Improved Lightpath (Wavelength) Routing in Large WDM Networks

Weifa Liang and Xiaojun Shen

***Abstract*—We address the problem of efficient circuit switching in wide area networks. The solution provided is based on finding optimal routes for lightpaths and semilightpaths. A lightpath is a fully optical transmission path, while a semilightpath is a transmission path constructed by chaining several lightpaths together, using wavelength conversion at their junctions. The problem thus is to find an optimal lightpath/semilightpath in the network in terms of the cost of wavelength conversion and the cost of using the wavelengths on links. In this paper, we first present an efficient algorithm for the problem which runs in time $O(k^2n + km + kn\log(kn))$, where n and m are the number of nodes and links in the network, and k is the number of wavelengths. We then analyze that the proposed algorithm requires $O(d^2nk_0^2 + mk_0\log n)$ time for a restricted version of the problem in which the number of available wavelengths for each link is bounded by k_0 and $k_0 = o(n)$, where d is the maximum in-degree or out-degree of the network. It is surprising to have found that the time complexity for this case is independent of k. It must be mentioned that our algorithm can be implemented efficiently in the distributed computing environment. The distributed version requires $O(kn)$ time and $O(km)$ messages. Compared with a previous $O(k^2n + kn^2)$ time algorithm, our algorithm has the following advantages. 1) We take into account the physical topology of the network which makes our algorithm outperform the previous algorithm. In particular, when k is small [e.g., $k = O(\log n)$] and $m = O(n)$, our algorithm runs in time $O(n\log^2 n)$, while the previous algorithm runs in time $O(n^2\log n)$. 2) Since our algorithm has high locality, it can be implemented on the network distributively.**

***Index Terms*—Algorithm design and analysis, graph theory, optical networks, optimal semilightpaths, routing algorithms.**

I. Introduction

THE EMERGING optical network offers the possibility of interconnecting hundreds of thousands of users, covering local to wide area, and providing capacities exceeding substantially those conventional networks. The network promises data transmission rates several orders of magnitudes higher than current electronic networks. The key to high speed in the network is to maintain the signal in optical form rather than traditional electronic form. The high bandwidth of fiber-optic links is utilized through *wavelength-division multiplexing* (WDM) technology which supports the propagation of multiple laser beams through a single fiber-optic link provided that each laser beam uses a distinct optical wavelength. The major applications of the network are video conferencing, scientific visualization, real-time medical imaging, supercomputing, and distributed computing [2], [12], [13]. A comprehensive overview of its physical theory and applications of this technology can be found in the books by Green [8] and McAulay [11].

One major topic related to WDM networks is the routing issue. The message transfer in the networks is through first establishing the *lightpath* and then proceeding the transfer. Lightpaths thus provide a powerful approach to utilize the vast available bandwidth in WDM networks [1], [5], [10]. A lightpath is an all-optical transmission path between two nodes in the network, implemented by assigning a unique wavelength throughout the path. Data transmitted through a lightpath do not need wavelength conversion or electronic processing at intermediate nodes. Therefore, lightpaths enable an efficient utilization of the optical bandwidth in WDM networks, and reduce electronic processing delay at the intermediate nodes, thereby improving reliability and the quality of the services provided to data communications.

While transmitting all traffic between every pair of nodes over lightpaths is desirable, it is not generally feasible to establish lightpaths between every pair of nodes and accommodate all the traffic by lightpaths, due to physical constraints such as limited number of wavelengths, limited number and tunability of optical transceivers at each node as well as lightwave dispersions that limit the physical length of a lightpath. Additionally, given the network conditions, a single optical wavelength may not be available between a given source and destination because some of the resources are already occupied by existing lightpaths. To cope with the above limits, Chlamtac *et al.* [4] introduced the *semilightpath* concept in which a transmission path is obtained by establishing and chaining several lightpaths together. Thus, the wavelength conversions on a semilightpath are required at some intermediate nodes, but generally not at every node. In this case, the lightpath is a special case of the semilightpath when the number of intermediate nodes on the path with wavelength conversion is zero.

The objective of this paper is to present algorithms for finding an efficient routing semilightpath between a given source and destination such that the cost sum of links and nodes on the path is minimum in terms of the following cost measurements: 1) the cost for traversing a link on some wavelength and 2) the cost for wavelength conversion when the path has to switch to a different wavelength at some intermediate nodes.

Clearly, if we only consider the first cost factor, the problem becomes the traditional single-source shortest path problem

Paper approved by D. P. Taylor, the Editor for Signal Design, Modulation, and Detection of the IEEE Communications Society. Manuscript received November 18, 1997; revised January 21, 2000.

W. Liang is with the Department of Computer Science, Australian National University, Canberra, ACT 0200 Australia (e-mail: wliang@cs.anu.edu.au).

X. Shen is with the Computer Science Telecommunications Program, University of Missouri at Kansas City, Kansas City, MO 64110 USA (e-mail: xshen@cstp.umkc.edu).

Publisher Item Identifier S 0090-6778(00)07535-8.

which has been well studied. The additional cost for wavelength conversion at nodes, however, makes the direct application of the single-source shortest path algorithm to the network graph inappropriate. In this paper, we show that, through a series of proper transformations, the problem can be reduced to a single-source shortest path problem.

The above problem was first formalized by Chlamtac *et al.* [4]. They presented an $O(k^2n + kn^2)$ time algorithm for it, where n is the number of nodes and k is the number of wavelengths in the network. Their algorithm is optimal when $k \geq n$ and the network is a dense network, e.g., $m = O(n^2)$, where m is the number of links in the network. Here, we point out that their algorithm has a minor error. That is, they use adjacency matrix to represent the auxiliary graph WG of kn nodes which takes $O(k^2n^2)$ time for the initial assignment. Therefore, the graph WG only can be represented by adjacency lists which takes $O(kn(k+n)) = O(k^2n + kn^2)$ time because there are at most $O(k+n)$ adjacent nodes for every node in WG. Moreover, their algorithm is a centralized algorithm. It may not be suitable for distributed computing environments. Besides, their algorithm did not take into account some important network parameters such as the number of links m and the maximum degree d of nodes in the network. Since the considered network is a large wide area network, it is usually a sparse, planar or approximate planar graph, thus, $m = O(n)$. In practice d is usually a constant. Even if d is not a constant, it should be a very slowly increasing function of n. For example, $d = O(\log n)$. If taking these network parameters into account, we may ask whether there is a faster algorithm for the problem. Motivated by these concerns, in this paper we answer this question by presenting a faster algorithm after taking the above network parameters into consideration.

In this paper, we first present an efficient algorithm for the problem, which requires $O(k^2n + km + kn\log(kn))$ time. We then give a distributed implementation of the proposed algorithm. The communication and time complexities of the distributed version are $O(km)$ and $O(kn)$, respectively. We finally introduce a restricted version of the problem in which the number of wavelengths assigned to every link is bounded by k_0. Assume that k_0 is strictly less than k, and in most cases k_0 is a very slowly increasing sublinear function of n even if k is quite large (e.g., $k > n$). For this latter case, the proposed algorithm takes $O(d^2nk_0^2 + mk_0\log n)$ time. It is surprising to have found that the time complexity for this case is independent of k. Compared with a previous $O(k^2n + kn^2)$ time algorithm for the problem, our algorithm has the following advantages. i) We take into account the physical topology of the network which makes our algorithm outperform the previous algorithm in most cases. In particular, when k is small [e.g., $k = O(\log n)$] and $m = O(n)$, our algorithm runs in time $O(n\log^2 n)$, while the previous algorithm runs in time $O(n^2\log n)$. ii) Since our algorithm has high locality, it can be efficiently implemented on the network distributively.

