Maximizing Charging Satisfaction of Smartphone Users via Wireless Energy Transfer

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Abstract—Smartphones now become an indispensable part of our daily life. However, maintaining a smartphone's continuing operation consumes lots of battery energy. For example, a fully-charged smartphone usually cannot support its continuing operation for a whole day. A fundamental issue on a smartphone is its energy issue. That is, how to prolong the lifetime of a smartphone so that it can run as long as possible to meet its user needs. Wireless energy transfer has been demonstrated as a promising technique to address this issue. In this paper, we study a novel smartphone charging problem, through wireless chargers deployed on public commuters, e.g., subway trains, to charge energy-critical smartphones when their users take subway trains to work or go home. Since the amounts of residual energy of different smartphones are significantly different, the charging satisfactions of different users are essentially different. In this paper, we formulate this charging satisfaction problem as a novel optimization problem that schedules the limited number of wireless chargers on subway trains to charge energy-critical smartphones such that the overall charging satisfaction of smartphone users is maximized, for a given monitoring period (e.g., one day). For this problem, we first devise a $\frac{1}{3}$-approximation algorithm if the travel trajectory of each smartphone user is given. We then propose an online algorithm to deal with dynamic energy-critical smartphone charging requests. We also propose a nontrivial distributed scheduling algorithm for a variant of the problem where the global knowledge of user energy information is unknown. We finally evaluate the performance of the proposed algorithms through experimental simulations, using a real dataset of subway-taking in San Francisco. The experimental results show that the proposed algorithms are very promising, and over 90 percent of energy-critical user smartphones can be satisfactorily charged in a one-day monitoring period.

Index Terms—Smartphone, energy charging, wireless energy transfer, subway trains, charging satisfaction maximization, approximation algorithm, online algorithm, distributed algorithm

1 Introduction

With the advance on micro-electronic technology and wireless communication, more and more people nowadays rely heavily on portable mobile devices, such as smartphones, tablets, and Apple watches, for entertainment and business purposes. Especially, smartphones now become an indispensable part of our daily life. The *eMarketer* reported that there were more than 4 billion smartphone users globally in 2014 and this number is expected to grow to 5 billion in 2017 [30]. However, smartphones are very energy-hungry, and a fully-charged smartphone usually cannot support its continuing operation for a whole day, even if the battery technology for smartphones has made substantial progress in the past decades to make smartphone batteries last much longer and have higher power densities [26]. For example, the lifetime of iPhone 6s is only about 10 hours for Internet usage [9]. Also, it is reported that 62 percent of smartphones have less than 20 percent of residual power, 33 percent of smartphones go below 10 percent power, and 12 percent of smartphones run out of their power completely at the end of a day [26]. The limited energy capacities of smartphones bring their users many inconveniences. Some users cannot continue using their smartphones any more later of the day (e.g., afternoon) due to the energy depletions of their smartphones. Others get to turn off some valuable yet energy-consuming functionalities such as GPS, 3G/4G, and Wi-Fi installed in their smartphones, to prolong the lifetimes of the smartphones. Consequently, smartphone users cannot make use of many features provided by smartphones such as Twitter, google maps, Email, YouTube, eBay, Pinterest, etc. Alternatively, a smartphone can be charged by a portable charger if needed. It however is inconvenient for its user to bring the charging device with him/her all time. A fundamental problem of smartphone charging thus is: is there any convenient way to charge energy-critical smartphones so that their users can enjoy all applications provided by the smartphones at all times without worrying whether there is enough energy left and turning off some useful mechanisms (e.g., 4G)? In this paper we tackle this challenge by using wireless chargers installed on subway trains to charge energy-critical smartphones.

The recent breakthrough on wireless energy transfer technology based on strongly coupled magnetic resonances has drawn lots of attentions in the research community [14], [15], [22], [32], [37]. Kurs et al. demonstrated that it is possible to achieve an approximate 40 percent efficiency of wireless power transfer for powering a 60 watts light bulb within two meters without any wire lines [14]. Engineers at Intel further achieved a 75 percent efficiency of wireless

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energy transfer for transferring 60 watts of power over a distance of up to two to three feet [12]. This technology has many advantages in comparison with other wireless charging technologies, including high wireless energy transfer efficiency over a mid-range, immunity to the neighboring environment, no requirement of line-of-sight or any alignment, and charging multiple mobile devices simultaneously [15], [28]. Also, it is reported that the technology is safe to human beings since it is not radiative [28]. Furthermore, an industry standard group A4WP, including Qualcomm Corp., Samsung Corp, etc., has applied the principle of magnetic resonance to develop a wireless energy transfer system over distance for consumer electronics [21]. Several commercial products of wireless energy transfer technology now are available in the market, e.g., smartphones, electric vehicles, and sensors [10], [20], [21], [28]. It thus is envisioned smartphones supporting wireless energy transfer will be pervasive in the near future and the wireless energy transfer market is expected to grow from just \$216 million in 2013 to \$8.5 billion in 2018 [29].

The wireless energy transfer technique will revolutionize the way people charge their smartphones. Subway is one of the most popular transportation means in modern metropolitan cities including New York, London, Tokyo, Hong Kong, Beijing, etc., where people take a subway train to work or go home. For example, the average daily subway ridership in New York alone is about 5.5 million in a weekday [25], and the number in Beijing even reaches over 10 million [24]. Let us consider an application scenario where there are multiple chargers deployed on every subway train and every charger can charge a smartphone via wireless energy transfer if the smartphone is within the charging range of the charger, e.g., 2 to 3 meters. When a smartphone user takes on a subway train, his/her smartphone can send a charging request automatically if its residual energy falls below a given threshold, e.g., only 20 percent energy left, assuming that the user pays the subway company some fee for such a service, e.g., by the amount of energy charged to the user. Once the request is received, a charger nearby the user is allocated to charge the smartphone wirelessly.

In this paper we consider a charging satisfaction maximization problem on subway trains as follows. People take on a subway train at one station and take off at another station, or they may interchange for another train. Therefore, each user has several opportunities to get his/her smartphone charged when he/she is on a subway train. Furthermore, some people will take more stations than others before they take off. Since there are only limited numbers of chargers installed on subway trains and the number of charging requests may be far more than the sum of the service capacities of all chargers, it thus is desirable to 'fairly' replenish energy to the requested smartphones such that as many smartphone users as possible are satisfied. Otherwise, some users with sufficiently residual energy will be fully charged while others with barely left energy will miss the charging opportunities and run out of their energy very soon. We thus model the charging satisfaction of each requested user as a non-decreasing submodular function of the amount of energy charged to the smartphone of the user, where a submodular function usually is used to characterize the diminishing return property. For example, the charging satisfaction of a user v_j is $f_j(B_j) = \log_2(RE_j + B_j + 1) - \log_2(RE_j + 1)$, where RE_j is the residual energy of user v_j before the user is charged and B_j is the amount of energy charged to the user. The charging satisfaction maximization problem is to allocate the chargers to replenish energy to smartphones of requested users for a given monitoring period (e.g., one day), such that the sum of charging satisfaction of all users is maximized.

Comparing with the solution that offers wired outlets to charge smartphones, there are two significant advantages in deploying wireless energy chargers to replenish energy to smartphones on subway trains. First, it is much more convenient to charge smartphones, since users do not need to connect their smartphones to the outlets with wires, and nearby wireless chargers will be automatically allocated to charge their smartphones wirelessly. Second, smartphone users can be fairly charged. In the solution of offering wired outlets, a smartphone user may have to wait for an unoccupied outlet a long time if all outlets are being used by other users, since an occupied user usually will not release his outlet until his smartphone has been charged a large amount of energy (compared with his smartphone battery capacity). As a result, the waiting user may miss his charging opportunity before he takes off the train, even if the residual energy of his smartphone is very low. In contrast, wireless chargers can identify energy-emergent smartphones and charge them so that the sum of the charging satisfactions of all users is maximized.

The charging satisfaction maximization problem is very challenging, due to the following constraints on charging smartphones by wireless chargers. (i) Each user may be within the charging ranges of multiple chargers but only one charger will be allowed to charge the user at any time; (ii) the number of users within the charging range of a charger is usually larger than the charging capacity of the charger and thus only a subset of the users can be chosen for charging; (iii) the energy transfer efficiency between a charger and a user decreases with the physical distance between them; and (iv) some users enjoy many charging opportunities while others may have only a few of them. We thus must address following subproblems: (a) when should each user be charged? and (b) which charger should be allocated to charge the user?

Unlike existing solutions to prolonging smartphone lifetimes by forcing their users to turn off some useful yet energy-consuming functionalities (e.g., 3G/4G) or offering inconvenient wired outlets to charge the smartphones on subway trains, we make use of wireless energy chargers installed on subway trains to charge energy-critical smartphones so that the smartphones can be charged in a convenient way (i.e., wireless charging) and their users can enjoy all applications provided by smartphones at all times. We also study the problem of 'fairly' charging energy-critical smartphone users, by proposing a novel charging satisfaction metric and efficient charging scheduling algorithms.

The main contributions of this paper are as follows. We are the first to consider the use of wireless energy chargers installed on subway trains to charge energy-critical smartphones of users when their users take subway trains. To fairly charge energy-critical smartphones, we first formulate a novel optimization problem of allocating wireless chargers to charge the smartphones for a given monitoring period, such that the overall charging satisfaction of smartphone users is maximized. We then devise a $\frac{1}{3}$-approximation algorithm for the problem if the travel trajectory of each user is given. We also propose an online algorithm to deal with dynamic energy-critical smartphone charging requests. Furthermore, we develop a novel distributed scheduling algorithm for a variant of the problem when the global knowledge of user energy information is not known in advance. We finally evaluate the performance of the proposed algorithms, using a real dataset of subway-taking in San Francisco. Experimental results show that the proposed algorithms are very promising, and as high as 87.4, 87.4 and 90 percent of energy-critical users can be charged through the solutions delivered by the proposed distributed, online, and approximation algorithms, respectively.

The rest of the paper is organized as follows. Section 2 reviews related work. Section 3 introduces preliminaries and defines the charging satisfaction maximization problem precisely. Sections 4 and 5 propose approximation and online algorithms for the problem with and without the knowledge of travel trajectory of each user, respectively. Section 6 devises a novel distributed algorithm. Section 7 evaluates the algorithm performance, and Section 8 concludes the paper.

2 Related Work

The wireless power transfer technology based on strongly magnetic resonances has drawn a lot of attentions in many areas, such as smartphones [21], [28], electric vehicles [10], [41], wireless sensor networks [16], [17], [31], [33], [34], [35], etc. For example, Zhu et al. [41] studied the problem of scheduling n electric vehicles (EVs) to K deployed charging stations in a road network such that each EV is fully charged and the average time spent on charging EVs is minimized, where the time for charging an EV includes its travel time to its assigned charging station, the queuing time, and the actual charging time. Unlike their work, in this paper each to-be-charged smartphone of a user is not required to be fully charged. Instead, the sum of charging satisfactions of all smartphone users should be maximized.

