Efficient Scheduling of Multiple Mobile Chargers for Wireless Sensor Networks

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***Abstract*—In this paper, we study the deployment of multiple mobile charging vehicles to charge sensors in a large-scale wireless sensor network for a given monitoring period so that none of the sensors will run out of energy, where sensors can be charged by the charging vehicles with wireless energy transfer. To minimize the network operational cost, we first formulate a charging scheduling problem of dispatching multiple mobile charging vehicles to collaboratively charge sensors such that the sum of travelling distance (referred to as the service cost) of these vehicles for this monitoring period is minimized, subject to that none of the sensors will run out of energy. Due to NP-hardness of the problem, we then propose a novel approximation algorithm with a guaranteed approximation ratio, assuming that the energy consumption rate of each sensor does not change for the given monitoring period. Otherwise, we devise a heuristic algorithm through modifications to the approximation algorithm. We finally evaluate the performance of the proposed algorithms via experimental simulations. Simulation results show that the proposed algorithms are very promising, which can reduce the service cost by up to 20% in comparison with the service costs delivered by existing ones.**

***Index Terms*—Approximation algorithms, combinatorial optimization problems, mobile chargers, periodic charging cycles, rechargeable sensor networks, wireless energy transfer.**

I. Introduction

Wireless sensor networks (WSNs) have played an important role in many monitoring and surveillance applications including environmental sensing, target tracking, structural health monitoring, etc. [1], [14], [19], [39]. As conventional sensors are powered by batteries, the limited battery capacity obstructs the large-scale deployment of WSNs. Although there are many energy-aware approaches developed in the past decade to reduce sensor energy consumptions or balance energy expenditures among sensors [2], [3], [9], [23], [30], the lifetime of WSNs remains a main performance bottleneck in their real deployments, since wireless data transmission consumes substantial sensor energy.

To mitigate the limited energy problem in sensor networks, researchers proposed many different efficient approaches. One method is to enable sensors to harvest ambient energy from their surroundings such as solar energy, vibration energy, wind energy, etc. [11], [13], [26]. However, the temporally and spatially varying nature of renewable energy resources makes the prediction of sensor energy harvesting rates very difficult. For instance, it is shown that the energy generating rates in sunny, cloudy and shadowy days can vary by up to three orders of magnitude in a solar harvesting system [22]. Moreover, the harvesting energy sources are intermittent and not always available. Such unpredictability and intermittency pose enormous challenges in the efficient usage of harvested energy for various monitoring or surveillance tasks.

A recent breakthrough in the wireless power transfer technique based on strongly coupled magnetic resonances has drawn plenty of attentions in the research community [15], [16]. Kurs *et al.* demonstrated that it is possible to achieve an approximate 40% efficiency of wireless power transfer for powering a $60W$ light bulb from a distance of two meters without any wire lines and plugs [15]. Industry research further achieved a 75% efficiency of wireless power transfer for transferring $60W$ of power over a distance of up to two to three feet [10]. Several commercial products based on the wireless energy transfer technology now are available in markets such as sensors [20], RFIDs [31], cell phones [21], and auto vehicles [5]. It is reported that the wireless energy transfer market is expected to grow from just \$216 million in 2013 to \$8.5 billion in 2018 [32]. Armed with this advanced technology, sensors can be charged at steady and high charging rates. Another breakthrough in the ultra-fast charging battery materials further fuels the feasibility of the wireless power transfer technique. Scientists from MIT implemented an ultra-fast charging in material $LiFePO_4$, which can be charged at a rate as high as 400 *Coulombs* per second [12]. The duration of fully-charging a battery thus can be shortened to a few seconds. Therefore, wireless power charging is a very promising technique to prolong the lifetime of WSNs. In this paper, we employ multiple mobile chargers (i.e., charging vehicles) to replenish sensor energy in a large-scale WSN for a given monitoring period T so that

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none of the sensors will run out of energy, where each sensor can be charged by a mobile charger in its vicinity with the wireless power transfer technique. Since each sensor consumes its energy on data sensing, data transmission, data reception, etc., the sensor may need to be charged multiple times to avoid its energy depletion for the period of T.

Most existing studies on sensor charging scheduling employ mobile chargers to charge all sensors periodically [25], [34], [35] or charge only the sensors that will run out of energy very soon [7], [18], [24], [29], [37], [38], [41]. One major disadvantage of these studies is that the total travelling distance of the mobile chargers for charging sensors in the entire monitoring period can be very long, which may not be necessary, as the energy consumption rates of different sensors usually are significantly different. For example, the sensors near to the base station have to relay data for other remote sensors, their energy consumption rates thus are much higher than that of the others [17]. Therefore, the naive strategy of charging all sensors per charging tour will significantly increase the total travelling distance of the mobile chargers. Similarly, the charging strategy that schedules the mobile chargers to charge only the life-critical sensors also suffers from the same problem as these life-critical sensors may be far away from each other in the monitoring area.