The rest of this paper is arranged as follows. In Section II we first describe the network model and some basic parameters related to the network. We then define the optimal semilightpath problem and introduce the cost measurement for semilightpaths. In Section III we first present an efficient algorithm for the problem. We then give a distributed implementation of the proposed algorithm. We finally make a comparison between our centralized algorithm and the previously centralized algorithm. In Section IV we introduce a restricted version of the problem which we call *special optimal semilightpath problem*, for which our algorithm runs in time $O(d^2nk_0^2 + mk_0\log(mk_0))$. We conclude our discussion in Section V.

II. Network Model and Problem Statement

The optical network is modeled by a *directed graph* $G = (V, E)$, where V and E are a set of nodes (vertices) and a set of directed links (edges) of the network. Let $n = |V|$ and $m = |E|$. Let $d_{\text{in}}(G, v)$ and $d_{\text{out}}(G, v)$ be the in-degree and out-degree of v in G. Define $d_{\text{in}} = \max\{d_{\text{in}}(G, v) | v \in V\}$, $d_{\text{out}} = \max\{d_{\text{out}}(G, v) | v \in V\}$, and $d = \max\{d_{\text{in}}, d_{\text{out}}\}$ the *maximum degree* of G. It is well known that $\sum_{v\in V} d_{\text{in}}(G, v) = \sum_{v\in V} d_{\text{out}}(G, v) = m$. Clearly, $m \leq dn$. Note that the undirected version of the network can be modeled by replacing an undirected link with two oppositely directed links.

Suppose that a set $\Lambda = \{\lambda_1, \lambda_2, \cdots, \lambda_k\}$ of wavelengths is available for the network. By the definition of Chlamtac *et al.* [4], the *cost structure* of using the resources in the network is represented as follows. For each link e and wavelength λ_i a nonnegative weight $w(e, \lambda_i)$ is associated, representing the "cost" of using wavelength λ_i on link e. If λ_i is not available on the link, then the weight is infinite. The "cost" of wavelength conversion is modeled via cost factors of the form $c_v(\lambda_p, \lambda_q)$, which is the cost of wavelength conversion at node v from wavelength λ_p to wavelength λ_q. For some given v, p and q, if the conversion at v is not available, then $c_v(\lambda_p, \lambda_q)$ is infinite. If the two wavelengths are equal (i.e., $p = q$), then $c_v(\lambda_p, \lambda_p) = 0$. The defined wavelength conversion cost accommodates the general case where the conversion cost depends on the nodes and the wavelengths involved.

A *semilightpath* $\mathcal{P}$ is a sequence $e_1, e_2, \cdots, e_l$ of links such that the tail of e_{i+1} coincides with the head of e_i, $i = 1, \cdots, l$. Further, a wavelength $\lambda_{j_i} \in \Lambda(e_i)$ is associated with each e_i, this is the wavelength used by the path on link e_i. The *cost* $C(\mathcal{P})$ of the semilightpath $\mathcal{P}$ is thus defined as follows. Denote by $head(e)$ and $tail(e)$ the head and the tail of a directed link e, which are the two endpoints of e. Then, the cost is

$$C(\mathcal{P}) = \sum_{i=1}^{l} w(e_i, \lambda_{j_i}) + \sum_{i=1}^{l-1} c_{head(e_i)}\left(\lambda_{j_i}, \lambda_{j_{i+1}}\right), \qquad (1)$$

where the first sum in Equation (1) is the cost of traversing the links and the second sum is the cost of wavelength conversion at intermediate nodes.

The *optimal semilightpath problem* is then defined as follows. Given a directed network $G = (V, E)$ and a set Λ of available wavelengths, let $\Lambda(e) \subseteq \Lambda$ be the set of available wavelengths associated with link $e \in E$. Suppose the wavelength conversion function at each node $v \in V$ is also given. Let s and t be two nodes in G which are the *source* and the *destination* respectively, the problem is to find a semilightpath from s to t such that the path cost defined in Equation (1) is minimized. For this problem, not only do we need to find such an optimal semilightpath, but we also need to assign every link e on the path a specific, valid wavelength $\lambda(e) \in \Lambda(e)$ and to set the wavelength conversion switch at every intermediate node on the path if necessary.

As remarked by Chlamtac *et al.* in [4], it is not forbidden in general for a semilightpath to visit a node more than once, using different wavelengths (which can be seen from Fig. 5). It is expected, however, that in practical cases the cost structure will exclude such cases from being optimal, but we do not have to exclude them *a priori*. The case where the same link is traversed using the same wavelength more than once is automatically excluded from the potential optimal solutions, since it can be shortened by a shortcut.

III. The Optimal Semilightpath Problem

In this section we present an efficient algorithm for the problem. The basic idea behind our algorithm is to transform the problem to a single-source shortest path problem in an auxiliary directed graph $G_{s,t}$ which will be defined later. Then, a solution of $G_{s,t}$ corresponds to a solution for the optimal semilightpath problem on G exactly. Note that a previously known algorithm for the problem by Chlamtac *et al.* [4] also first constructs an auxiliary graph WG which they call the *wavelength graph* of G, then finds a solution in WG which corresponds to a solution of the original problem. However, the construction of $G_{s,t}$ is totally different from WG. Later we will see that the performance of our algorithm is much better than theirs.

A. An Efficient Algorithm

Given the network G, there is a set $\Lambda(e)$ $(\subseteq\Lambda)$ of available wavelengths for each link $e \in E$. We first construct a directed *multigraph* $G_M = (V_M, E_M, w)$ as follows. $V_M = V$, and for each link $e = \langle u, v\rangle$ in E, construct $|\Lambda(e)|$ parallel directed links in E_M from u to v, each of which is associated with a distinct wavelength $\lambda \in \Lambda(e)$ and the weight $w(e, \lambda)$. Thus, G_M has $|V_M| = |V| = n$ nodes and $m_1 = |E_M| = \sum_{e\in E} |\Lambda(e)|$ directed links. Obviously $m_1 \leq km$.

Let $E_{\rm in}(G, v)$ and $E_{\rm out}(G, v)$ be the set of incoming links and the set of outgoing links of v in G, respectively. By the definition, $head(e) = v$ for each $e \in E_{\rm in}(G, v)$ and $tail(e') = v$ for each $e' \in E_{\rm out}(G, v)$. Now, for each node $v \in V_M$, it is obvious that

$$\begin{aligned}|E_{\rm in}(G_M, v)| &+ |E_{\rm out}(G_M, v)|\\ &= \sum_{e\in E_{\rm in}(G,v)\cup E_{\rm out}(G,v)} |\Lambda(e)|\\ &\leq k(d_{\rm in}(G, v) + d_{\rm out}(G, v)) \leq 2kd.\end{aligned}$$

Let $\Lambda_{\rm in}(G_M, v)$ be the set of wavelengths for all links in $E_{\rm in}(G_M, v)$, i.e., a wavelength λ belongs to $\Lambda_{\rm in}(G_M, v)$ if there is an $e \in E_{\rm in}(G_M, v)$ which has been assigned the wavelength λ. Similarly, $\Lambda_{\rm out}(G_M, v)$ is defined as the set of wavelengths for all links in $E_{\rm out}(G_M, v)$.