There are extensive studies in adopting wireless energy replenishment to prolong the lifetime of wireless sensor networks (WSNs) [16], [17], [31], [35], [36]. Xie et al. [31] employed a wireless charging vehicle to periodically visit each sensor in a WSN, where the charging vehicle can charge each sensor wirelessly when the vehicle travels in the vicinity of the sensor. On the other hand, Xu et al. [35], [36] and Liang et al. [16], [17] employed multiple charging vehicles to replenish sensors energy in WSNs, in which Xu et al. studied the problem of finding a series of charging scheduling for the charging vehicles to maintain the perpetual operations of the WSN for a given monitoring period such that the total travel distance of the vehicles for the period is minimized [35], [36], and Liang et al. considered the problem of dispatching the minimum number of charging vehicles to charge a set of to-be-charged sensors, assuming that the energy capacity of each charging vehicle is bounded [16], [17]. Unlike these studies, in this paper the wireless chargers on subway trains are fixed and each smartphone may not be fully charged. In fact, the amount of energy received by a smartphone relies on the duration of its user on trains and how much residual energy it has.

There are several closely related studies in partial executions for interactive services such as web search, where a request can be partially executed and the response quality to it improves with increasing resources but diminishing utility gain margins [8], [38], [39], [40]. He et al. [8] considered the problem of allocating processing time to competing service requests so that the total quality gained by executing all requests is maximized. Zheng et al. [40] investigated an extended version of the problem in [8], by taking into account both the multi-resource sharing and execution parallelism. Xu et al. [38] studied the problem of scheduling partially-executed requests in data centers such that the cost of the total energy consumption is minimized, subject to the Service Level Agreement (SLA) on the response quality to each executed request. Zheng et al. [39] considered the problem of scheduling interactive jobs in a data center with multiple identical machines so that the utility sum of all jobs is maximized. Unlike these studies of request execution on a single data center [8], [40] or a user's request traffic can be arbitrarily split among all data centers [38], in this paper we investigate scheduling multiple wireless chargers, rather than a single wireless charger, and each smartphone can be charged by only one charger, instead of being arbitrarily split among chargers, at any time. Furthermore, although the problem considered in [39] seems similar to that in this paper, in terms of there being N machines in [39] and K chargers in this paper, the N machines are identical with each having a processing capacity of one and each job is allowed to be allocated to every one of the N machines, the N machines thus can be considered as a single virtual machine with a more powerful processing capacity of N. Contrarily, the charging efficiencies (i.e., processing capacities) of different chargers for a smartphone user significantly vary and each smartphone can be allocated to only the wireless chargers nearby, since the energy transfer efficiency decreases with the increase of the distance between a charger and a smartphone. Therefore, the problem considered in this paper thus is essentially different from those in [8], [38], [39], [40], and the proposed algorithms for partial requests are inapplicable to the problem considered in this paper.

There are several fairness metrics that are used to measure the fairness in resource allocation, such as Jain's fairness metric [11], the max-min fairness [3], and the proportional fairness [13]. In *Jain's fairness metric* [11], the value of $J(x_1, x_2, \ldots, x_n)$ $(= \frac{(\sum_{i=1}^{n} x_i)^2}{n \times \sum_1^n x_i^2})$ is used to measure the fairness, where $x_1, x_2, \ldots, x_n$ are the amounts of energy charged to users $v_1, v_2, \ldots, v_n$, respectively. It reaches the maximum when all users are charged with the same amount of energy. In the *max-min fairness* [3], the value of $min_{i=1}^{n}\{x_i\}$ is used to measure fairness and reaches the maximum when all users are replenished with the same amount of energy, i.e., $x_1 = x_2 = \cdots = x_n$. Finally, in the *proportional fairness* [13], given the energy demands $d_1, d_2, \ldots, d_n$ of the n users, the value of $\min_{i=1}^{n}\{\frac{x_i}{d_i}\}$ is used to measure fairness, and it achieves the maximum when $\frac{x_1}{d_1} = \frac{x_2}{d_2} = \cdots = \frac{x_n}{d_n}$. It can be seen that none of these three mentioned fairness metrics can be used to measure the charging satisfactions of smartphone users. The rationale behind is as follows. On one hand, in both Jain's and the max-min fairness metrics, they achieve their maximums if each user is charged with the same amount of energy, however, they neglected an important fact, that is, different users may have different amounts of residual energy, and energy-critical users are willing to be charged more energy than the others. On the other hand, the proportional fairness ignores that energy consumption rates of different users may be significantly different, and users with low energy consumption rates will feel satisfied if only small amounts of energy are charged to their smartphones, while those users who consume their energy very quickly will require to be charged with large amounts of energy. Instead, in this paper we use a general submodular function to characterize the diminishing return property of user charging satisfaction, by incorporating the amounts of residual energy of users, the amounts of energy charged to users, and their energy consumption rates. We will use this submodular function as the fairness metric on user charging satisfactions.

3 Preliminaries

In this section, we first present the system model, then propose a novel model for characterizing user charging satisfaction, and finally define charging satisfaction maximization problems and show the NP-hardness.

3.1 System Model

We assume that the subway system in a metropolis consists of multiple subway trains and there are K chargers $C_1, C_2, \ldots, C_K$ deployed at K different places on subway trains. We consider the charging scheduling of the K chargers for a given monitoring period (e.g., 1 day), and we divide the period into T equal time slots with each time slot lasting δ units (e.g., 1 minute). We index the T time slots by $1, 2, \ldots, T$. Assume that each charger C_i has a charging capacity c_i, i.e., it can charge up to c_i smartphones at the same time, where $c_i \geq 1$ is a positive integer. We further assume that the output power of charger C_i is P_i^o (W). Denote by d_{ijt} the Euclidean distance between charger C_i and smartphone v_j at time slot t, $1 \leq i \leq K$, $1 \leq j \leq n$ and $1 \leq t \leq T$. Following the seminal work of Kurs et al. [14], the energy transfer efficiency μ_{ijt} of charger C_i charging smartphone v_j decreases with the increase of distance d_{ijt}. For example, Xie et al. [31] showed that

$$\mu_{ijt} = -0.0958 d_{ijt}^2 - 0.0377 d_{ijt} + 1, \tag{1}$$

where $0 \leq \mu_{ijt} \leq 1$. The reception power P_{ijt} of smartphone v_j from charger C_i at time slot t thus is

$$P_{ijt} = \mu_{ijt} \times P_i^o. \tag{2}$$

To ensure that the reception power P_{ijt} is large enough to charge smartphone v_j, we assume that charger C_i can replenish energy to smartphone v_j if d_{ijt} is no more than a maximum charging range D so that the energy transfer efficiency μ_{ijt} is no less than a threshold γ. For example, assume that $\mu_{ijt} \geq \gamma = 20$ percent. The maximum charging range then is $D = 2.7$ m by Eq. (1).

Assume that there are n_c users taking the subway for the period of T, in which n ($\leq n_c$) of them are required to be charged at some time, and each user can be charged by only one charger at each time slot. Let $V = \{v_1, v_2, \ldots, v_n\}$ be the set of the n users. Denote by E_j^{max} the battery capacity of the smartphone of each user $v_j \in V$. In the following, we use smartphone v_j and user v_j interchangeably. Assume user v_j sends a charging request at time slot t_j^S in a train and will take off the train at time slot t_j^F, clearly $t_j^S < t_j^F$. Then, user v_j can be charged within the time interval from t_j^S to t_j^F. We further assume that the location of each user does not change during every time slot, but it is allowed to change at different time slots. We define the travel trajectory of user v_j between time slots t_j^S and t_j^F as the set of locations of the user on the train at every time slot in time interval $[t_j^S, t_j^F)$.

We also assume that each wireless charger is capable to measure the distance between the charger and a user when the user sends a charging request or his/her location changes at some time during his/her journey on the subway, through an indoor positioning technique, such as angle of arrival (AoA), time of arrival (ToA), received signal strength indication (RSSI), etc [18], [19]. In case that chargers cannot measure the distances or the measured distances are not accurate enough, every charger can periodically charge smartphone users nearby in a very short period and measure the energy transfer efficiencies. Assume that the duration of the period is much shorter than the entire monitoring period T, and thus can be ignored.

Denote by RE_j the residual energy of the smartphone of user v_j when the user sends a charging request with $1 \leq j \leq n$. Let binary variable x_{ijt} indicate whether charger C_i charges smartphone v_j at time slot t, i.e., $x_{ijt} = 1$ if smartphone v_j is charged by charger C_i at time slot t; $x_{ijt} = 0$, otherwise, where $1 \leq i \leq K$, $1 \leq j \leq n$, and $1 \leq t \leq T$. The amount of energy charged into the smartphone of user v_j when he/she takes off the subway is

$$B_j = \min\{\sum_{t=t_j^S}^{t_j^F - 1} \sum_{i=1}^{K} P_{ijt} \times \delta \times x_{ijt},\ E_j^{max} - RE_j\},\ \forall v_j \in V, \tag{3}$$

where $\sum_{t=t_j^S}^{t_j^F-1} \sum_{i=1}^{K} P_{ijt} \times \delta \times x_{ijt}$ is the amount of effective energy consumed by chargers for charging user v_j, $(E_j^{max} - RE_j)$ is the maximum possible amount of energy that can be charged to user v_j, P_{ijt} is the reception power of user v_j from charger C_i at time slot t, δ is the duration of each time slot, and E_j^{max} and RE_j are the energy capacity and residual energy before charging of user v_j, respectively.

3.2 User Charging Satisfaction

In this section we model the charging satisfaction of every user v_j, by incorporating the amount of residual energy RE_j of user v_j when the user sends a charging request, the amount of energy B_j charged to user v_j, and his/her average energy consumption rate ρ_j. Before we proceed, we introduce non-decreasing submodular functions, which usually are used to characterize the diminishing return property [6].

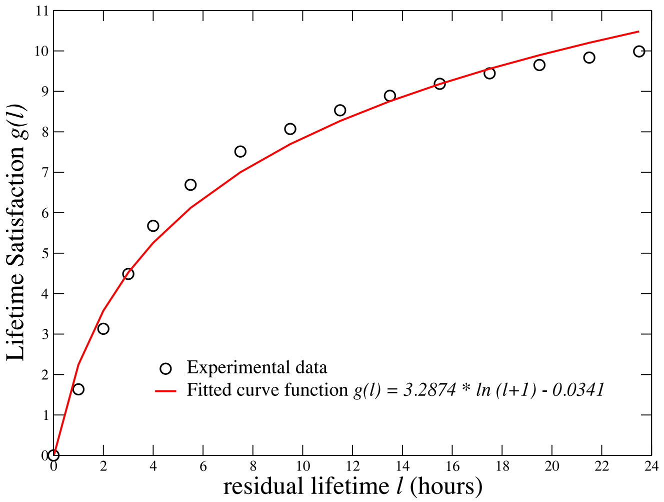


Fig. 1. The lifetime satisfaction function $g(l)$ of the residual lifetime l of a smartphone user.

