, E_v)$ **for every** $v \in V$ **and** G_M**.**
4. **Find a shortest path from** s' **to** t'' **in** $G_{s,t}$**.**
5. **Find an optimal semilightpath in** G **using the result in Step 4.**

3.1.1 The Construction of $G_M(V_M, E_M)$

A directed *multigraph* $G_M = (V_M, E_M, w)$ **is constructed as follows.** $V_M = V$**, and for each link** $e = \langle u, v \rangle$ **in** E **construct** $|\Lambda(e)|$ **parallel directed links in** E_M **from** u **to** v**, and each of them is assigned a distinct wavelength** $\lambda \in \Lambda(e)$ **and the weight** $w(e, \lambda)$**.**

As a result, G_M **has** $|V_M| = |V| = n$ **nodes and** $m_1 = |E_M| = \Sigma_{e \in E} |\Lambda(e)| \leq km$ **directed links.**

3.1.2 The Construction of $G_v(X_v, Y_v, E_v)$

A directed weighted bipartite graph $G_v = (X_v, Y_v, E_v, \omega_1)$ is constructed as follows, for every node $v \in V_M$.

- **For each $\lambda \in \Lambda_{in}(G_M, v)$, there is a corresponding node x in X_v.**
- **For each $\lambda' \in \Lambda_{out}(G_M, v)$, there is a corresponding node y in Y_v.**
- **There is a directed link $e = \langle x, y \rangle \in E_v$ from $x \in X_v$ to $y \in Y_v$ iff one of the following conditions holds. (i) $\lambda = \lambda'$. The weight of e is assigned $\omega_1(e) = C_v(\lambda, \lambda) = 0$; (ii) $\lambda \neq \lambda'$ and the wavelength conversion from λ to λ' at v is allowed. The weight of e is assigned $\omega_1(e) = C_v(\lambda, \lambda')$.**

3.1.3 The Construction of $G_{s,t}$

Next, another directed, weighted auxiliary graph $G_{s,t} = (V'_{s,t}, E'_{s,t}, \omega_3)$ **is constructed, where** $V'_{s,t} = \cup_{v \in V_M}(X_v \cup Y_v) \cup \{s', t''\}$**. Let** $\langle u, v \rangle \in E_M$ **be any directed link with wavelength** λ**. Suppose that** $u' \in Y_u$ **and** $v' \in X_v$ **are the nodes in** G_u **and** G_v **which are derived from nodes** u **and** v **of** G **and** λ**. Then,** $\langle u', v' \rangle \in E'_{s,t}$ **and the weight of this link is** $\omega_2(\langle u', v' \rangle) = w(\langle u, v \rangle, \lambda)$**.**

3.2 An Example

Assume that $\Lambda = \{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$ is the set of wavelengths in G. The wavelengths on every link in G are defined as follows. $\Lambda(\langle 1,2\rangle) =$