For each node $v \in V_M$, we construct a directed weighted bipartite graph $G_v = (X_v, Y_v, E_v, \omega_1)$ as follows. For each $\lambda \in \Lambda_{\rm in}(G_M, v)$, there is a corresponding node x in X_v, and for each $\lambda' \in \Lambda_{\rm out}(G_M, v)$, there is a corresponding node y in Y_v. There is a directed link $e = \langle x, y\rangle \in E_v$ from $x \in X_v$ to $y \in Y_v$ if and only if one of the following conditions holds. i) $\lambda = \lambda'$. The weight of e is assigned $\omega_1(e) = C_v(\lambda, \lambda) = 0$; ii) $\lambda \neq \lambda'$ and the wavelength conversion from λ to λ' at v is allowed. The weight of e is assigned $\omega_1(e) = C_v(\lambda, \lambda')$.

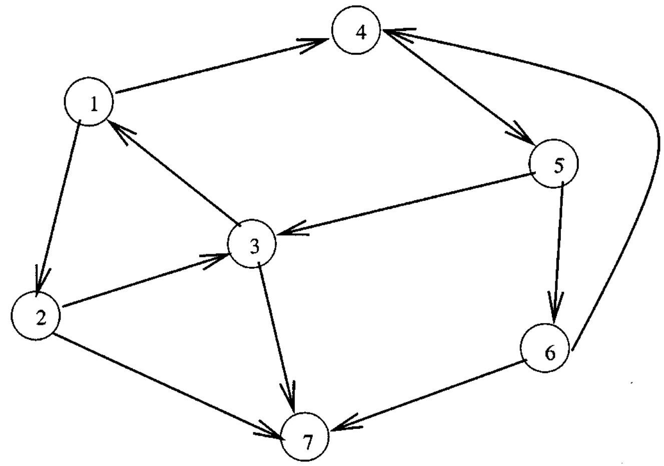

Fig. 1. The network $G = (V, E)$.

Observation 1: Let $G_v(X_v, Y_v, E_v, \omega_1)$ be the weighted bipartite graph defined as above. Then, $|X_v| + |Y_v| = |\Lambda_{\rm in}(G_M, v)| + |\Lambda_{\rm out}(G_M, v)| \leq 2k$ and $|E_v| \leq k^2$.

Next we construct another directed, weighted auxiliary graph $G' = (V', E', \omega_2)$ from G_M as follows. $V' = \bigcup_{v\in V_M}(X_v \cup Y_v)$. Let $\langle u, v\rangle \in E_M$ be any directed link with wavelength λ. Suppose that $u' \in Y_u$ and $v' \in X_v$ are the nodes in G_u and G_v which are derived from nodes u and v of G and λ. Then, $\langle u', v'\rangle \in E'$ and the weight of this link is $\omega_2(\langle u', v'\rangle) = w(\langle u, v\rangle, \lambda)$.

Let $E_{\rm org}$ be the set of links of G' obtained from E_M by the above transformation, then $|E_{\rm org}| = |E_M| = \sum_{e\in E} |\Lambda(e)| \leq km$. Define $E' = \bigcup_{v\in V_M} E_v \cup E_{\rm org}$, and $\omega_2(e) = \omega_1(e)$ for every $e \in \bigcup_{v\in V_M} E_v$.

Observation 2: Let $G'(V', E', \omega_2)$ be the directed weighted graph defined as above. Then,

$$\begin{aligned}|V'| &= \sum_{v\in V_M} (|X_v| + |Y_v|)\\ &= \sum_{v\in V_M} (|\Lambda_{\rm in}(G_M, v)| + |\Lambda_{\rm out}(G_M, v)|) \leq 2kn\end{aligned}$$

and

$$|E'| = \sum_{v\in V_M} |E_v| + |E_{\rm org}| \leq k^2 n + m_1 \leq k^2 n + km.$$

Since we are dealing with a large wide area network, which is sparse in practice [i.e., it has a large n, a small m, e.g., $m = O(n)$, and a bounded degree d, etc.]. Therefore, we will use the adjacency lists to represent G.

Observation 3: Let $G'(V', E', \omega_2)$ be the directed weighted graph defined as above. Then, G' can be constructed in $O(k^2 n + km)$ time and space if G' is represented by adjacency lists.

In the following, we give an example to illustrate the graph constructions. Assume that the network G is defined by Fig. 1. There is a set $\Lambda = \{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$ of available wavelengths

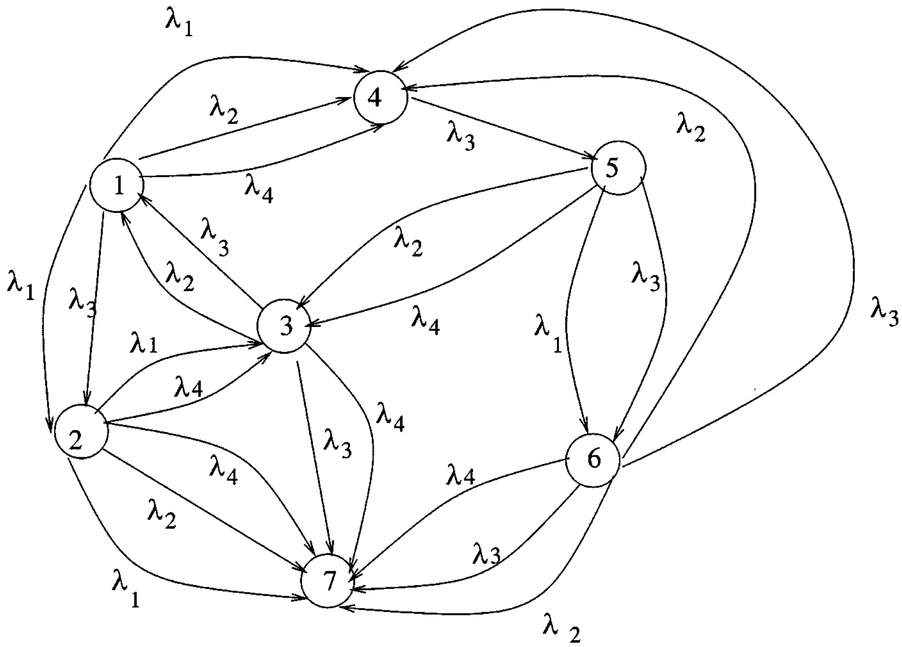

Fig. 2. The auxiliary graph $G_M = (V_M, E_M, w)$.

in G. The available wavelengths on every link in G are defined as follows.