Definition 1. *Let E be a finite set and z be a real-valued function with $z: 2^E \mapsto \mathcal{R}^{\geq 0}$, function z is a non-decreasing submodular function if and only if it has the following three properties [6]. (i) $z(\emptyset) = 0$; (ii) Non-decreasement: $z(S_1) \leq z(S_2)$ for any two sets $S_1, S_2 \subseteq E$ with $S_1 \subseteq S_2$; and (iii) Diminishing return property (submodularity): $z(S_1 \cup \{e\}) - z(S_1) \geq z(S_2 \cup \{e\}) - z(S_2)$ for any two sets S_1 and S_2 with $S_1 \subseteq S_2 \subset E$ and $e \in E \setminus S_2$.*

Note that the average energy consumption rates of different users may significantly vary, since there are various types of smartphones, e.g., smartphones manufactured by different companies, and the user behaviors of using their smartphones are different, e.g., different frequencies of using smartphones. Recall that B_j is the amount of energy charged to user v_j. Then, the smartphone operational time of user v_j can be prolonged from $\frac{RE_j}{\rho_j}$ to $\frac{RE_j+B_j}{\rho_j}$ after the amount of energy B_j has been charged into the smartphone.

Since smartphone users are sensitive to the residual operational time of their smartphones, we here use a non-decreasing submodular function $g(l_j)$ to characterize the lifetime satisfaction of a user v_j for a residual operational time l_j of his/her smartphone. For example, we have conducted a questionnaire about the lifetime satisfaction function $g(l_j)$ and the feedback we received from 123 smartphone users can be approximated by Eq. (4), where each user was asked to give a score (from 0 point to 10 points) for a given residual lifetime l of his/her smartphone and $0 \leq l \leq 24$, see Fig. 1,

$$g(l) = 3.2874 \times \ln(l+1) - 0.0341. \tag{4}$$

Given the residual energy RE_j of user v_j and his/her average energy consumption ρ_j, we model the satisfaction $f_j(B_j)$ of user v_j for charging an amount of energy B_j as

$$f_j(B_j) = g\left(\frac{RE_j + B_j}{\rho_j}) - g(\frac{RE_j}{\rho_j}\right), \tag{5}$$

where $\frac{RE_j}{\rho_j}$ and $\frac{RE_j+B_j}{\rho_j}$ are the residual lifetimes of user v_j before and after charging an amount of energy B_j, respectively. Note that our model $f_j(B_j)$ of characterizing the charging satisfaction has following important properties.

- (i) A user v_j is more satisfied if more energy is replenished to his/her smartphone.
- (ii) If two users v_1 and v_2 have the same energy consumption rate (i.e., $\rho_1 = \rho_2$) but different amounts of residual energy RE_1 and RE_2 (assuming that $RE_1 < RE_2$), then user v_1 is more satisfied than user v_2 if they both are charged with the same amount of energy.
- (iii) If two users v_1 and v_2 have the same residual lifetime (i.e., $\frac{RE_1}{\rho_1} = \frac{RE_2}{\rho_2}$) but different energy consumption rates ρ_1 and ρ_2 (assuming that $\rho_1 < \rho_2$), then user v_1 is more satisfied than user v_2 if they both are charged with the same amount of energy B. The rationale behind is that the prolonged operational time $\frac{B}{\rho_1}$ of user v_1 is longer than that $\frac{B}{\rho_2}$ of user v_2, i.e., $\frac{B}{\rho_1} > \frac{B}{\rho_2}$.

3.3 Problem Definitions

Since the number of to-be-charged smartphones usually is larger than the sum of the charging capacities of all chargers on trains, we consider scheduling the chargers to charge energy-critical smartphones for a given monitoring period consisting of T time slots, such that the overall charging satisfaction of smartphone users is maximized, where we say a smartphone is energy critical if its residual energy is below its defined energy threshold.

We distinguish our discussions into two different cases: offline charging scheduling and online charging scheduling. In the *offline charging scheduling*, the travel trajectory of each to-be-charged user v_j from time slot t_j^S that user v_j sends his/her charging request to time slot t_j^F that the user takes off the subway is given, and such knowledge can be obtained through tracking the train-taking history of user v_j or from the ticket information of user v_j when he/she bought the ticket at a subway station. In the *online charging scheduling*, we assume that the user travel trajectory information is not available, due to personal security and privacy concerns.

Given K chargers $C_1, C_2, \ldots, C_K$ deployed on subway trains with charging capacities $c_1, c_2, \ldots, c_K$, respectively, n to-be-charged users $v_1, v_2, \ldots, v_n$ with user v_j sending his/her charging request at time slot t_j^S, and the travel trajectories of these n users, *the offline charging satisfaction maximization problem* is allocating the K chargers to charge the n to-be-charged smartphones for a given monitoring period T, so that the accumulative charging satisfaction of all smartphone users is maximized, i.e., our objective is to

$$maximize \sum_{j=1}^{n} f_j(B_j), \tag{6}$$

subject to constraints (2), (3), (5), and following constraints

$$\sum_{j=1}^{n} x_{ijt} \leq c_i, \quad 1 \leq i \leq K, \ 1 \leq t \leq T, \tag{7}$$

$$\sum_{i=1}^{K} x_{ijt} \leq 1, \quad 1 \leq j \leq n, \ 1 \leq t \leq T, \tag{8}$$

$$x_{ijt} \in \{0,1\}, \ 1 \leq i \leq K, 1 \leq j \leq n, 1 \leq t \leq T, \tag{9}$$

$$x_{ijt} = 0, \quad if t < t_j^S, t \geq t_j^F, or d_{ijt} > D, \tag{10}$$

where constraint (7) ensures that each charger C_i can charge no more than c_i users at each time slot t, constraint (8) ensures that each user v_j can be charged by no more than one charger at each time slot t, constraint (9) ensures that each user v_j is either charged by a charger C_i at time slot t or not, and constraint (10) ensures that each user v_j will not be charged by any charger C_i when the user have not sent a charging request (i.e., $t < t_j^S$) or have taken off (i.e., $t \geq t_j^F$), or the distance d_{ijt} between them is longer than the maximum charging range D (i.e., $d_{ijt} > D$).

The online charging satisfaction maximization problem can be similarly defined as follows. The problem is to allocate the K chargers to charge the n to-be-charged smartphones for a given period T without the knowledge of the travel trajectory of each user in future, so that the sum of charging satisfaction of all smartphone users is maximized.

The travel trajectory information of each user can be used to significantly improve the user charging satisfaction, as users with plenty of charging opportunities can be distinguished from those with only a few charging opportunities. For example, assume that there are only two lifetime-critical users v_1 and v_2 within the charging range of a charger C_i and the residual lifetimes of v_1 and v_2 at some time slot t are 20 and 10 minutes, respectively. We further assume that user v_1 will take off the train very soon (e.g., 5 minutes later) while user v_2 will take a longer trip on the train. For this scenario, a charging allocation algorithm $\mathcal{A}$ without the user trajectory information may allocate charger C_i to charge only user v_2 since the residual lifetime of user v_2 is less than user v_1. As a result, user v_1 will miss his/her only charging opportunity while user v_2 will be charged into a large amount of energy and the residual lifetime of user v_2 is prolonged from 10 minutes to, for example, 4 hours. Then, the sum of charging satisfactions of users v_1 and v_2 when they take off the trains by algorithm $\mathcal{A}$ is $0 + 3.2874 \times \ln(4+1)$ $-3.2874 \times \ln(10/60+1) = 4.8$ by Eqs. (4) and (5). Contrarily, another algorithm $\mathcal{B}$ with the trajectory information may allocate charger C_i to charge user v_1 before he/she takes off the train and then assign charger C_i to charger user v_2 after user v_1 has taken off the train. As a result, the residual lifetimes of users v_1 and v_2 when they take off the trains are prolonged to, for example, $20 + 30 = 50$ minutes and $4 - \frac{30}{60} = 3.5$ hours, respectively. Then, the sum of charging satisfaction of the two users by algorithm $\mathcal{B}$ is $3.2874\times$ $(\ln(50/60+1) - \ln(20/60+1) + \ln(3.5+1) - \ln(10/60+1))$ $= 5.9 > 4.8$.

For both the offline and online charging satisfaction maximization problems, we assume that there is a server on each subway train and each charger can communicate with the server. Then the server has the global knowledge of user energy information and the distances between users and chargers. The server thus can execute a scheduling algorithm to find charging scheduling to chargers, the chargers then perform the charging. However, sometimes the server may not be installed at each subway train or chargers cannot communicate with the server directly. It thus is desirable to devise a scheduling algorithm that operates distributively. We thus define the *distributed charging satisfaction maximization problem* as to assign the K chargers to charge users for a period of T so that the accumulative charging satisfaction of users is maximized, under the constraint that every charger has only the knowledge of energy information and distances of the users within its maximum charging range and chargers cannot communicate with each other.

3.4 NP-Hardness

Theorem 1. *The offline charging satisfaction maximization problem is NP-hard.*

Proof. We show the NP-hardness of the problem by reducing the decision version of the bin packing problem to a special case of the problem of concern. Given k bins with each having a capacity S and a list of n items with sizes $a_1, a_2, \ldots, a_n$, respectively, the decision version of *the bin packing problem* is to decide whether there is a way to pack the n items into k bins such that the total size of items packed into each bin is no more than the bin capacity S [27].

Given k bins with each having capacity S and n items with sizes $a_1, a_2, \ldots, a_n$, we construct an instance of the offline charging satisfaction maximization problem as follows.

There is only $K = 1$ wireless charger with a charging capacity $c = 1$ and k to-be-charged users $v_1, v_2, \ldots, v_k$ on the subway, and the energy consumption rate ρ_j of each user v_j is one, i.e., $\rho_j = 1$. Also, the maximum amount $E_j^{max} - RE_j$ of energy that can be charged to each user v_j is S, i.e., $E_j^{max} - RE_j = S$, $1 \leq j \leq k$. Furthermore, there are $T = n$ time slots in the monitoring period and the amount of energy that can be charged to each user v_j at time slot t is a_t, $1 \leq t \leq T$. In addition, we assume that the charging satisfaction function $f_j(B_j)$ is a linear function of the amount of charged energy B_j, which is a special submodular function, i.e., $g(l_j) = l_j$ and $f_j(B_j) = g(\frac{RE_j+B_j}{\rho_j}) - g(\frac{RE_j}{\rho_j}) = B_j$ as $\rho_j = 1$. We can see that the offline charging satisfaction maximization problem in this special case is to allocate the charger to charge the k to-be-charged users for a period of $T = n$ time slots so that the accumulative amount of energy charged to the k users is maximized, subject to that the total amount of energy charged to each user v_j is no more than $E_j^{max} - RE_j = S$. Given the maximum amount of energy OPT of the offline charging satisfaction maximization problem in this special case, it can be seen that there is a solution to pack the n items to k bins such that the total weight of the items packed into each bin is no more than the bin capacity S if $OPT = \sum_{t=1}^{n} a_t$; and there is not such a solution if $OPT < \sum_{t=1}^{n} a_t$. Since the bin packing problem is NP-hard [27], the offline charging satisfaction maximization problem is NP-hard, too. □

4 Algorithm for the Offline Charging Satisfaction Maximization Problem

In this section, we propose a novel $\frac{1}{3}$-approximation algorithm for the offline charging satisfaction maximization problem. We also analyze the approximation ratio and time complexity of the proposed algorithm.