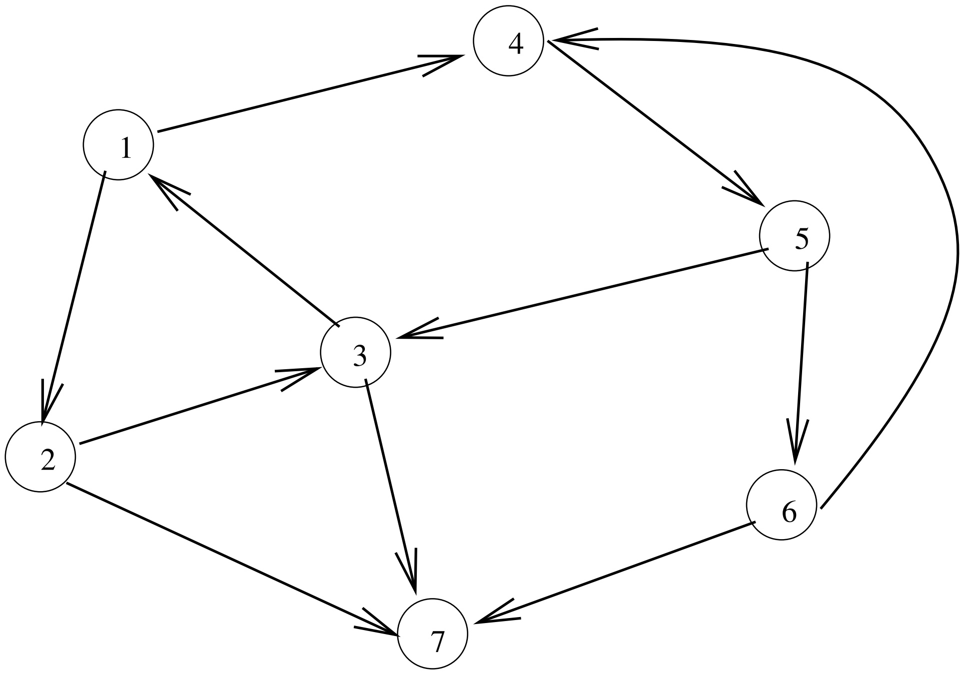

Figure 2: The network $G = (V, E)$.

$\{\lambda_1, \lambda_3\}$**,** $\Lambda(\langle 1,4\rangle) = \{\lambda_1, \lambda_2, \lambda_4\}$**,** $\Lambda(\langle 2,3\rangle) = \{\lambda_1, \lambda_4\}$**,** $\Lambda(\langle 2,7\rangle) = \{\lambda_1, \lambda_2, \lambda_3\}$**,** $\Lambda(\langle 3,1\rangle) = \{\lambda_2, \lambda_3\}$**,** $\Lambda(\langle 3,7\rangle) = \{\lambda_3, \lambda_4\}$**,** $\Lambda(\langle 4,5\rangle) = \{\lambda_3\}$**,** $\Lambda(\langle 5,3\rangle) = \{\lambda_2, \lambda_4\}$**,**

The auxiliary graph G_M of G is described as follows (see Fig. 3).

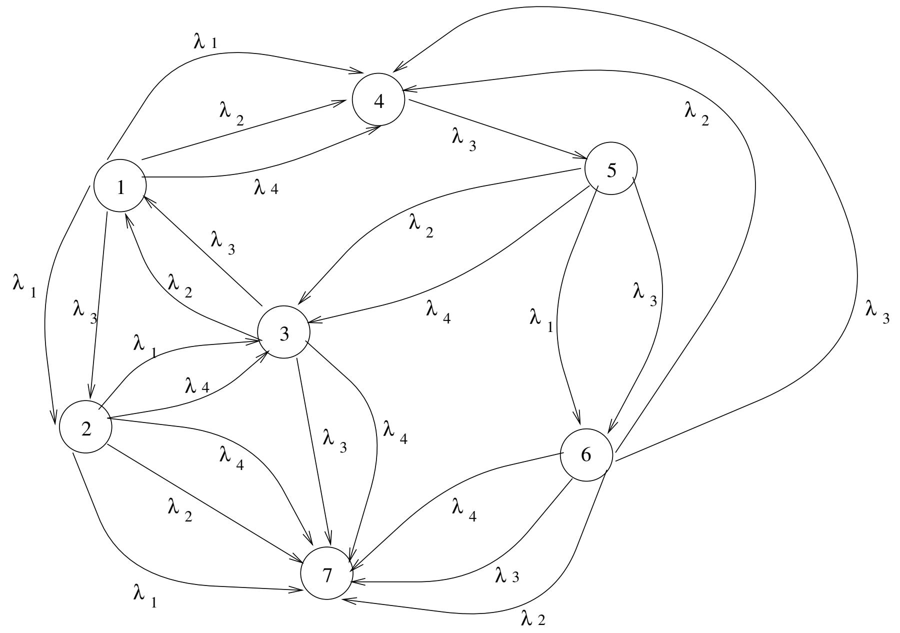

Figure 3: The auxiliary graph $G_M = (V_M, E_M, w)$.

$$\Lambda_{in}(G_M, 1) = \{\lambda_2, \lambda_3\}, \qquad \Lambda_{out}(G_M, 1) = \{\lambda_1, \lambda_2, \lambda_3, \lambda_4\},$$

$$\Lambda_{in}(G_M, 2) = \{\lambda_1, \lambda_3\}, \qquad \Lambda_{out}(G_M, 2) = \{\lambda_1, \lambda_2, \lambda_4\},$$

$$\Lambda_{in}(G_M, 3) = \{\lambda_1, \lambda_2, \lambda_4\}, \qquad \Lambda_{out}(G_M, 3) = \{\lambda_2, \lambda_3, \lambda_4\},$$