$$\begin{aligned}&\Lambda(\langle 1, 2\rangle) = \{\lambda_1, \lambda_3\}, \quad \Lambda(\langle 1, 4\rangle) = \{\lambda_1, \lambda_2, \lambda_4\},\\ &\Lambda(\langle 2, 3\rangle) = \{\lambda_1, \lambda_4\}, \quad \Lambda(\langle 2, 7\rangle) = \{\lambda_1, \lambda_2, \lambda_3\},\\ &\Lambda(\langle 3, 1\rangle) = \{\lambda_2, \lambda_3\}, \quad \Lambda(\langle 3, 7\rangle) = \{\lambda_3, \lambda_4\},\\ &\Lambda(\langle 4, 5\rangle) = \{\lambda_3\}, \quad \Lambda(\langle 5, 3\rangle) = \{\lambda_2, \lambda_4\},\\ &\Lambda(\langle 5, 6\rangle) = \{\lambda_1, \lambda_3\}, \quad \Lambda(\langle 6, 4\rangle) = \{\lambda_2, \lambda_3\},\\ &\Lambda(\langle 6, 7\rangle) = \{\lambda_2, \lambda_3, \lambda_4\}.\end{aligned}$$

Then, the auxiliary graph G_M of G is described as follows (see Fig. 2). Thus

$$\begin{aligned}&\Lambda_{\text{in}}(G_M, 1) = \{\lambda_2, \lambda_3\}, \quad \Lambda_{\text{out}}(G_M, 1) = \{\lambda_1, \lambda_2, \lambda_3, \lambda_4\},\\ &\Lambda_{\text{in}}(G_M, 2) = \{\lambda_1, \lambda_3\}, \quad \Lambda_{\text{out}}(G_M, 2) = \{\lambda_1, \lambda_2, \lambda_4\},\\ &\Lambda_{\text{in}}(G_M, 3) = \{\lambda_1, \lambda_2, \lambda_4\}, \quad \Lambda_{\text{out}}(G_M, 3) = \{\lambda_2, \lambda_3, \lambda_4\},\\ &\Lambda_{\text{in}}(G_M, 4) = \{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}, \quad \Lambda_{\text{out}}(G_M, 4) = \{\lambda_3\},\\ &\Lambda_{\text{in}}(G_M, 5) = \{\lambda_3\}, \quad \Lambda_{\text{out}}(G_M, 5) = \{\lambda_1, \lambda_2, \lambda_3, \lambda_4\},\\ &\Lambda_{\text{in}}(G_M, 6) = \{\lambda_1, \lambda_3\}, \quad \Lambda_{\text{out}}(G_M, 6) = \{\lambda_2, \lambda_3, \lambda_4\},\\ &\Lambda_{\text{in}}(G_M, 7) = \{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}, \quad \Lambda_{\text{out}}(G_M, 7) = \emptyset.\end{aligned}$$

Based on G_M, the bipartite graph G_v for every $v \in V_M$ $(= V)$ is then constructed. Fig. 3 shows how to construct the bipartite graph $G_3 = (X_3, Y_3, E_3, \omega_1)$ at node 3, where a node labeled by "(v, λ_j)" represents that the node is obtained from node v of G_M and wavelength λ_j. From Fig. 3 we can see that there is no link from a node in X_3 labeled by $(3, \lambda_2)$ to a node in Y_3 labeled by $(3, \lambda_3)$, which means that the wavelength conversion from λ_2 to λ_3 at node 3 is not allowed.

Assume that G_v for every $v \in V$ has been constructed, the construction of G' is easy. Fig. 4 shows a subgraph of G' induced by the nodes in G_1 and G_3. Note that there are two directed links $\langle u, v\rangle$ and $\langle p, q\rangle$ between the nodes in G_3 and G_1 in Fig. 4, which are obtained from the parallel links $\langle 3, 1\rangle$ of G_M labeled by λ_2 and λ_3, respectively. By the definition, $\{\langle u, v\rangle, \langle p, q\rangle\} \subset E_{\text{org}}$.

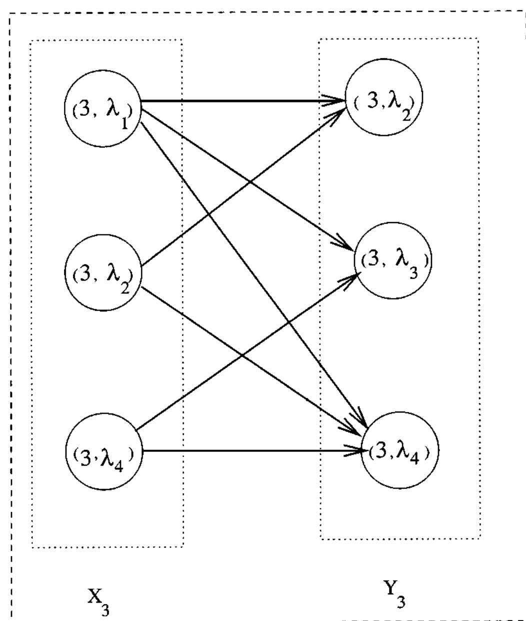

Fig. 3. The auxiliary graph $G_3 = (X_3, Y_3, E_3, \omega_1)$ at node 3.

Now, we consider how to find an optimal semilightpath of G from s to t. First, we construct a directed, weighted auxiliary graph $G_{s,t} = (V'_{s,t}, E'_{s,t}, \omega_3)$ as follows. $V'_{s,t} = V' \cup \{s', t''\}$ and

$$E'_{s,t} = E' \cup \{\langle s', u\rangle | u \in Y_s\} \cup \{\langle v, t''\rangle | v \in X_t\}),$$

where X_t and Y_s are the subsets of nodes in the bipartite graphs G_s and G_t. For every $e \in E'$, $\omega_3(e) = \omega_2(e)$. The links $\langle s', u\rangle$ and $\langle v, t''\rangle$ are assigned weight zeros, i.e., $\omega_3(\langle s', u\rangle) = 0$, $\omega_3(\langle v, t''\rangle) = 0$. Obviously, the number of nodes and links in $G_{s,t}$ are no more than $2kn+2$ and $k^2n+2k+km$, respectively.