4.1 Algorithm

The basic idea behind the algorithm is as follows. It proceeds the charging allocation iteratively. Within each iteration, a pair $(C_{i^*}^{t^*}, v_{j^*}^{t^*})$ with the maximum increased satisfaction among all possible pairs is chosen, where charger C_{i^*} is allocated to charge user v_{j^*} at time slot t^*. In the following, we elaborate the approximation algorithm.

Given K chargers $C_1, C_2, \ldots, C_K$ with charging capacities $c_1, c_2, \ldots, c_K$, respectively, the n to-be-charged users $v_1, v_2, \ldots, v_n$ with user v_j sending a charging request at time slot t_j^S and taking off the subway at time slot t_j^F. Recall that the residual energy of user v_j is RE_j and its energy consumption rate is ρ_j. Also, the travel trajectory of the user is given. The algorithm proceeds as follows.

Let $\mathcal{C} = \{C_1, C_2, \ldots, C_K\}$ and $V = \{v_1, v_2, \ldots, v_n\}$. We first construct a bipartite graph $G_t = (\mathcal{C}_t, V_t, E_t)$ for each time slot t, where $\mathcal{C}_t$ is the set of chargers at time slot t (i.e., $\mathcal{C}_t = \mathcal{C}$), V_t is the set of users that have charging opportunities on the subway at time slot t (i.e., $V_t = \{v_j | v_j \in V, t_j^S \leq t < t_j^F\}$), where $t = 1, 2, \ldots, T$. For each charger $C_i^t \in \mathcal{C}_t$ and each user $v_j^t \in V_t$, there is an edge (C_i^t, v_j^t) in E_t if their Euclidean distance d_{ijt} at time slot t is no more than the maximum charging range D of charger C_i^t, i.e., $d_{ijt} \leq D$. Then, the reception power P_{ijt} of charger C_i^t charging user v_j^t at time slot t is calculated by Eq. (2), and the amount of energy B_{ijt} charged to user v_j is $B_{ijt} = P_{ijt} \times \delta$, where δ is the duration of every time slot.

We then allocate the K chargers to charge the n users for the given period T iteratively. Let re_j be the amount of residual energy of user v_j after allocating some chargers to charge the user. Also, let c_i^t be the maximum residual number of users that charger C_i^t can charge at time slot t. Initially, $re_j = RE_j$, where RE_j is the residual energy of user v_j before any charging, $1 \leq j \leq n$, $c_i^t = c_i$, $1 \leq i \leq K$, and $1 \leq t \leq T$. At each iteration, for each edge $(C_i^t, v_j^t) \in G_t$, recall that B_{ijt} is the amount of energy that can be charged to user v_j^t if charger C_i^t charges the user at time slot t, where $1 \leq t \leq T$. The amount of increased satisfaction of user v_j then is

$$\Delta(B_{ijt}) = g\left(\frac{re_j + B_{ijt}}{\rho_j}) - g(\frac{re_j}{\rho_j}\right), \tag{11}$$

by Eq. (5). We identify an edge $(C_{i^*}^{t^*}, v_{j^*}^{t^*})$ from the T graphs $G_1, G_2, \ldots, G_T$ such that the increased satisfaction of charging some user v_j^t by a charger C_t^i at time slot t is maximized, i.e., $(C_{i^*}^{t^*}, v_{j^*}^{t^*}) = \arg\max_{(C_i^t, v_j^t) \in G_1 \cup G_2 \cup \ldots \cup G_T} \{\Delta(B_{ijt})\}$. We then allocate charger C_{i^*} to charge user v_{j^*} at time slot t^*. We also increase the residual amount of energy re_{j^*} of user v_{j^*} by $B_{i^*j^*t^*}$ and reduce the maximum residual number of users $c_{i^*}^{t^*}$ that charger $C_{i^*}^{t^*}$ can charge at time slot t^* by one. Furthermore, we remove the incident edges of user node $v_{j^*}^{t^*}$ from graph G_{t^*} since user v_{j*} can be charged by no more than one charger at time slot t^*, and remove the incident edges of charger node $C_{i^*}^{t^*}$ from graph G_{t^*} if $c_{i^*}^{t^*}$ has been decreased to zero as the number of users allocated to charger $C_{i^*}^{t^*}$ at time slot t^* now reaches its charging capacity c_i. The approximation algorithm continues until no edges are left in any of the T graphs $G_1, G_2, \ldots, G_T$. We detail the algorithm in `Algorithm` 1.

Algorithm 1. `ApproAlg`

Input: K deployed chargers $C_1, C_2, \ldots, C_K$ with charging capacities $c_1, c_2, \ldots, c_K$, n to-be-charged smartphone users, the residual energy, energy consumption rate, and travel trajectory of each user, and a given period T.

Output: A charging allocation $\mathcal{A}$ that assigns the chargers to the users for the period T such that the sum of charging satisfaction of the users is maximized.

Construct a bipartite graph $G_t = (\mathcal{C}_t, V_t, E_t)$ for each time slot t, where $\mathcal{C}_t$ is the set of the K chargers, V_t is the set of the users at time slot t, there is an edge (C_i^t, v_j^t) in E_t if the distance between charger C_i^t and user v_j^t at time slot t is no more than D with $1 \leq t \leq T$;
For each edge (C_i^t, v_j^t) in the T graphs, compute the amount of energy B_{ijt} that can be charged to user v_j^t if allocating charger C_i^t to charge the user at time slot t by Eq. (2), where $1 \leq i \leq K$, $1 \leq j \leq n$, and $1 \leq t \leq T$;
$\mathcal{A} \leftarrow \emptyset$; /* the charging allocation */
$re_j \leftarrow RE_j, 1 \leq j \leq n$; /* residual energy of user v_j */
$c_i^t \leftarrow c_i, 1 \leq i \leq K, 1 \leq t \leq T$;
while there is an edge in any of the T graphs $G_1, G_2, \ldots, G_T$ **do**
  For each edge $(C_i^t, v_j^t) \in G_1 \cup G_2 \cup \cdots \cup G_T$, compute the increased satisfaction $\Delta(B_{ijt})$ of user v_j Eq. (11);
  Find an edge $(C_{i^*}^{t^*}, v_{j^*}^{t^*})$ such that $(C_{i^*}^{t^*}, v_{j^*}^{t^*}) = \arg\max_{(C_i^t, v_j^t)} \in G_1 \cup G_2 \cup \cdots \cup G_T \{\Delta(B_{ijt})\}$;
  $\mathcal{A} \leftarrow \mathcal{A} \cup \{(C_{i^*}, v_{j^*}, t^*)\}$;
  $re_{j^*} \leftarrow re_{j^*} + B_{i^*j^*t^*}$,
  $c_{i^*}^{t^*} \leftarrow c_{i^*}^{t^*} - 1$;
  Remove the incident edges of $v_{j^*}^{t^*}$ from graph G_{t^*};
  Remove the incident edges of charger node $C_{i^*}^{t^*}$ from graph G_{t^*} if $c_{i^*}^{t^*}$ decreases to zero;
end while
return $\mathcal{A}$.

4.2 Algorithm Analysis

We now analyze the approximation ratio of the proposed `Algorithm` 1. We start by introducing the definition of *matroids* as follows [6].

A matroid $\mathcal{M}$ is a pair of $(E, \mathcal{F})$ that meets three properties, where E is a finite set and $\mathcal{F}$ is a family of subsets of E, i.e., $\mathcal{F} \subseteq 2^E$. (i) $\emptyset \in \mathcal{F}$; (ii) the hereditary property: $S_1 \subseteq S_2$ and $S_2 \in \mathcal{F}$ imply that $S_1 \in \mathcal{F}$ for any two subsets $S_1, S_2 \subseteq E$; (iii) the independent set exchange property: for any two sets $S_1, S_2 \in \mathcal{F}$, if $|S_1| < |S_2|$, then, there is an element $e \in S_2 \setminus S_1$ such that $S_1 \cup \{e\} \in \mathcal{F}$.

We have the following important lemma, which is the cornerstone of the approximation ratio analysis.

Lemma 1. *[6] Let E be a finite set and $\mathcal{F}$ be a non-empty collection of subsets of E which has the property that $S_1 \subseteq S_2 \subseteq E$ and $S_2 \in \mathcal{F}$ imply that $S_1 \in \mathcal{F}$. Given a non-decreasing submodular function $z : 2^E \mapsto \mathcal{R}^+$, a greedy heuristic always delivers a $\frac{1}{k+1}$-approximate solution to the problem $\max_{S\subseteq E}\{z(S) : S \in \mathcal{F}\}$, assuming that $(E, \mathcal{F})$ is described by the intersection of k matroids, where k is a positive integer.*

We then show that the objective function $\sum_{j=1}^{n} f_j(B_j)$ is a submodular function by the following lemma.

Lemma 2. *Function $\sum_{j=1}^{n} f_j(B_j)$ is a non-decreasing submodular function, where $f_j(B_j) = g(\frac{RE_j+B_j}{\rho_j}) - g(\frac{RE_j}{\rho_j})$, $g(\cdot)$ is a given non-decreasing submodular function, RE_j is the residual energy of user v_j before charging, ρ_j is the average energy consumption rate, and B_j is the amount of energy charged to user v_j for the period of T.*

Proof. We only need to prove that $f_j(B_j)$ is a non-decreasing submodular function for each j with $1 \le j \le n$. Then, $\sum_{j=1}^{n} f_j(B_j)$ is a non-decreasing submodular function, as it is a non-negative, linear combination of submodular functions.