The long total travelling distance of mobile chargers can result in prohibitively high energy consumptions of mobile chargers on their mechanical movements. It is reported that the most fuel-efficient vehicle has an energy consumption of 600 kJ per km (i.e., 27 kWh per 100 miles) [28] while the energy capacity of a regular sensor battery is 10.8 kJ [25]. This implies that the amount of energy consumed by the vehicle travelling for one kilometer is equivalent to the amount of energy used for charging as many as 55 ($\approx 600\ \text{kJ}/10.8\ \text{kJ}$) sensors. Since WSNs usually are deployed for long-term environmental sensing, target tracking, and structural health monitoring [1], [14], [19], [39], the monitoring area of a WSN can be very large (e.g., several square kilometers) [6], [14], the mobile chargers by the existing studies consume a large proportion of their energy on travelling, rather than on sensor charging, thereby leading to a very high cost of network operations.

Unlike existing studies that ignore the energy consumption of mobile chargers on travelling for charging sensors, in this paper we propose efficient charging scheduling algorithms to dispatch multiple mobile chargers for sensor charging in a large-scale WSN for a long-term monitoring period T, so that not only none of the sensors runs out of energy but also the total travelling distance of all mobile chargers for the period of T is minimized. As energy consumption rates of different sensors may significantly different, different sensors require different charging frequencies during period T, the challenges of scheduling the mobile chargers are: (1) when should we activate a charging round to dispatch the mobile chargers to replenish sensor energy? (2) which sensors should be included in each charging round? (3) given a set of to-be-charged sensors, which sensors should be charged by which mobile charger? (4) what is the charging order of the sensors assigned to each mobile charger? In this paper, we will tackle these challenges by first formulating a novel optimization problem, and then devising an efficient approximation algorithm with a performance guarantee and a heuristic algorithm for the problem, depending on whether the energy consumption rate of each sensor is fixed or not for the given monitoring period.

The main contributions of this paper can be summarized as follows. We first formulate a novel service cost minimization problem of finding a series of charging schedulings of multiple mobile chargers to maintain the perpetual operations of sensors for a given monitoring period T such that the total travelling distance of all mobile chargers is minimized. This objective is critical to reducing the WSN maintenance cost. Due to NP-hardness of the problem, we then devise an approximation algorithm with a provable approximation ratio if energy consumption rates of sensors are fixed during the monitoring period. Otherwise, we propose a heuristic solution through modifications to the approximate solution. We finally conduct extensive experiments by simulations to evaluate the algorithm performance. Experimental results demonstrate that the proposed algorithms are very promising, which can reduce the service cost by up to 20% in comparison with the ones delivered by existing algorithms. To the best of our knowledge, this is the first approximation algorithm for scheduling multiple mobile chargers to charge sensors in a given monitoring period if the energy consumption rate of each sensor does not fluctuate in this period. Otherwise, a novel heuristic solution is proposed.

The rest of the paper is organized as follows. Section II reviews related work. Section III introduces preliminaries. Section IV devises an algorithm for a q-rooted TSP problem, which will be served as a subroutine of the proposed algorithms. Sections V and VI propose approximation and heuristic algorithms for the problem under fixed and variable sensor energy consumption rates, respectively. Section VII evaluates the algorithm performance, and Section VIII concludes the paper.

II. Related Work

The wireless power transfer technology based on strongly magnetic resonances has drawn a lot of attentions and researchers adopted the wireless energy replenishment to prolong the lifetime of WSNs [7], [18], [24], [25], [29], [33]–[36], [41]. Most existing studies jointly considered data flow routing and sensor energy replenishment. For instance, Shi *et al.* [25] proposed to replenish sensor energy in a WSN by employing a wireless charging vehicle to periodically visit each sensor. They formulated a problem of maximizing the ratio of the vacation time of the charging vehicle to the renewable energy cycle time, by considering both data flow routing and the charging time of each sensor, assuming that the data generation rate of each sensor does not change over time. They also extended their work to more general settings that the charging vehicle can charge multiple sensors simultaneously [33], or the vehicle can replenish sensor energy and collect sensor data at the same time [34], [35]. Zhao *et al.* [41] considered a joint design of data gathering and energy replenishment by exploiting sink mobility. To this end, for every fixed interval, they first chose a set of to-be-charged sensors, and then deliver a data gathering solution, such that the network utility is maximized while maintaining perpetual network operations. They later extended their work by taking the energy consumptions of data sensing and reception into consideration [7]. The consideration of joint data flow routing and energy replenishment in aforementioned studies may be applicable if the sensing data can be collected later; otherwise, this method may not work for real-time monitoring. In addition, this joint method suffers from other drawbacks in real WSN deployments such as preventing data aggregation at relay nodes [18].