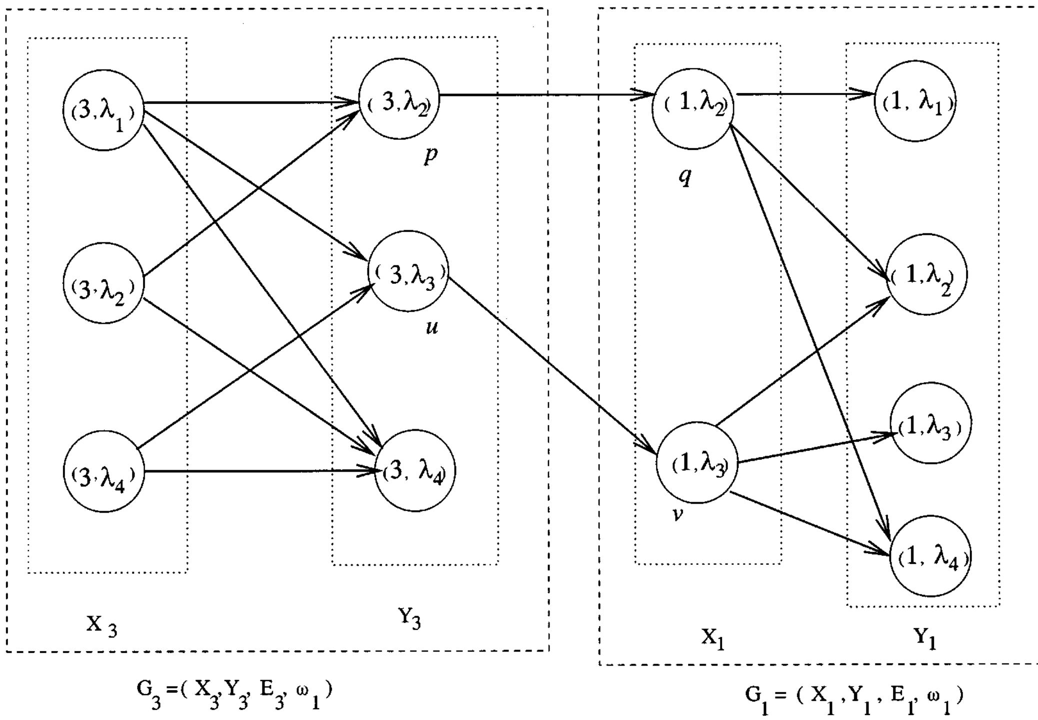

Fig. 4. A subgraph of G' induced by the nodes in G_1 and G_3.

The construction of $G_{s,t}$ results in a straightforward one to one mapping between a single-source shortest path of $G_{s,t}$ from s' to t'' and an optimal semilightpath of G from s to t. Let $\mathcal{P}_{s',t''}$ be a shortest path of $G_{s,t}$ from s' to t'' and $e'_1, e'_2, \cdots, e'_l$ be the link sequence of $\mathcal{P}_{s',t''}$. Then, for every i, $1 \le i \le l$, if i is even, $i \ne 1$, $i \ne l$, and e'_i corresponds to a directed link e_j of G with weight $\omega_3(e'_i) = w(e_j, \lambda)$, then e_j is assigned wavelength λ; otherwise, e'_i corresponds to a wavelength conversion at a node v from wavelength λ to wavelength λ', if the weight of e'_i is $\omega_3(e'_i) = c_v(\lambda, \lambda')$, i.e., set the switch at node v from λ to λ'. Therefore, we have the following theorem.

Theorem 1: Given a directed network $G(V, E)$ and a pair of nodes s and t, assume that each link e of G has been assigned with a set $\Lambda(e) \subseteq \Lambda$ of available wavelengths, and every node has been given the wavelength conversion function. There is an algorithm for finding an optimal semilightpath from s to t in G. The algorithm requires $O(k^2 n + km + kn\log(kn))$ time.

Proof: By the above discussion, we first construct a directed, weighted auxiliary graph $G_{s,t} = (V'_{s,t}, E'_{s,t}, \omega_3)$, which can be done in $O(k^2 n + km)$ time clearly. Let $G_{s,t}$ be represented by adjacency lists. It is already shown that the number of nodes and the number of links in $G_{s,t}$ are no more than $2kn + 2$ and $k^2 n + 2k + km$, respectively. From the book by Cormen *et al.* [6] (see p. 530), we know that finding a shortest path in a graph H between two nodes takes $O(m' + n'\log n')$ time if the Fibonacci heap technique due to Fredman and Tarjan [7] is employed, where H has n' nodes and m' edges. Notice that the algorithm actually constructs a shortest path tree rooted at the source node, i.e., the shortest paths from the root to all other reachable nodes have been constructed in this amount of time. So, our algorithm can be implemented in $O(k^2 n + km + kn\log(kn))$ time by the Fibonacci heap technique. The theorem then follows. □

Corollary 1: Given a directed network $G(V, E)$ in which every link $e \in E$ has been assigned a set $\Lambda(e)$ of available wavelengths, and every node has been given a wavelength conversion function, there is an algorithm for finding all pairs optimal semilightpaths in G. The algorithm requires $O(k^2 n^2 + kmn + kn^2\log(kn))$ time.

Proof: Let us recall the construction of $G_v(X_v, Y_v, E_v, \omega_1)$ for every $v \in V_M$ $(= V)$. First, another weighted bipartite graph $G'_v = (X'_v, Y'_v, E'_v, \omega_1)$ is constructed. $X'_v = X_v \cup \{v'\}$, $Y'_v = Y_v \cup \{v''\}$, and

$$E'_v = E_v \cup \{\langle v', u\rangle | u \in Y_v\} \cup \{\langle u, v''\rangle | u \in X_v\}$$

and all links in $E'_v - E_v$ are assigned weight zeros. Then, an auxiliary directed, weighted graph $G_{\text{all}} = (V_{\text{all}}, E_{\text{all}}, \omega_3)$ is constructed as follows. $V_{\text{all}} = \bigcup_{v\in V}(X'_v \cup Y'_v)$ and $E_{\text{all}} = \bigcup_{v\in V} E'_v \cup E_{\text{org}}$. For every link $e \in E'_v$, $\omega_3(e) = \omega_1(e)$; and for every $e' \in E_{\text{org}}$, $\omega_3(e') = \omega_2(e')$. Now, it is clear that a shortest path in G_{all} from s' to t'' correspond to an optimal semilightpath in G from s to t. There are $O(n^2)$ pairs of nodes in G. So, we need to construct a shortest tree rooted at v' for every $v \in V$. The construction of each such a tree takes $O(k^2 n + km + kn\log(kn))$ time by Theorem 1 because $|V_{\text{all}}| = |V'| + 2n \le 2kn + 2n = 2n(k+1)$ and

$$\begin{aligned}|E_{\text{all}}| &= |E'| + \sum_{v\in V}(|X_v| + |Y_v|)\\ &\le |E'| + 2kn \le k^2 n + km + 2kn.\end{aligned}$$

There are n such trees, the corollary then follows. □

Note that the optimal semilightpath found by our algorithm and Chlamtac *et al.* [4] may go through a node v in G many times. Fig. 5 illustrates such a case, and Fig. 6 shows the

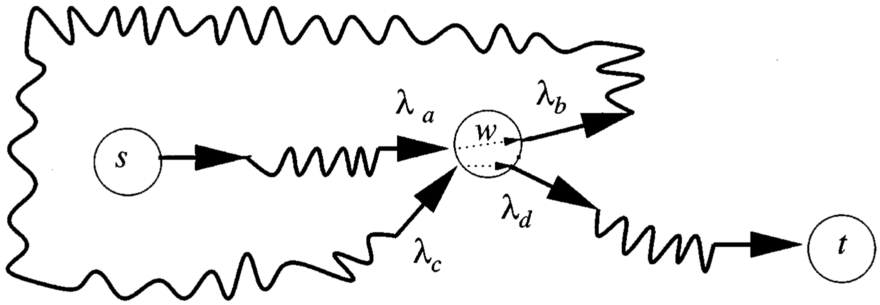

Fig. 5. The corresponding path in G from s to t.