To this end, we show that function $f_j(B_j)$ meets the three properties of submodular functions (see Definition 1 in Section 3.2). Let $E = E_1 \cup E_2 \cup \cdots \cup E_T$. We first show that (i) $f_j(\emptyset) = 0$. In this case, we can see that the amount of energy charged to user v_j is zero, i.e., $B_j = 0$. Then, $f_j(B_j) = f_j(0) = 0$. We then show that function $f_j(B_j)$ meets property (ii) of submodular functions, i.e., $f_j(S_1) \le f_j(S_2)$ for any two charging allocations $S_1, S_2 \subseteq E$ with $S_1 \subseteq S_2$. We can see that the amount of energy charged to user v_j by charging allocation S_1 is no more than that by allocation S_2, i.e., $f_j(S_1) \le f_j(S_2)$. We finally prove that function $f_j(B_j)$ satisfies property (iii) of submodular functions, i.e., $f_j(S_1 \cup \{e\}) - f_j(S_1) \ge f_j(S_2 \cup \{e\}) - f_j(S_2)$ for any two charging allocations S_1 and S_2 with $S_1 \subseteq S_2 \subset E$ and $e = (C_i^t, v_j^t) \in E \setminus S_2$. Denote by $B_j^{S_1}$, $B_j^{S_1\cup\{e\}}$, $B_j^{S_2}$, and $B_j^{S_2\cup\{e\}}$ the amounts of energy charged to user v_j by charging allocations S_1, $S_1 \cup \{e\}$, S_2, $S_2 \cup \{e\}$, respectively. Since $S_1 \subseteq S_2$, we have $B_j^{S_1} \le B_j^{S_2}$. Then, we know that $g(\frac{RE_j+B_j^{S_1}}{\rho_j}) \le g(\frac{RE_j+B_j^{S_2}}{\rho_j})$ as $g(.)$ is a non-decreasing function. It can be seen that the increased replenished energy $B_j^{S_1\cup\{e\}} - B_j^{S_1}$ by charging user v_j with charger C_i at time slot t from charging allocations S_1 to $S_1 \cup \{e\}$ is no less than that $B_j^{S_2\cup\{e\}} - B_j^{S_2}$ from charging allocations S_2 to $S_2 \cup \{e\}$, i.e., $B_j^{S_1\cup\{e\}} - B_j^{S_1} \ge B_j^{S_2\cup\{e\}} - B_j^{S_2}$, since $B_j^{S_1} \le B_j^{S_2}$ and the amount of energy charged to user v_j in any charging allocation is no more than $E_j^{max} - RE_j$ by Eq. (3), where $e = (C_i^t, v_j^t)$, E_j^{max} and RE_j are the energy capacity and residual energy before charging of user v_j, respectively. In summary, we have

$$\begin{aligned} f_j(S_1 \cup \{e\}) - f_j(S_1) &= g\left(\frac{RE_j + B_j^{S_1\cup\{e\}}}{\rho_j}\right) - g\left(\frac{RE_j + B_j^{S_1}}{\rho_j}\right) \\ &= g\left(\frac{RE_j + B_j^{S_1}}{\rho_j} + \frac{B_j^{S_1\cup\{e\}} - B_j^{S_1}}{\rho_j}\right) - g\left(\frac{RE_j + B_j^{S_1}}{\rho_j}\right) \\ &\ge g\left(\frac{RE_j + B_j^{S_1}}{\rho_j} + \frac{B_j^{S_2\cup\{e\}} - B_j^{S_2}}{\rho_j}\right) - g\left(\frac{RE_j + B_j^{S_1}}{\rho_j}\right) \end{aligned} \tag{12}$$

$$\begin{aligned} &\ge g\left(\frac{RE_j + B_j^{S_2}}{\rho_j} + \frac{B_j^{S_2\cup\{e\}} - B_j^{S_2}}{\rho_j}\right) - g\left(\frac{RE_j + B_j^{S_2}}{\rho_j}\right) \\ &= g\left(\frac{RE_j + B_j^{S_2\cup\{e\}}}{\rho_j}\right) - g\left(\frac{RE_j + B_j^{S_2}}{\rho_j}\right) = f_j(S_2 \cup \{e\}) - f_j(S_2), \end{aligned} \tag{13}$$

where In Eq. (12) holds since $B_j^{S_1\cup\{e\}} - B_j^{S_1} \ge B_j^{S_2\cup\{e\}} - B_j^{S_2}$ and $g(.)$ is a non-decreasing function, and In Eq. (13) holds as $\frac{RE_j+B_j^{S_1}}{\rho_j} \le \frac{RE_j+B_j^{S_2}}{\rho_j}$ and $g(.)$ is a submodular function. Therefore, $\sum_{j=1}^{n} f_j(B_j)$ is a non-decreasing submodular function. The lemma then follows. □

We finally analyze the approximation ratio of `Algorithm` 1 by the following theorem.

Theorem 2. *There is a $\frac{1}{3}$-approximation algorithm for the offline charging satisfaction maximization problem, which takes $O(nT^2\log(nT) + KT)$ time, where n is the number of to-be-charged users, K is the number of deployed chargers, and T is the number of time slots in a given monitoring period.*

Proof. Since the objective function $\sum_{j=1}^{n} f_j(B_j)$ of the problem is a non-decreasing submodular function by Lemma 2, in the following, we show that constraints (7) and (8) can be represented by $k = 2$ matroids. Then, following Lemma 1, it can be seen that `Algorithm` 1 delivers a $\frac{1}{k+1} = \frac{1}{3}$-approximate solution.

Recall that there is a bipartite graph $G_t = (\mathcal{C}_t, V_t, E_t)$ for each time slot t with $1 \le t \le T$. Let $X = \mathcal{C}_1 \cup \mathcal{C}_2 \cup \ldots \cup \mathcal{C}_T$ and $Y = V_1 \cup V_2 \cup \ldots \cup V_T$. Recall that $E = E_1 \cup E_2 \cup \ldots \cup E_T$. Note that E is the set of the feasible charging allocations defined by constraints (9) and (10). We define a set system $\mathcal{M}_X = (E, \mathcal{F}_X)$ on the edge set E, where $\mathcal{F}_X$ is a family of subsets of E such that, for each edge set $S \in \mathcal{F}_X$ ($S \subseteq E$), the number of edges in S sharing the same endpoint C_i^t is no more than c_i for each charger node $C_i^t \in X$, and c_i is the maximum number of users that charger C_i can charge at time slot t. Similarly, we define another set system $\mathcal{M}_Y = (E, \mathcal{F}_Y)$, where $\mathcal{F}_Y$ is a family of subsets of E such that, for each edge set $S \in \mathcal{F}_Y$ ($S \subseteq E$), no two edges in S have the same endpoint in Y. Following the definitions of set systems $\mathcal{M}_X$ and $\mathcal{M}_Y$, it can be seen that constraints (7) and (8) are represented by $\mathcal{M}_X$ and $\mathcal{M}_Y$, respectively. In the following we only show that $\mathcal{M}_X$ is a matroid by meeting the three properties of matroids. The claim that $\mathcal{M}_Y$ is a matroid can be shown similarly, omitted.

(1) $\mathcal{M}_X = (E, \mathcal{F}_X)$ meets property (i) of a matroid that $\emptyset \in \mathcal{F}_X$. (2) Given any two sets $S_1, S_2 \subseteq E$, assume that

$S_1 \subseteq S_2$ and $S_2 \in \mathcal{F}_X$. Following the definition of $\mathcal{F}_X$ and the fact $S_2 \in \mathcal{F}_X$, it can be seen that the number of edges in S_2 sharing the same endpoint C_i^t is no more than c_i, for each charger node $C_i^t \in X$. The number of edges in S_1 sharing the same endpoint C_i^t then is no more than c_i due to $S_1 \subseteq S_2$. Thus, $S_1 \subseteq S_2$ and $S_2 \in \mathcal{F}_X$ imply that $S_1 \in \mathcal{F}_X$, meeting property (ii) of a matroid. (3) Given any two sets $S_1, S_2 \in \mathcal{F}_X$, assume that $|S_1| < |S_2|$. Denote by $N_S(C_i^t)$ the set of edges in S sharing endpoint $C_i^t \in X$ for any set $S \in \mathcal{F}_X$. We note that $\{N_S(C_i^t)\}_{C_i^t \in X}$ is a partitioning of set S, since $\bigcup_{C_i^t \in X} N_S(C_i^t) = S$ and $N_S(C_i^t) \cap N_S(C_j^{t'}) = \emptyset$ for $C_i^t \neq C_j^{t'}$, due to that each of the T graphs is a bipartite graph and no edge between nodes C_i^t and $C_j^{t'}$. As $S_1, S_2 \in \mathcal{F}_X$ and $|S_1| < |S_2|$, there must be a node $C_i^t \in X$ such that the number of edges in S_1 sharing endpoint C_i^t is strictly less than that in S_2, i.e., $|N_{S_1}(C_i^t)| < |N_{S_2}(C_i^t)|$. Otherwise, $|N_{S_1}(C_i^t)| \geq |N_{S_2}(C_i^t)|$ for each node $C_i^t \in X$. Then, $|S_1| = \sum_{C_i^t \in X} |N_{S_1}(C_i^t)| \geq \sum_{C_i^t \in X} |N_{S_2}(C_i^t)| = |S_2|$, which contradicts the assumption that $|S_1| < |S_2|$. Since $|N_{S_1}(C_i^t)| < |N_{S_2}(C_i^t)| \leq c_i$, there is an edge (C_i^t, v_j^t) in $N_{S_2}(C_i^t) \setminus N_{S_1}(C_i^t)$. We then add this edge to S_1. It is obvious that $|N_{S_1}(C_i^t) \cup \{(C_i^t, v_j^t)\}| \leq c_i$. Also, note that adding edge (C_i^t, v_j^t) in S_1 does not increase the number of edges that share the same endpoint $C_j^{t'}$ for each node $C_j^{t'} \in X$ with $C_j^{t'} \neq C_i^t$, as the endpoint v_j^t of edge (C_i^t, v_j^t) does not belong to set X. Therefore, set $S_1 \cup \{(C_i^t, v_j^t)\} \in \mathcal{F}_X$, meeting property (iii) of a matroid. Therefore, $\mathcal{M}_X = (E, \mathcal{F}_X)$ is a matroid. It can be seen that the offline charging satisfaction maximization problem can be cast as a non-decreasing submodular function maximization problem, subject to the constraints of $k = 2$ matroids: $\mathcal{M}_X$ and $\mathcal{M}_y$, `Algorithm` 1 thus delivers a $\frac{1}{k+1} = \frac{1}{3}$-approximate solution by Lemma 1.