There are several recent studies adopting on-demand sensor energy replenishment [18], [24], [29], [36]–[38]. For example, Xu *et al.* [36] considered the problem of scheduling K mobile chargers to replenish a set of to-be-charged sensors such that the length of the longest charging tour among the K charging tours of the K mobile chargers is minimized, for which they proposed constant approximation algorithms. Ren *et al.* [24] studied the employment of a single mobile charger to charge on-demand sensors under the travel distance constraint. Liang *et al.* [18] proposed an approximation algorithm for minimizing the number of mobile charging vehicles needed for charging a set of to-be-charged sensors, subject to the energy capacity constraint on each mobile vehicle. Wang *et al.* [29] developed a hybrid approach for scheduling multiple mobile chargers to charge sensors: active and passive energy replenishment. Unlike these mentioned studies, we here consider the charging scheduling of mobile chargers for a given period T, rather than only at a specific time point. Furthermore, we are the first to propose efficient scheduling algorithms to minimize the total travelling distance of multiple mobile chargers for maintaining the perpetual operations of sensor networks, which can significantly reduce the network operational and maintenance costs.

III. Preliminaries

In this section, we first present the network model and energy consumption models, then introduce notations and notions, and finally define the problems and show their NP-hardness.

A. Network Model

We consider a wireless sensor network consisting of n sensors, which are randomly deployed in a 2-D space. Let V be the set of sensors. Each sensor $v_i \in V$ generates sensing data with a rate of b_i (in bps). Also, each sensor v_i is powered by a rechargeable battery with energy capacity B_i. There is one stationary base station in the network. We assume that there is a routing protocol for sensing data collection that relays sensing data from sensors to the base station through multihop relays. For example, each sensor uploads its sensing data to the base station via the path with the minimum energy consumption. Assume that the entire network monitoring period is T (T typically is long, e.g., several months, even years). Since each sensor consumes its energy on data sensing, processing, transmission and reception, it is required to be charged multiple times to avoid its energy depletion during T.

In this paper, we employ q wireless mobile chargers to replenish energy to sensors in the network, where mobile charger l is located at depot r_l, $1 \leq l \leq q$. With loss of generality, let $R = \{r_1, r_2, \ldots, r_q\}$ be the set of depot locations of the q mobile chargers. To determine charging trajectories of the q mobile chargers, we define a weighted, undirected graph $G = (V \cup R, E; w)$, where for any two distinct nodes (sensors or depots) u and v in $V \cup R$, there is an edge $e = (u, v) \in E$ between them with their Euclidean distance being the weight $w(e)$ of edge e. Each time mobile charger l is dispatched to charge some sensors, it always starts from and ends at its depot r_l for recharging itself or refuelling its petrol. In other words, each charging tour of a mobile charger l in G is a *closed tour* including depot r_l. For any closed tour C in G, denote by $w(C)$ the weighted sum of the edges in C, i.e., $w(C) = \sum_{e \in E(C)} w(e)$. We assume that each mobile charger has enough energy to charge the sensors assigned to it in each charging tour [25], [33]–[35]. We consider a point-to-point charging, i.e., to efficiently charge a sensor by a mobile charger, the mobile charger must be in the vicinity of the sensor [15] and the sensor will be charged to its fully capacity.

We assume that the duration of the q mobile chargers per charging round that includes the time for charging sensors and their travelling time is several orders of magnitude less than the lifetime of a fully-charged sensor. The rationale behind the assumption is as follows. Once a sensor is fully charged, its lifetime can last from several weeks to months until its next charging, since the sensor energy can be well managed through various existing energy conservation techniques, e.g., duty cycling [2]. On the other hand, the q mobile chargers can collaboratively finish a charging round within a few hours, since sensor batteries can be made with ultra-fast charging battery materials [12], [27]. For example, in 2009 scientists from MIT implemented an ultra-fast charging, in which a battery can be fully charged within a few seconds [12]. In 2014, scientists from Nanyang Technological University developed a new lithium-ion battery that can be charged up to 70% in only 2 minutes, while the new battery has other advantages, such as a longer lifespan of over 20 years and easy manufacturing [27]. We thus envision that ultra-fast charging batteries will be commercialized in the near future and will be widely used for smartphones, sensors, electric vehicles, etc. Therefore, we ignore the time spent by the q mobile chargers per charging round. Note that [8], [40] and [41] also adopted the similar assumption.