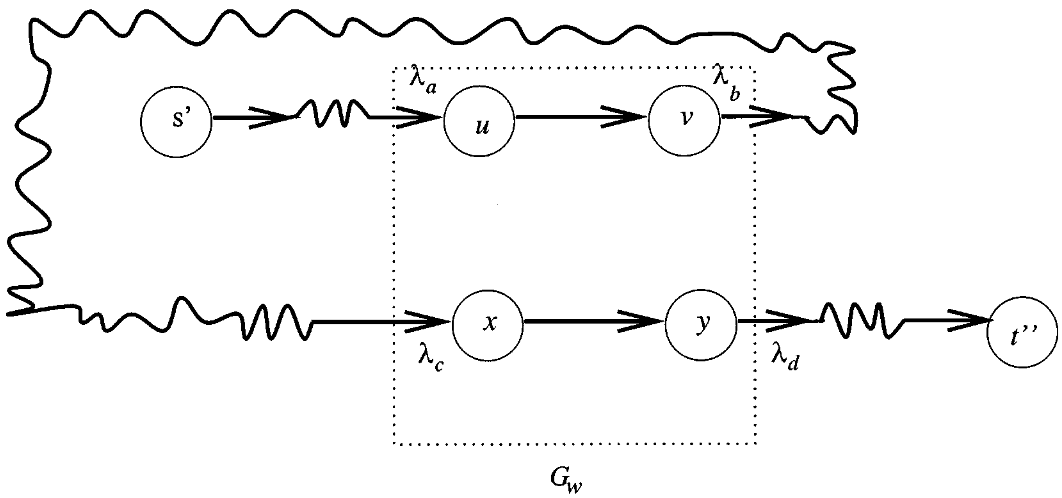

Fig. 6. The shortest path in $G_{s,t}$ from s' to t''.

corresponding shortest path in $G_{s,t}$ from s' to t'' of the path, which means the switch at v is switched several times during the routing. This is allowed in our model as remarked in the end of Section II. Through putting more restrictions on the cost structure, this situation can be avoided. Two such restrictions are given as follows.

Recall that $E_{\text{in}}(G, v)$ and $E_{\text{out}}(G, v)$ are the sets of incoming links and outgoing links of v in G. Let

$$\Lambda_{\text{in}}(G, v) = \bigcup_{e \in E_{\text{in}}(G, v)} \Lambda(e)$$

and

$$\Lambda_{\text{out}}(G, v) = \bigcup_{e \in E_{\text{out}}(G, v)} \Lambda(e).$$

Restriction 1: For any $\lambda_p \in \Lambda_{\text{in}}(G, v)$ and $\lambda_q \in \Lambda_{\text{out}}(G, v)$, $c_v(\lambda_p, \lambda_q)$ is well defined.

Restriction 1 says that for any node v, if it can receive a message using wavelength λ_p and transmit the message using wavelength λ_q, then the wavelength conversion from λ_p to λ_q in it can be carried out. Clearly, this restriction is reasonable.

Restriction 2:

$$\max_{v \in V, \lambda_p \in \Lambda_{\text{in}}(G, v), \lambda_q \in \Lambda_{\text{out}}(G, v)} \{c_v(\lambda_p, \lambda_q)\} < \min_{e \in E, \lambda \in \Lambda} \{w(e, \lambda)\} \tag{2}$$

Restriction 2 says that the local wavelength conversion cost at any node is always less than the transfer cost using any wavelength on any link. We think this is also reasonable because we are dealing with a large wide area network.

Theorem 2: If the network $G(V, E)$ satisfies all conditions defined before and the two restrictions, then the optimal semilightpath in G from s to t goes through each node v of G only once.

Proof: We show that the situation appeared in Fig. 5 would not happen if the two restrictions defined hold. Then, the corresponding path in G goes through every node in it only once. Suppose that the case in Fig. 5 occurs, that means, the shortest path $\mathcal{P}$ in $G_{s,t}$ from s' to t'' consists of the following subpaths: path $P_{s',u}$, link $\langle u, v \rangle$ which is a switch at w in G actually, path $P_{v,y}$, and path $P_{y,t''}$. We now consider the link $\langle u, y \rangle$ which is existent by Restriction 1. The cost of $\langle u, y \rangle$ is $c_w(\lambda_a, \lambda_d)$. By Restriction 2, $c_w(\lambda_a, \lambda_d)$ is less than the cost of the segment of the shortest path consisting of link $\langle u, v \rangle$ and $P_{v,y}$. Since $P_{v,y}$ contains at least one link induced from a link in G, then, another path from s' to t'' consisting of $P_{s',u}$, link $\langle u, y \rangle$, and $P_{y,t''}$ has less cost than that of $\mathcal{P}$. This contradicts that $\mathcal{P}$ is the shortest path in $G_{s,t}$ from s' to t''. Therefore, the situation in Fig. 5 would not occur, and the theorem then follows. □

B. A Distributed Algorithm

Usually an optical network can be decomposed into two separate networks in functionality. One is called *data network* which is used to transfer large volume data such as image data and databases etc., by its optical fibers. The other is called *control network* which is used for high level protocol controls such as finding an optimal semilightpath/lightpath and setting the optical switches along the path. Because the size of each message used for these controls is usually not large, people make use of the electronic network to implement high level protocol controls. Since we are dealing with large wide area networks, in practice it is not realistic to design a centralized algorithm to find an optimal semilightpath for the networks. Instead, a distributed algorithm seems more appropriate to accommodate the task. In most cases, if there is a request for establishing a lightpath/semilightpath, which happens quite often in on-line mode, a goal is to seek an efficient distributed algorithm for finding an optimal semilightpath for every pair of request on the networks. Here, we provide an answer for this purpose.

What we do is to embed the ideal network $G_{s,t}$ into the physical network G first. We then simulate $G_{s,t}$ using G. Now, we claim that the construction of $G_{s,t}$ has high locality, which is explained as follows. In the original network, $G(V, E)$, a weighted, bipartite graph $G_v(X_v, Y_v, E_v, \omega_1)$ is constructed, for every node v if $v \neq s$ and $v \neq t$. A weighted, bipartite graph $G_s = (X_s \cup \{s'\}, Y_s, E_s \cup \{\langle s', u\rangle | u \in Y_s\}, \omega_1)$ at node s is also constructed, where all links in $\{\langle s', u\rangle | u \in Y_s\}$ are assigned weight zeros. Similarly, the weighted, bipartite graph G_t at node t can be constructed

$$G_t = (X_t, Y_t \cup \{t''\}, E_t \cup \{\langle u, t''\rangle | u \in X_t\}, \omega_1)$$

where all links in $\{\langle u, t''\rangle | u \in X_t\}$ are assigned weight zeros. Each physical link $e \in E$ of G serves as $|\Lambda(e)|$ links of $G_{s,t}$. As a result, $G_{s,t}$ is constructed and represented distributively, i.e., every node v of G holds all adjacency lists of the nodes in G_v of $G_{s,t}$. The problem then becomes to find a shortest path in $G_{s,t}$ from s' to t''. Now, we use G to simulate $G_{s,t}$, i.e., each node of G is actually a subgraph of $G_{s,t}$. While the single-source shortest path problem is a well-studied problem in the distributed computing environment, there are many efficient algorithms to accommodate it like the algorithm by Chandy and Misra [3]. Therefore, we have the following theorem.