We finally analyze the time complexity of `Algorithm` 1. We assume that the number of chargers on a subway that can charge each user at each time slot is bounded by a constant, since chargers usually are not densely deployed. Then, the number of edges in the T graphs $G_1, G_2, \ldots, G_T$ is $\sum_{t=1}^{T} O(|V_t|) \times O(1) = \sum_{t=1}^{T} O(|V|) = O(nT)$ and we can construct the T graphs in time $\sum_{t=1}^{T}(K + |V_t|) + O(nT) = O((K+n)T)$. The time complexity of `Algorithm` 1 depends on the data structure we adopt in its implementation. To quickly find the charging allocation $\mathcal{A}$, we here adopt the priority queue – a max-heap [4] in the **while** loop of `Algorithm` 1. Before the **while** loop, we associate each edge (C_i^t, v_j^t) in the T graphs $G_1, G_2, \ldots, G_T$ with a key Δ_{ijt}, which is the increased overall satisfaction by allocating charger C_i to charge user v_j at time slot t, i.e., $\Delta_{ijt} = g\left(\frac{RE_j + B_{ijt}}{\rho_j}\right) - g\left(\frac{RE_j}{\rho_j}\right)$ by Eq. (5). Following [4], we can build a max-heap H in time $O(nT)$ and the heap includes all edges in the T graphs $G_1, G_2, \ldots, G_T$. For each edge (C_i^t, v_j^t) in heap H, there is an associated boolean variable b_{ijt} for it, which indicates whether the edge has been removed from the heap or not. Initially, b_{ijt} = '*false*'. Within each iteration of the **while** loop, we can find the edge $(C_{i^*}^{t^*}, v_{j^*}^{t^*})$ such that $(C_{i^*}^{t^*}, v_{j^*}^{t^*}) = \arg\max_{(C_i^t, v_j^t) \in G_1 \cup G_2 \cup \cdots \cup G_T} \{\Delta(B_{ijt})\}$ in time $O(\log(nT))$. If variable $b_{i^*j^*t^*}$ indicates that edge $(C_{i^*}^{t^*}, v_{j^*}^{t^*})$ has already been removed (i.e., $b_{i^*j^*t^*}$ = '*true*') or the number of users allocated to charger $C_{i^*}^{t^*}$ in the charging allocation $\mathcal{A}$ at time slot t^* has already reached to its charging capacity c_{i^*}, we simply remove the edge from heap H. Otherwise, we add an allocation (C_{i^*}, v_{j^*}, t^*) to $\mathcal{A}$. Then, we can remove the incident edges of user node $v_{j^*}^{t^*}$ in heap H in time $O(1)$ by assigning the boolean variables b_{ijt}s of these edges '*true*'. Note that after we have allocated charger C_{i^*} to charge user v_{j^*} at time slot t^*, the increased overall satisfaction Δ_{ij^*t} by allocating a charger C_i to charge user v_{j^*} at time slot $t \in \{1, 2, \ldots, T\} \setminus \{t^*\}$ decreases, since the amount of residual energy of user v_{j^*} has been increased by $B_{i^*j^*t^*}$ due to the allocation (C_{i^*}, v_{j^*}, t^*). Assume that re_{j^*} is the residual energy of user v_{j^*} after the allocation. Following Eq. (5), the value of Δ_{ij^*t} is updated by $\Delta_{ij^*t} = g(\frac{re_{j^*} + B_{ij^*t}}{\rho_{j^*}}) - g(\frac{re_{j^*}}{\rho_{j^*}})$ for each edge $(C_i^t, v_{j^*}^t)$, where $1 \leq i \leq K$ and $t \in \{1, 2, \ldots, T\} \setminus \{t^*\}$. Therefore, we have $O(T)$ such updates. Since the keys Δ_{ij^*t}s of $O(T)$ edges $(C_i^t, v_{j^*}^t)$s in heap H have been decreased, we can maintain the max-heap property of H in time $O(T\log(nT))$. Therefore, the time complexity of `Algorithm` 1 is $O((K+n)T) + O(nT) + O(nT) * O(T\log(nT)) = O(nT^2\log(nT) + KT)$. □

5 ALGORITHM FOR THE ONLINE CHARGING SATISFACTION MAXIMIZATION PROBLEM

In the previous section, we proposed an approximation algorithm for the offline charging satisfaction maximization problem, assuming that the travel trajectory of each user is given. However, such knowledge sometimes may not be available due to personal security and privacy concerns. In this section, we study the online charging satisfaction maximization problem without the knowledge of user trajectories, by developing a heuristic algorithm for it.

5.1 Online Algorithm

The basic idea behind the algorithm is that it finds a charging allocation with only the residual energy information of to-be-charged users provided so that the sum of the charging satisfaction of users at each time slot is maximized. To this end, we reduce the problem to the maximum weight matching problem, and an exact solution to the latter in turn returns a feasible solution to the former.

The online algorithm is invoked at the beginning of every time slot for the entire period T and a charging allocation $\mathcal{A}_t$ will be delivered by the algorithm at every time slot t with $1 \leq t \leq T$. As a result, the union of charging allocations $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_T$ forms a feasible solution to the problem. Specifically, assume that the charging allocations $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_{t-1}$ for the previous $t-1$ time slots have been obtained, we now find a charging allocation $\mathcal{A}_t$ at time slot t to maximize the sum of charging satisfaction of the users. Recall that V_t is the set of users that have charging opportunities at time slot t, i.e., $V_t = \{v_j | v_j \in V, t_j^S \leq t < t_j^F\}$. Denote by re_j^t

the amount of residual energy of user v_j at the beginning of time slot t. We consider the charging satisfaction maximization problem at time slot t, that is, how to allocate the K chargers to charge the users in V_t, so that the sum of charging satisfaction of the users at this time slot is maximized.

We first construct a bipartite graph $G_t = (\mathcal{C}, V_t, E_t; w_t)$, where $\mathcal{C}$ is the set of the K chargers, there is an edge (C_i, v_j) in edge set E_t if the Euclidean distance between charger C_i and user v_j is no more than the maximum charging range D. For each edge $(C_i, v_j) \in E_t$, its weight $w_t(C_i, v_j)$ is the net satisfaction by allocating charger C_i to charge user v_j at time slot t, i.e., $w_t(C_i, v_j) = g(\frac{re_j^t + B_{ijt}}{\rho_j}) - g(\frac{re_j^t}{\rho_j})$, where $\frac{re_j^t}{\rho_j}$ and $\frac{re_j^t + B_{ijt}}{\rho_j}$ are the residual lifetimes of user v_j before and after charging v_j at time slot t, respectively, re_j^t is the residual energy of the user at the beginning of time slot t, B_{ijt} is the amount of energy that can be charged to the user, and ρ_j is its energy consumption rate.

We then construct another bipartite graph $G'_t = (\mathcal{C}', V_t, E'_t; w'_t)$ from $G_t = (\mathcal{C}, V_t, E_t; w_t)$ as follows. For each charger C_i in $\mathcal{C}$, there are c_i 'virtual charger' nodes $C_{i,1}, C_{i,2}, \ldots, C_{i,c_i}$ in $\mathcal{C}'$, which have the same location as charger C_i in graph G_t. Thus, the charging capacity of each 'virtual charger' is exactly one. Also, for each edge (C_i, v_j) in graph G_t, there are c_i edges $(C_{i,1}, v_j), (C_{i,2}, v_j), \ldots, (C_{i,c_i}, v_j)$ in edge set E'_t, and each of these c_i edges has the same weight as the original edge (C_i, v_j), i.e., $w'_t(C_{i,k}, v_j) = w_t(C_i, v_j)$ with $1 \le k \le c_i$.

Consider the maximum weight matching problem in $G'_t = (\mathcal{C}', V_t, E'_t; w'_t)$, which aims to find a matching M such that the weighted sum of edges in M is maximized. Given a maximum weight matching M in G'_t, a charging allocation $\mathcal{A}_t$ for the online charging satisfaction maximization problem can then be derived, by adding (C_i, v_j, t) to $\mathcal{A}_t$ for each matched edge $(C_{i,k}, v_j)$ in matching M. The detailed algorithm is given in `Algorithm 2`.

Algorithm 2. `OnlineAlg`

Input: K deployed chargers $C_1, C_2, \ldots, C_K$ with charging capacities $c_1, c_2, \ldots, c_K$, n_t to-be-charged users $v_1, v_2, \ldots, v_{n_t}$ with their amounts of residual energy re_j^t and energy consumption rates ρ_j at time slot t

Output: A charging allocation $\mathcal{A}_t$ assigning the chargers to charge users at time slot t so that the sum of user charging satisfactions is maximized

1: Construct a bipartite graph $G_t = (\mathcal{C}, V_t, E_t; w_t)$, where $\mathcal{C} = \{C_1, C_2, \ldots, C_K\}$, $V_t = \{v_1, v_2, \ldots, v_{n_t}\}$, there is an edge (C_i, v_j) in E_t if $d_{ijt} \le D$ and $w_t(C_i, v_j) = g(\frac{re_j^t + B_{ijt}}{\rho_j}) - g(\frac{re_j^t}{\rho_j})$ for each edge $(C_i, v_j) \in E_t$;
2: Construct another graph $G'_t = (\mathcal{C}', V_t, E'_t; w'_t)$ from graph $G_t = (\mathcal{C}, V_t, E_t; w_t)$, where there are c_i virtual charger nodes $C_{i,1}, C_{i,2}, \ldots, C_{i,c_i}$ in $\mathcal{C}'$ for charger C_i in $\mathcal{C}$, there are c_i edges $(C_{i,1}, v_j), (C_{i,2}, v_j), \ldots, (C_{i,c_i}, v_j)$ in E'_t for each edge $(C_i, v_j) \in E_t$, $w'_t(C_{i,k}, v_j) = w_t(C_i, v_j)$, and $1 \le k \le c_i$;
3: Find a maximum weight matching M in graph G'_t;
4: $\mathcal{A}_t \leftarrow \emptyset$; /* the charging allocation for time slot t*/
5: For each matched edge $(C_{i,k}, v_j) \in M$, add (C_i, v_j, t) to $\mathcal{A}_t$;
6: **return** $\mathcal{A}_t$.

5.2 Algorithm Analysis

We now analyze the time complexity of `Algorithm 2` through the following theorem.

Theorem 3. *There is an algorithm for the online charging satisfaction maximization problem, which takes $O((n+K)^2 \log (n+K))$ time for charging scheduling at each time slot t with $1 \le t \le T$, where n is the number of to-be-charged users and K is the number of chargers.*

Proof. We first show that `Algorithm 2` delivers a feasible solution $\mathcal{A}_t$. Note that there are c_i virtual charger nodes $C_{i,1}, C_{i,2}, \ldots, C_{i,c_i}$ in graph G'_t for each charger C_i in graph G_t and the sets of user nodes in graphs G'_t and G_t are the same. Since the charging allocation $\mathcal{A}_t$ is constructed from the maximum weight matching M in graph G'_t, we know that each user in allocation $\mathcal{A}_t$ is assigned to only one charger and the number of users assigned to each charger C_i in $\mathcal{A}_t$ is no more than its charging capacity c_i. Therefore, `Algorithm 2` delivers a feasible solution $\mathcal{A}_t$ at each time slot t with $1 \le t \le T$.

We then analyze the time complexity of `Algorithm 2`, which is dominated by finding the matching M in graph G'_t. Let $n_t = |V_t| + |\mathcal{C}'|$ and $m_t = |E'_t|$ be the number of nodes and edges in graph G'_t, respectively. `Algorithm 2` can find the matching M in time $O(m_t n_t + n_t^2 \log n_t) = O(n_t^2 \log n_t) = O((n + Kc_{max})^2 \log (n + Kc_{max})) = O((n + K)^2 \log (n+K))$ by applying an algorithm in [7] and noticing that $O(m_t) = O(n_t)$ and $n_t = O(n + Kc_{max})$, where $c_{max} = \max_{i=1}^{K}\{c_i\} = O(1)$. □

6 Algorithm for the Distributed Charging Satisfaction Maximization Problem

So far we assumed that there is a server on each subway train and each charger can communicate with the server. As a result, the server can execute a scheduling algorithm to find a solution of charging allocations to the chargers. However, there may not be such a server on the board or the chargers cannot communicate with the server. It thus is desirable to devise a scheduling algorithm that operates in a distributed way. In this section, we propose a novel distributed algorithm for the problem.

6.1 Distributed Algorithm

The distributed algorithm finds a charging allocation $\mathcal{A}_t$ at the beginning of each time slot t with $1 \le t \le T$. Recall that V_t is the set of users that have charging opportunities at time slot t, i.e., $V_t = \{v_j | v_j \in V, t_j^S \le t < t_j^F\}$. Denote by re_j^t the amount of residual energy of user $v_j \in V_t$ at the beginning of time slot t. Also, let $N(C_i)$ be the set of users within the maximum charging range D of each charger C_i, i.e., $N(C_i) = \{v_j | v_j \in V_t, d_{ijt} \le D\}$, and $N(v_j)$ be the set of chargers that can charge user v_j, i.e., $N(v_j) = \{C_i | C_i \in \mathcal{C}, d_{ijt} \le D\}$.