The bipartite graph G_v for every $v \in V_M$ $(= V)$ is then constructed. Fig. 4 shows how to construct the bipartite graph $G_3 = (X_3, Y_3, E_3)$ at node 3, where a node labeled by "(v, λ_j)" represents that the node is obtained from node v of G_M and wavelength λ_j. From Fig. 4 we can see that there is no link from a node in X_3 labeled by $(3, \lambda_2)$ to a node in Y_3 labeled by $(3, \lambda_3)$, which means that the wavelength conversion from λ_2 to λ_3 at node 3 is not allowed.

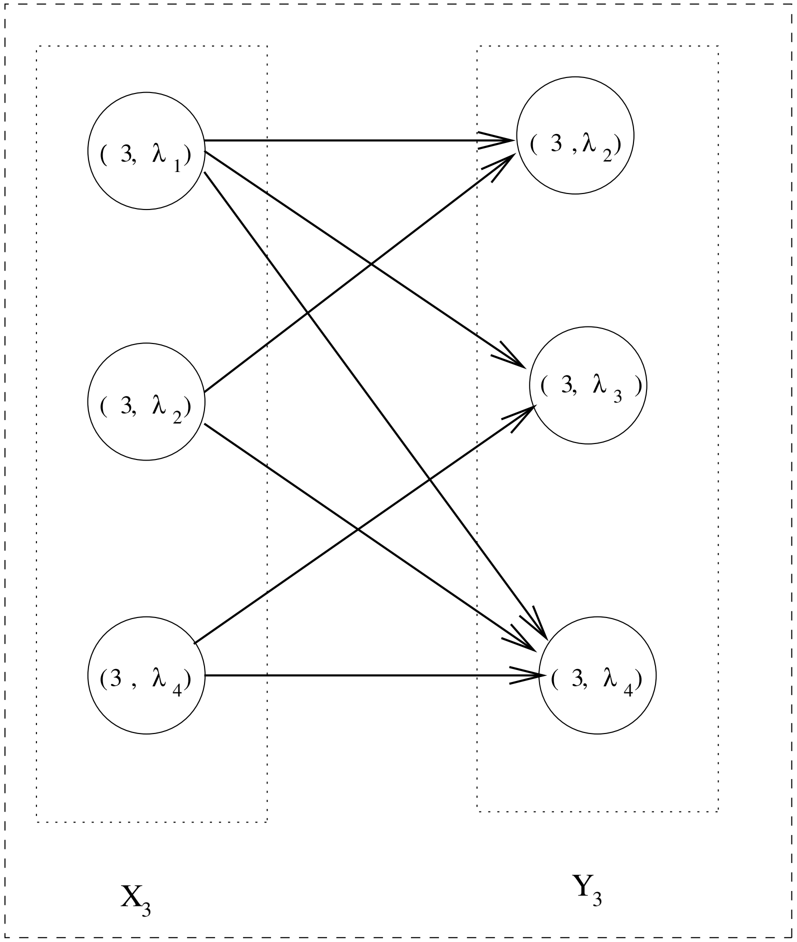

Figure 4: The auxiliary graph $G_3 = (X_3, Y_3, E_3)$ at node 3.

Fig. 5 shows a subgraph of $G_{s,t}$ induced by the nodes in G_1 and G_3. Note that there are two directed links $\langle u, v\rangle$ and $\langle p, q\rangle$ between the nodes in G_3 and G_1 in Fig. 5, which are obtained from the parallel links $\langle 3, 1\rangle$ of G_M labeled by λ_2 and λ_3.