Theorem 3: Given a directed network $G(V, E)$ and a pair of nodes s and t, assume that every link e in G has been assigned a set $\Lambda(e)$ of available wavelengths, and every node has been given a wavelength conversion function. There is an efficient distributed algorithm for finding an optimal semilightpath from s to t. The communication and time complexities of the proposed algorithm are $O(km)$ and $O(kn)$, respectively, on the distributed computational model.

Proof: Since the local computation is negligible in the distributed computational model, the construction of $G_{s,t}$ can be done in constant time. Then, G is used to simulate $G_{s,t}$. That is, each node of G simulates a subgraph of $G_{s,t}$ with at most $2k+1$ nodes. It is already known that the communication and time complexities for finding a shortest path between two nodes in H are $O(m')$ and $O(n')$, respectively [3], where H contains n' nodes and m' edges. Therefore, the communication and time complexities for our problem are $O(km)$ and $O(kn)$, because the links in $\bigcup_{v\in V} E_v$ in $G_{s,t}$ are *virtual links* which are located inside of physical nodes of G. By the definition of this model, the communication costs on these links are negligible. □

Corollary 2: Given a directed network $G(V, E)$, assume that each link e of G has been assigned a set $\Lambda(e)$ of available wavelengths, and every node has been given a wavelength conversion function. There is a distributed algorithm for finding all pairs optimal semilightpaths. The communication and the time complexities of the algorithm are $O(k^2n^2)$ and $O(k^2n^2)$ on the distributed computational model.

Proof: We first construct the graph G_{all} which has been defined in the proof body of Corollary 1. The construction of G_{all} takes constant time because the local computational time is negligible in the distributed computational model. We already knew that a shortest path in G_{all} from s' to t'' corresponds to an optimal semilightpath in G from s to t. Therefore, finding all pairs shortest paths in G_{all} between n^2 pairs of nodes corresponds to finding all pairs shortest paths in G. By Haldar's algorithm, finding all pairs shortest paths in a graph H with n' nodes can be done in $O(n'^2)$ time using $O(n'^2)$ messages [9]. Following the same approach in the proof of Theorem 3, we embed G_{all} to G, i.e., we use G to simulate G_{all}. It is clear that all pairs shortest paths of G_{all} can be found in $O(k^2n^2)$ time using $O(k^2n^2)$ messages, because every node in G accommodates a subgraph of G_{all} with at most $2k+2$ nodes. □

C. Comparison to a Previously Known Algorithm

We compare our centralized algorithm with a previously centralized algorithm by Chlamtac *et al.* [4] for the optimal semilightpath problem. Both algorithms are based on the decision tree model, in which only algebraic operations are allowed on the weights. In the following, we compare the time complexity of our algorithm with theirs. We conclude that our algorithm outperforms theirs in most cases.

Let $\mathcal{A}_1$ and $\mathcal{A}_2$ be two algorithms for solving the same problem, and let $T_{\mathcal{A}_1}$ and $T_{\mathcal{A}_2}$ be the execution times of $\mathcal{A}_1$ and $\mathcal{A}_2$. Then, we say that $\mathcal{A}_2$ improves $\mathcal{A}_1$ by a *factor* of $\Omega(\rho)$ if $T_{\mathcal{A}_1} = \Omega(\rho T_{\mathcal{A}_2})$. Now, let $T_{CFZ} = O(kn^2 + k^2n)$ be the time complexity of their algorithm and let $T_c = O(k^2n + km + kn\log(kn))$ be the time complexity of the proposed algorithm. Then, if $k = \Omega(n)$, $T_c = T_{CFZ} = O(k^2n)$ since $m \le dn \le n^2$, which means both algorithms have the same worst time complexity. Otherwise, $T_c = O(k^2n + km + kn\log(kn))$ and $T_{CFZ} = O(kn^2)$. We have

$$\frac{T_{CFZ}}{T_c} = \frac{c'kn^2}{k^2n + km + kn\log(kn)} \geq \frac{c''n}{\max\{k, d, \log n\}} \tag{3}$$

where c' and c'' are constants and $m \le dn$.

We now analyze the inequity (3) by the following two cases.

1) $\max\{k, d\} \le c'''\log n$ with c''' is constant, then $T_{CFZ}/T_c = \Omega(n/\log n)$. This means that our algorithm has improved their algorithm by an $\Omega(n/\log n)$ factor.
2) $\max\{k, d\} \ge c'''\log n$, then $T_{CFZ}/T_c = \Omega(n/\max\{k, d\})$. This means that our algorithm

has improved their algorithm by an $\Omega(n/\max\{k, d\})$ factor. Note that $d < n$, $k < n$, and usually d is constant or a sublinear function of n which increases very slow because G is a large, sparse wide area network.

Overall, our algorithm has improved their algorithm by a factor of $\Omega(n/\max\{k, d, \log n\})$. Particularly, this improvement is significant when k and m are relatively small but n is relatively large. For example, when $m = O(n)$ and $k = O(\log n)$, our algorithm runs in $O(n \log^2 n)$ time only, while their algorithm takes $O(n^2 \log n)$ time.

As said before, in practice it is impossible to run such a centralized algorithm for a very large wide area network, while a distributed version seems more appropriate. Through the previous discussion, we know that our algorithm has high locality, it can be implemented in the network distributively, while the algorithm by Chlamtac *et al.* [4] seems difficult to do so.

IV. A Special Optimal Semilightpath Problem

In this section, we analyze the proposed algorithm for a special case with more restrictions. In the previous section, we assume that, associated with each directed link e, there is a set $\Lambda(e)$ $(\subseteq \Lambda)$ of wavelengths available. Though the number of wavelengths $|\Lambda|$ $(= k)$ in a network may be quite large, the number of transmitters and receivers (tuning) at each node usually is bounded, so is the number of wavelengths at every link. In Section III, we assume every node can switch $O(k^2)$ pairs of wavelengths. When $k = O(n)$, in practice this is impossible because the size n of the network is quite large. Based on this argument, in this section, we assume the number of available wavelengths, $|\Lambda(e)|$, associated with each link e, is bounded by k_0, i.e., $|\Lambda(e)| \le k_0$. We further assume that k_0 is either a constant or a function of n increasing very slow, i.e., $k_0 = o(n)$ even if $k = \Omega(n)$. Under this new restriction, the aim is to find an optimal semilightpath in the network G from s to t. Following the definitions in Section III, G_M and G_v constructed, for every $v \in V$. G_M has $|V_M|$ $(= |V| = n)$ nodes and $|E_M|$ $(= \sum_{e \in E} |\Lambda(e)| \le mk_0)$ links. G_v has the following property.

Observation 4: Let $G_v(X_v, Y_v, E_v, \omega_1)$ be the weighted, bipartite graph defined as before. Then

$$\begin{aligned}|X_v| + |Y_v| &= |\Lambda_{\text{in}}(G_M, v)| + |\Lambda_{\text{out}}(G_M, v)| \\ &\le d_{\text{in}}(G, v)k_0 + d_{\text{out}}(G, v)k_0 \le 2dk_0\end{aligned}$$

and

$$|E_v| \le k_0^2 d_{\text{in}}(G, v) d_{\text{out}}(G, v) \le d^2 k_0^2.$$

G' is then obtained from G_v for all $v \in V_M$ $(= V)$ and G_M. Therefore, we have the following observation.