At the beginning of every time slot t, the distributed algorithm proceeds the charging allocation iteratively. Within each iteration, it finds charging allocation for a subset of users in V_t. Every charger C_i first calculates the net satisfaction $\Delta(B_{ijt})$ of every user $v_j \in N(C_i)$ if it is allocated to charge user v_j at time slot t by Eq. (11). Charger C_i then chooses the top $k = \min\{c_i, |N(C_i)|\}$ users $v_1, v_2, \ldots, v_k$ in $N(C_i)$ by their net satisfactions, and sends each of them a 'ChargingPermission' message, where c_i is its charging capacity and $|N(C_i)|$ is the number of users within its maximum charging range. Every user $v_j \in V_t$ may or may not receive 'ChargingPermission' messages from chargers, which is distinguished into three cases: (i) user v_j does not receive any 'ChargingPermission' message, no action is needed; (ii) user v_j receives only one message from a charger $C_i \in N(v_j)$, the user sends a 'ChargingAcknowledgement' message to charger C_i; and (iii) user v_j receives multiple messages from the chargers in $N(v_j)$, the user then sends a 'ChargingAcknowledgement' message to a charger C_i with the maximum net satisfaction among the chargers, and sends a 'ChargingRejection' message to each of the chargers in $N(v_j) \setminus \{C_i\}$, as user v_j should be charged by no more than one charger at time slot t. Assume that every charger C_i receives k_{ack} 'ChargingAcknowledgement' messages and k_{rej} 'ChargingRejection' messages from the users in $N(C_i)$. Charger C_i then decreases its charging capacity by k_{ack} and removes the users who sent messages to it from $N(C_i)$, since these users have already been chosen to be charged by some chargers in this iteration and will not be considered in the next iterations. The distributed algorithm continues until the charging capacity of each charger C_i decreases to zero or there are no users left within its maximum charging range, i.e., $N(C_i) = \emptyset$. The detailed distributed algorithms at chargers and users are given in `Algorithms` 3 and 4, respectively.

Algorithm 3. `DistributedAlg (Each Charger` C_i `at Every Time Slot` t`)`

$V_c^t \leftarrow \emptyset$; /* the set of to-be-charged users at time slot t*/
Calculate the net satisfaction $\Delta(B_{ijt})$ of every user $v_j \in N(C_i)$ by Eq. (11);
$c_i^t \leftarrow c_i$; /*the residual charging capacity of charger C_i*/
while $c_i^t > 0$ and $N(C_i) \neq \emptyset$ **do**
  Choose the top $k = \min\{c_i^t, |N(C_i)|\}$ users $v_1, v_2, \ldots, v_k$ in $N(C_i)$ by their net satisfactions;
  Send each of the k users a 'ChargingPermission' message;
  for each user v_j of the k users **do**
    if charger C_i receives a 'ChargingAcknowledgement' message from user v_j **then**
      $V_c^t \leftarrow V_c^t \cup \{v_j\}$; /* user v_j will be charged */
      $c_i^t \leftarrow c_i^t - 1$; /* decrease its charging capacity */
    end if
  end for
  Remove the users who sent their messages to charger C_i from $N(C_i)$;
end while
Perform energy charging to users in V_c^t.

6.2 Algorithm Analysis

Theorem 4. *There is an algorithm for the distributed charging satisfaction maximization problem, it takes $O(n_i \log K)$ time and $O(K \log K)$ messages for charging scheduling at each time slot t with $1 \leq t \leq T$, where n_i is the number of users within the maximum charging range of charger C_i and K is the number of chargers.*

Proof. We first analyze the time complexity of the distributed algorithm. Notice that the charging capacity c_i of each charger C_i is bounded by a constant in the real life. Then, the execution of each **while** loop in `Algorithm` 3 takes $O(n_i + c_i) = O(n_i)$ time, assuming that the data structure of the maximum heap is adopted [4], where $n_i = |N(C_i)|$. We calculate how many **while** loops that `Algorithm` 3 will perform as follows. Denote by n_{max} the maximum number of chargers that can charge a user, i.e., $n_{max} = \max_{v_j \in V_t}\{|N(v_j)|\}$. Then, a user v_j will receive no more than n_{max} 'ChargingPermission' messages from chargers and will reply exact one 'ChargingAcknowledgement' message to one of the chargers within a **while** loop if user v_j receives some 'ChargingPermission' messages. Since the K chargers on subway will send $\sum_{i=1}^{K} \min\{c_i^t, N(C_i)\}$ 'ChargingPermission' messages to users, there will be no less than $\frac{\sum_{i=1}^{K} \min\{c_i^t, N(C_i)\}}{n_{max}}$ 'ChargingAcknowledgement' messages received from the chosen users. As a result, there are no more than $\sum_{i=1}^{K} \min\{c_i^t, N(C_i)\} \times (1 - \frac{1}{n_{max}})$ to-be-allocated users left after every **while** loop. If $n_{max} = 1$, `Algorithm` 3 then performs only one **while** loop. Otherwise ($n_{max} > 1$), `Algorithm` 3 performs the **while** loop $O(\log_{(1-\frac{1}{n_{max}})} \frac{1}{\sum_{i=1}^{K} \min\{c_i, N(C_i)\}}) = O(\log_{\frac{n_{max}}{n_{max}-1}} \sum_{i=1}^{K} c_i) = O(\log K)$ times, where $\sum_{i=1}^{K} c_i = O(K)$ and the value of n_{max} is a constant in the real life, since it is unlikely that there are many chargers densely deployed at a location. Therefore, the time complexity of the distributed algorithm is $O(n_i) \times O(\log K) = O(n_i \log K)$.

We then analyze the message complexity of the distributed algorithm. Within every **while** loop of `Algorithm` 3, the K chargers and the users in V_t will send $O(\sum_{i=1}^{K} \min\{c_i, N(C_i)\}) = O(K)$ messages. Thus, the message complexity of the algorithm is $O(K) \times O(\log K) = O(K \log K)$ at every time slot. □

7 Performance Evaluation

In this section, we evaluate the performance of the proposed algorithms, using a real dataset.

7.1 Experimental Settings

We consider the subway network in San Francisco in the States, which consists of 6 subway lines and 45 stations [1]. The running timetable of the subway trains is obtained from [1], which includes the arrival times of the stations of each train. For simplicity, we assume that the length, width, and height of each train are 100, 3.2, and 3.2 meters, respectively, and there are 300 seats along the two sides of each train [2]. We divide one day into equal length time slots with each time slot lasting $\delta = 1$ minute. We assume that the maximum charging range of each wireless charger is 2.7 meters [14]. We also assume that the charging capacity c_i of each charger C_i is 1 and the output power P_i^o of charger C_i is 10 watts. We deploy 41 wireless chargers along the two sides of each train, where Fig. 2 illustrates such a deployment in a two-dimensional space and the height of each deployed charger is 0.4 meters (at the position below seats on the train).

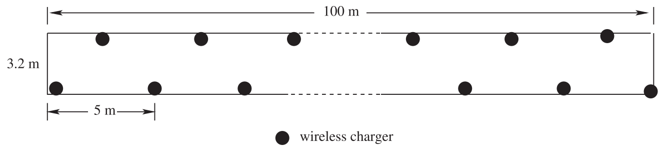


Fig. 2. The deployment of wireless chargers on each train.

Algorithm 4. `DistributedAlg` (Each User v_j at Every Time Slot t)

Assume that user v_j receives p 'ChargingPermission' messages from p chargers $C_1, C_2, \ldots, C_p$, respectively, where $C_i \in N(v_j)$;
Select the charger C_i with the maximum net satisfaction among the p chargers and reply it with a 'Charging-Acknowledgement' message;
Reply each of the chargers in $N(v_j) \setminus \{C_i\}$ with a 'Charging-Rejection' message.

We adopt a real subway-taking dataset of San Francisco in a weekday of November 2014, which specifies the number of users between each pair of stations [23]. As a result, the total number of users n_c in a day is 4,24,763. Also, we generate the boarding time on trains for each user by referring to the 2008 station profile study of San Francisco subway [23]. In addition, we assume that a user will randomly take a vacant seat if there are vacant seats available in the train. Otherwise, the user will randomly stand on the train.

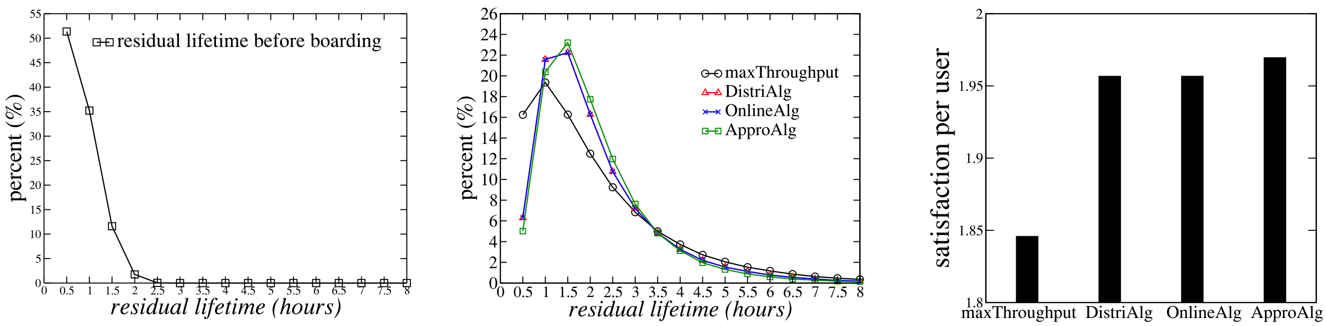
The battery capacity E_j^{max} of each user v_j's smartphone is randomly chosen from $20\ kJ\ (\approx 3.7\ V \times 1,500\ \text{mAh})$ to $40\ \text{kJ}(\approx 3.7\ \text{V} \times 3,000\ \text{mAh})$. Also, the energy consumption rate ρ_j of user v_j is randomly drawn from an interval $[0.5\ \text{W}, 1\ \text{W}]$. As a result, the lifetime of a fully charged smartphone can last from 5.5 ($\approx \frac{20\text{kJ}}{1\ \text{W}\times 3600\ \text{S}}$) hours to 22 ($\approx \frac{40\ \text{kJ}}{0.5\ \text{W}\times 3600\ \text{S}}$) hours. Furthermore, a fraction number α of users request to be charged (i.e., the number of to-be-charged users is $\alpha \times n_c$), $0 \leq \alpha \leq 1$. The residual energy RE_j of each to-be-charged user v_j when issuing a charging request is randomly chosen from an interval $[0, \beta \times E_j^{max}]$, where $0 \leq \beta \leq 1$.