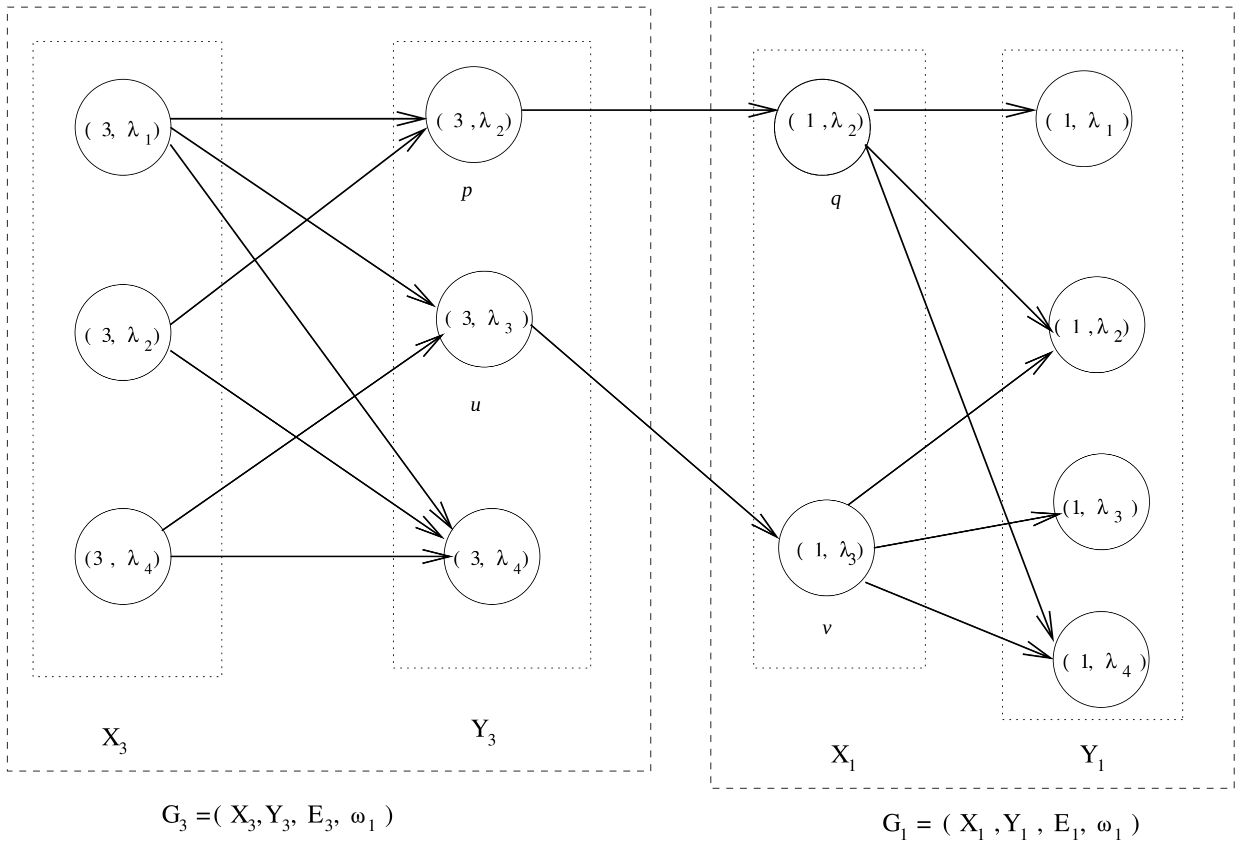

Figure 5: A subgraph of $G_{s,t}$ induced by the nodes in G_1 and G_3.

Theorem 1 ***Given a directed network $G(V,E)$ and a pair of nodes s and t, assume that each link e of G has been assigned with a set $\Lambda(e) \subseteq \Lambda$ of available wavelengths, and every node has been given the wavelength conversion function. There is an algorithm for finding an optimal semilightpath from s to t in G. The algorithm requires $O(k^2n + km + kn\log(kn))$ time.***

3.3 Comparison

Let $T_{CFZ} = O(kn^2 + k^2 n)$ **and** $T_c = O(k^2 n + km + kn\log(kn))$ **be the time complexity of their algorithm and the proposed algorithm.**