Observation 5: Let $G'(V', E', \omega_2)$ be the directed weighted graph defined before. Then

$$|V'| = \sum_{v \in V_M} (|X_v| + |Y_v|) \le \sum_{e \in E} |\Lambda(e)| \le mk_0$$

and

$$\begin{aligned}|E'| &= \sum_{v \in V_M} |E_v| + |E_{\text{org}}| \le d^2 n k_0^2 + m_1 \\ &\le d^2 n k_0^2 + mk_0.\end{aligned}$$

Finally, $G_{s,t}$ can be constructed from G'. The number of nodes and links in $G_{s,t}$ are no more than $mk_0 + 2$ and $d^2 n k_0^2 + mk_0 + 2dk_0$, respectively. Therefore, we have the following theorem.

Theorem 4: Given a directed network $G(V, E)$ and a pair of nodes s and t, assume that each link e of G has been assigned a set $\Lambda(e)$ of available wavelengths and $|\Lambda(e)| \le k_0$, and every node has been given a wavelength conversion function. There is an algorithm for finding an optimal semilightpath from s to t. The algorithm requires $O(d^2 n k_0^2 + mk_0 \log n)$ time assuming $k_0 = o(n)$.

By Theorem 4, we know that, even though the total number of wavelengths k in the network is quite large (e.g., $k > n$), the execution time of the algorithm only depends on the number of wavelengths k_0 at every link, the maximum degree d, and the number of nodes n, and the number of links m in the network. That is, the execution time of the proposed algorithm is independent of k. As said before, we are dealing with large wide area networks. In practice, such networks are usually sparse [i.e., $m = O(n)$], d is fixed or a slowly increasing function of n [e.g., $d = O(\log n)$], and k_0 is also fixed or a slowly increasing function of n [e.g., $k_0 = O(\log n)$]. Thus, the algorithm takes time $O(d^2 n k_0^2 + mk_0 + mk_0 \log(mk_0)) = O(d^2 n k_0^2 + mk_0 \log(mk_0)) = O(d^2 n k_0^2 + mk_0 \log n)$.

Following the same analysis in the previous section, we have the following theorem.

Theorem 5: Given a directed network $G(V, E)$ and a pair of nodes s and t, assume that every link e in G has been assigned a set $\Lambda(e)$ of available wavelengths and $|\Lambda(e)| \le k_0$, and every node has been given a wavelength conversion function. There is an efficient distributed algorithm for finding an optimal semilightpath from s to t. The communication and time complexities of the proposed algorithm are $O(mk_0)$ and $O(nk_0)$, respectively, on the distributed computational model.

Corollary 3: Given a directed network $G(V, E)$, assume that every link e in G has been assigned a set $\Lambda(e)$ of available wavelengths and $|\Lambda(e)| \le k_0$, and every node has been given a wavelength conversion function. There is a distributed algorithm for finding all pairs optimal semilightpaths. The communication and the time complexities of the algorithm are $O(n^2 k_0^2)$ and $O(n^2 k_0^2)$, respectively, on the distributed computational model.

V. Conclusions

In this paper, we have presented an improved algorithm for finding an optimal lightpath/semilightpath in wide area fiber-optic networks. The lightpath/semilightpath obtained has the minimum cost including the cost of each link on the path and the cost of wavelength conversion. In addition, we have also analyzed our algorithm for a restricted optimal semilightpath problem. Compared with a previous algorithm for the problem, our algorithm runs much faster in most cases. Also, a distributed version of the proposed algorithm is given in this paper. Its simplicity, time efficiency, and high locality make the proposed algorithm a competitive candidate to be used in large WDM networks.

REFERENCES

[1] A. S. Acampora, "Partially reconfigurable multihop lightwave networks," in *Proc. GLOBECOM*, San Diego, CA, Dec. 1990.

[2] A. Aggarwal, A. Bar-Noy, D. Coppersmith, R. Ramaswami, B. Schieber, and M. Sudan, "Efficient routing in optical networks," *J. ACM*, vol. 46, pp. 973–1001, 1996.

[3] K. M. Chandy and J. Misra, "Distributed computations on graphs: Shortest path algorithms," *Commun. ACM*, vol. 25, pp. 833–837, 1982.

[4] I. Chlamtac, A. Faragó, and T. Zhang, "Lightpath (wavelength) routing in large WDM networks," *IEEE J. Select. Areas Commun.*, vol. 14, pp. 909–913, 1996.

[5] I. Chlamtac, A. Ganz, and G. Karmi, "Lightpath communications: a novel approach to high bandwidth optical WDN's," *IEEE Trans. Commun.*, vol. 40, pp. 1171–1182, July 1992.

[6] T. H. Cormen, C. E. Leiserson, and R. L. Rivest, *Introduction to Algorithms*. Cambridge, MA: MIT Press, 1989.

[7] M. L. Fredman and R. E. Tarjan, "Fibonacci heaps and their uses in improved network optimization algorithms," *J. ACM*, vol. 34, pp. 596–615, 1987.

[8] P. E. Green, *Fiber-Optic Communication Networks*. Englewood Cliffs, NJ: Prentice-Hall, 1992.

[9] S. Haldar, "An 'all pairs shortest paths' distributed algorithm using $2n^2$ messages," *J. Algorithms*, vol. 24, pp. 20–36, 1997.

[10] J.-F. P. Labourdette and A. S. Acampora, "A multichannel multihop local lightwave network," in *Proc. GLOBECOM*, Tokyo, Japan, Nov. 1987.

[11] A. D. McAulay, *Optical Computer Architectures*. New York: Wiley, 1991.

[12] R. Ramaswami, "Multi-wavelength lightwave networks for computer communication," *IEEE Commun. Mag.*, vol. 31, pp. 78–88, 1993.

[13] R. J. Vitter and D. H. C. Du, "Distributed computing with high-speed optical networks," *IEEE Computer*, vol. 26, pp. 8–18, 1993.

Weifa Liang received the Ph.D. degree in computer science from the Australian National University, Canberra, Australia, in 1998. He received the M.E. degree from University of Science and Technology of China, in 1989, and the B.S. degree from Wuhan University, China, in 1984, both in computer science.

He is currently holding a Lecturing position in the Department of Computer Science at the Australian National University. His research interests include routing protocol design for high speed networks, parallel processing, parallel and distributed algorithms, data warehousing and OLAP, query optimization, and graph theory.

Xiaojun Shen received the Ph.D. degree in computer science from the University of Illinois at Urbana-Champaign, in 1989. He received the M.S. degree in computer science from the Nanjing University of Science and Technology, China, in 1982, and the B.S. degree in numerical analysis from the Qinghua University, Beijing, China, in 1968.

He is currently an Associate Professor in the Computer Science Telecommunications Program at the University of Missouri-Kansas City. His current research interests include interconnection networks, algorithms, and computer networking.