In addition to the three proposed algorithms `ApproAlg`, `OnlineAlg`, and `DistriAlg`, we also implement another charging allocation algorithm `maxThroughput`, which finds charging allocations such that the *sum of amounts of energy* charged to users is maximized. Similar to algorithm `OnlineAlg`, algorithm `maxThroughput` finds a charging allocation at each time slot t (i.e., in an online way). Unlike algorithm `OnlineAlg` that the weight of each edge (C_i, v_j) is the net satisfaction by allocating charger C_i to charge user v_j at time slot t, the weight of edge (C_i, v_j) in algorithm `maxThroughput` is the *amount of energy* charged to user v_j.

7.2 Performance Evaluation of Different Algorithms

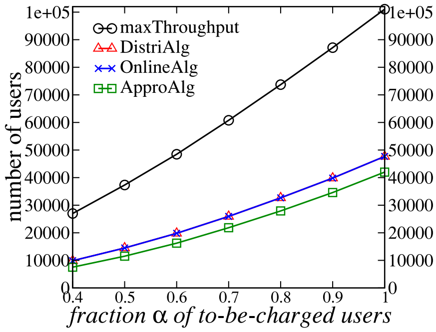
In the following we evaluate the performance of the three proposed algorithms `ApproAlg`, `OnlineAlg`, and `DistriAlg` against the benchmark algorithm `maxThroughput`, assuming half users on trains require to be charged, i.e., $\alpha = 0.5$. Fig. 3a plots the residual lifetime distribution of to-be-charged users when they send charging requests, from which it can be seen that most users have short residual lifetimes. For example, more than 50 percent of to-be-charged users have residual lifetime less than a half hour.

Fig. 3b demonstrates the residual lifetime distribution of users when they take off trains, from which it can be seen that the percentages of users with residual lifetime less than a half hour drop from 50 percent (see Fig. 3a) to 16, 6.3, 6.3, and 5 percent in the charging allocations delivered by algorithms `maxThroughput`, `DistriAlg`, `OnlineAlg`, and `ApproAlg`, respectively. As a result, algorithm `maxThroughput` can identify only a proportion of 68 percent ($= \frac{50-16}{50}$) lifetime-critical users, while algorithms `DistriAlg`, `OnlineAlg`, and `ApproAlg` can identify most energy-critical users, which are as high as 87.4 percent ($= \frac{50-6.3}{50}$), 87.4 percent, and 90 percent ($= \frac{50-5}{50}$), respectively. The rationale behind is that algorithm `maxThroughput` finds charging allocations only by the amounts of energy charged to users, it thus fails to identify energy-critical users. Unlike algorithm `maxThroughput`, algorithms `DistriAlg`, `OnlineAlg` and `ApproAlg` are able to identify energy-critical users, since the net satisfaction gain by charging these users are significantly larger than that by charging those with long residual lifetimes. On the other hand, the number of users with energy critical residual lifetimes by algorithm `ApproAlg` is less than that by algorithms `DistriAlg` and `OnlineAlg`, since the latter two algorithms find charging allocations without the knowledge of the travel trajectory of each user as the one by algorithm `ApproAlg`. Thus, algorithm `ApproAlg` can distinguish users with many charging opportunities from the users with only a few charging opportunities. Fig. 3b also shows that the number of users with residual lifetimes between 1 hour and 3 hours delivered

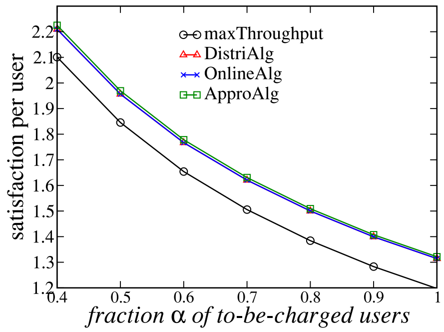


(a) residual lifetime distribution of to-be-charged users when boarding the trains.
(b) residual lifetime distribution of to-be-charged users when taking off the trains.
(c) the satisfaction per user delivered by different algorithms

Fig. 3. Performance of algorithms `maxThroughput`, `DistriAlg`, `OnlineAlg`, and `ApproAlg` when $\alpha = 0.5$ and $\beta = 0.1$.



(a) the number of users with residual lifetime less than a half hour when taking off the trains.



(b) satisfaction per user delivered by the four algorithms

Fig. 4. Performance of algorithms `maxThroughput`, `DistriAlg`, `OnlineAlg`, and `ApproAlg` by varying α from 0.4 to 1 when β is fixed at 0.1.

by the three algorithms `DistriAlg`, `OnlineAlg`, and `ApproAlg` are much larger than that by algorithm `maxThroughput`. In contrast, the numbers of users with residual lifetimes longer than 3 hours by the three algorithms are slightly less than that by algorithm `maxThroughput`, since the net satisfaction gain by charging the users with residual lifetime longer than 3 hours is marginal in the three algorithms.

Fig. 3c plots the charging satisfaction performance delivered by the four mentioned algorithms, from which it can be seen that the charging satisfaction per user delivered by algorithms `DistriAlg`, `OnlineAlg` and `ApproAlg` are around 6, 6, and 6.7 percent higher than that by algorithm `maxThroughput`, and the satisfaction delivered by algorithm `ApproAlg` is the highest one among them. Also, we can see from Fig. 3c that the performances of algorithms `DistriAlg` and `OnlineAlg` are almost identical. The rationale behind is that there are only a limited number chargers that can charge a smartphone user on a subway train. Then, the solutions found by the centralized algorithm (i.e., algorithm `OnlineAlg`) and the distributed algorithm (i.e., algorithm `DistriAlg`) are close to each other.

7.3 The Impact of the Number of to-be-Charged Users

We then study the impact of the number of to-be-charged users on algorithm performance, by varying α from 0.4 to 1. Fig. 4a plots the number of users with residual lifetimes less than a half hour when they take off trains by algorithms `maxThroughput`, `DistriAlg`, `OnlineAlg`, and `ApproAlg`, respectively, from which it can be seen that the numbers of users in the charging allocations delivered by algorithms `DistriAlg`, `OnlineAlg`, and `ApproAlg` are much less than that by algorithm `maxThroughput`. Furthermore, the number by algorithm `ApproAlg` always is the smallest one, which is only 88 percent of numbers of users by algorithm `OnlineAlg` and 41.5 percent of numbers of users by algorithm `maxThroughput` when all users request to be charged (i.e., $\alpha = 1$). The reason why algorithm `ApproAlg` outperforms algorithm `OnlineAlg` is that the former has the knowledge of user trajectories and more energy-critical users can be charged on time.

Fig. 4b shows that the satisfaction per user by algorithms `maxThroughput`, `DistriAlg`, `OnlineAlg`, and `ApproAlg` decreases with the increase of α, since every to-be-charged user will have less charging opportunities if there are more to-be-charged users on subway trains (i.e., a larger α). Also, the satisfaction by algorithm `ApproAlg` always is the highest one. Note that although the satisfaction per user delivered by algorithm `ApproAlg` is only slightly higher than that by algorithm `OnlineAlg` (about 0.65 percent), the charging allocations found by algorithm `ApproAlg` are much better than that by algorithm `OnlineAlg`, since there are much less numbers of users with energy critical residual lifetime by algorithm `ApproAlg` when users take off trains, which has already been shown in Fig. 4a.

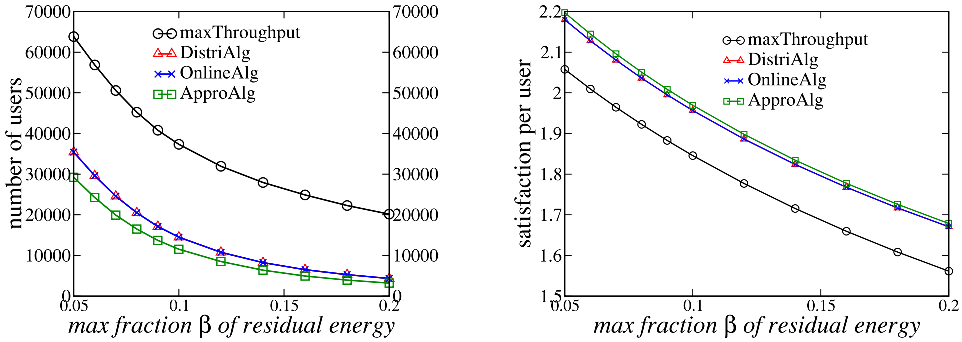
7.4 The Impact of Residual Energy Before Charging

We finally investigate the impact of residual energy of to-be-charged users when they take on trains, by varying the maximum fraction β of residual energy of users from 0.05 to 0.2, where the residual energy RE_j of user v_j before charging is randomly chosen from an interval $[0, \beta \times E_j^{max}]$ and E_j^{max} is the battery capacity of the smartphone of the user. Fig. 5a plots the number of users with residual lifetimes less than a half hour when they take off trains, from which it can be seen that the number of users by each of the four mentioned algorithms decreases with the increase of β, since there are more amounts of energy in the smartphones of these users before charging with a larger β. Also, the numbers of users by algorithms `DistriAlg`, `OnlineAlg`, and `ApproAlg` decrease to less than 5,000 while the number of users by algorithm `maxThroughput` is still more than 20,000 when $\beta = 0.2$. Again, the number of users by algorithm `ApproAlg` is the smallest one, which is around from 73.8 to 82.5 percent of that by algorithm `OnlineAlg` and even is about from 15.8 to 45.8 percent of that by algorithm `maxThroughput`.

Fig. 5b implies that the satisfaction per user by each of the four algorithms decreases with the increase of the value of β. The rationale behind is that a user is less satisfied for charging an amount of energy if the user has more energy in his/her smartphone before charging. Fig. 5b also shows the satisfactions per user by algorithm `ApproAlg` and `OnlineAlg` are around 6.4 and 7.1 percent higher than that by algorithm `maxThroughput`, respectively.

8 Conclusion

In this paper, we considered the use of wireless energy chargers installed on subway trains to charge energy-critical smartphones of users, through wireless energy transfer



(a) the number of users with residual lifetime less than a half hour when taking off the trains.

(b) satisfaction per user delivered by the four algorithms

Fig. 5. Performance of algorithms `maxThroughput`, `DistriAlg`, `OnlineAlg`, and `ApproAlg` by varying β from 0.05 to 0.2 when $\alpha = 0.5$.

while users take subway trains to work or go home. We first formulated a novel optimization problem that schedules wireless chargers to charge energy-critical smartphones for a given monitoring period, such that the overall charging satisfaction of smartphone users is maximized. We then devised a non-trivial $\frac{1}{3}$-approximation algorithm for the problem, assuming that the travel trajectory of each to-be-charged smartphone user is given. We also proposed an online algorithm to deal with dynamic energy-critical smartphone charging requests. Furthermore, we developed a distributed algorithm for the problem when the global knowledge of user energy information is not given. We finally evaluated the performance of the proposed algorithms, using a real dataset. Experimental results showed that the proposed algorithms are very promising, and as high as 87.4, 87.4, and 90 percent of energy-critical users can be charged on time in the solutions delivered by the proposed distributed, online, and approximation algorithms.

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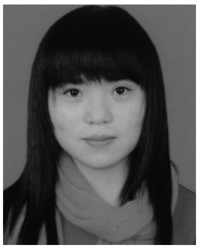

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