If $k = \Omega(n)$**,** $T_c = T_{CFZ} = O(k^2 n)$ **since** $m \le dn \le n^2$**, which means both algorithms have the same worst time complexity. Otherwise,**

$$\frac{T_{CFZ}}{T_c} = \frac{c' kn^2}{k^2 n + km + kn\log(kn)} \ge \frac{c'' n}{\max\{k, d, \log n\}} \tag{2}$$

where c' **and** c'' **are constants and** $m \le dn$**.**

Overall, our algorithm has improved their algorithm by a factor of $\Omega(\frac{n}{\max\{k,d,\log n\}})$**. Particularly, this improvement is significant when** k **and** m **are relatively small but** n **is relatively large. For example, when** $m = O(n)$ **and** $k = O(\log n)$**, our algorithm runs in** $O(n\log^2 n)$ **time only, while their algorithm takes** $O(n^2 \log n)$ **time.**

4. The Distributed Implementation

Since we are dealing with large wide area networks, in practice it is not realistic to design a centralized algorithm to find an optimal semilightpath for the networks. Instead, a distributed algorithm seems more appropriate to accommodate the task.

4.1 The Running Modes of Optical Networks

An optical network can be decomposed into two separate networks in functionality logically.

- **One is called** *data network* **which is used to transfer large volume data such as image data and databases etc, using its optical fibers.**

- **The other is called** *control network* **which is used for high level protocol controls such as finding an optimal semilightpath/lightpath and setting the optical switches along the path.**

4.2 The Basic Idea of the Distributed Algorithm

The idea is to embed the ideal network $G_{s,t}$ into the physical network G first, then use G to simulate $G_{s,t}$, i.e., each node of G is actually a subgraph of $G_{s,t}$. While the single-source shortest path problem is a well studied problem in the distributed computing environment, there are many efficient algorithms to accommodate it like the algorithm by Chandy and Misra. Note that the construction of $G_{s,t}$ has high locality.

Theorem 2 ***Given a directed network*** $G(V, E)$ ***and a pair of nodes*** s ***and*** t***, assume that every link*** e ***in*** G ***has been assigned a set*** $\Lambda(e)$ ***of available wavelengths, and every node has been given a wavelength conversion function. There is an efficient distributed algorithm for finding an optimal semilightpath from*** s ***to*** t***. The communication and time complexities of the proposed algorithm are*** $O(km)$ ***and*** $O(kn)$ ***respectively, on the distributed computational model.***

5. The Special Optimal Semilightpath Problem

Though the number of wavelengths $|\Lambda|$ $(= k)$ in a network may be quite large, the number of transmitters and receivers (tuning) at each node usually is bounded, so is the number of wavelengths at every link. When $k = O(n)$, in practice this is impossible because the size n of the network is quite large. Based on this argument, here we assume the number of available wavelengths, $|\Lambda(e)|$, associated with each link e, is bounded by k_0. We further assume that k_0 is either a constant or a function of n increasing very slow.

Theorem 3 ***Given a directed network*** $G(V, E)$ ***and a pair of nodes*** s ***and*** t***, assume that each link*** e ***of*** G ***has been assigned a set*** $\Lambda(e)$ ***of available wavelengths and*** $|\Lambda(e)| \leq k_0$***, and every node has been given a wavelength conversion function. There is an algorithm for finding an optimal semilightpath from*** s ***to*** t***. The algorithm requires*** $O(d^2nk_0^2 + mk_0 \log n)$ ***time assuming*** $k_0 = o(n)$***.***

As said before, we are dealing with large wide area networks. In practice such networks are usually sparse (i.e. $m = O(n)$**),** d **is fixed or a slowly increasing function of** n **(e.g.,** $d = O(\log n)$**), and** k_0 **is also fixed or a slowly increasing function of** n **(e.g.** $k_0 = O(\log n)$**). Thus, the algorithm takes time** $O(d^2nk_0^2 + mk_0 + mk_0 \log(mk_0)) = O(d^2nk_0^2 + mk_0 \log(mk_0)) = O(d^2nk_0^2 + mk_0 \log n)$**.**

6. Conclusions

- **An improved algorithm for finding an optimal lightpath/semilightpath in wide area fiber optic networks has been proposed.**
- **A distributed version has also been given.**
- **A special case of the optimal semilightpath problem has been studied.**

6. Conclusions (cont.)

Compared with a previous algorithm for the problem, our algorithm runs much faster, and can be implemented in the distributed computational environment. Its simplicity, time efficiency, and high locality make the proposed algorithm a competitive candidate to be used in large WDM networks.