B. Energy Consumption Models

Each sensor will consume its energy on data sensing, data transmission, and data reception, and the energy consumption models for these three components are shown in (1)–(3), respectively [17]

$$P_{\text{sense}} = \lambda \times b_i \tag{1}$$

$$P_{Tx} = \left(\beta_1 + \beta_2 d_{ij}^{\alpha}\right) \times b_i^{Tx} \tag{2}$$

$$P_{Rx} = \gamma \times b_i^{Rx} \tag{3}$$

where b_i (in bps) is the data sensing rate of sensor v_i, b_i^{Tx} and b_i^{Rx} are the data transmission rate and the reception rate of sensor v_i, respectively, d_{ij} is the Euclidean distance between sensors v_i and v_j, α is a constant that is equal to 2 or 4, and the values of other parameters are as follows [17]:

$$\begin{aligned} \lambda &= 60 \times 10^{-9}\ J/b \\ \beta_1 &= 45 \times 10^{-9}\ J/b \\ \beta_2 &= 10 \times 10^{-12}\ J/b/m^2, \text{ when } \alpha = 2 \\ \text{or } \beta_2 &= 1 \times 10^{-15}\ J/b/m^4, \text{ when } \alpha = 4 \\ \gamma &= 135 \times 10^{-9}\ J/b. \end{aligned}$$

C. Notations and Notions

A charging scheduling of q mobile chargers is to dispatch each of the q mobile chargers from its depot to collaboratively visit a set of to-be-charged sensors in the current round, and each charger will return to its depot after finishing its charging tour. Assume that at time t_j, let closed tours $C_{j,1}, C_{j,2}, \ldots, C_{j,q}$ be the charging tours of the q mobile chargers, where tour $C_{j,l}$ of mobile charger l contains its depot r_l and $1 \le l \le q$. Let $\mathcal{C}_j = \{C_{j,1}, C_{j,2}, \ldots, C_{j,q}\}$ be the set of the q tours at time t_j. Notice that it is likely that some tours $C_{j,l}$s may contain none of the sensors, and if so, $V(C_{j,l}) = \{r_l\}$ and $w(C_{j,l}) = 0$. For the sake of simplicity, we represent each charging scheduling by a 2-tuple $(\mathcal{C}_j, t_j)$, where all sensors in tour $C_{j,l} \in \mathcal{C}_j$ will be charged to their full energy capacities by mobile charger l, all the q mobile chargers are dispatched at time t_j, and $0 < t_j < T$. Denote by $V(C_{j,l})$ and $V(\mathcal{C}_j)$ the set of nodes in $C_{j,l}$ and $\mathcal{C}_j$, respectively. Then, $V(\mathcal{C}_j) = \cup_{l=1}^{q} V(C_{j,l})$.

The charging cycle of a sensor $v_i \in V$ is the duration between its two consecutive chargings, and its *maximum charging cycle* τ_i is the maximum duration in which it will not run out of its energy. Since different WSNs adopt different sensing and routing protocols, different sensors may have different energy consumption rates and different maximum charging cycles. If the energy consumption rate of each sensor $v_i \in V$ does not vary for the period of T, denote by ρ_i and τ_i its energy consumption rate and maximum charging cycle, then $\tau_i = B_i/\rho_i$, where B_i is the energy capacity of sensor v_i and the energy consumption rate ρ_i of sensor v_i usually is determined by the data generation rate of the sensor and the sum of data rates from other sensors that the sensor must forward to the base station [2]. It is obvious that sensors with shorter maximum charging cycles need to be charged more frequently than sensors with longer maximum charging cycles. Since each time the q mobile chargers are dispatched to charge a set of sensors, they will consume their electricity or petrol, thereby incurring a service cost. We thus define *the service cost* of the q mobile chargers as *the sum of their travel distances* for charging sensors for the period of T.

D. Problem Definitions

We note that not every sensor must be replenished in each charging round as the energy consumption rates of different sensors may be significantly different. Therefore, a naive strategy of charging all sensors per round will increase the service cost substantially. Also, as some to-be-charged sensors and their nearest depots in a large-scale sensor network can be far away from each other, it is crucial to schedule the q mobile chargers by taking both the maximum charging cycles and the geographical locations of the sensors into account.

Given a metric complete graph $G = (V \cup R, E)$ with q mobile chargers located at q depots in R, a distance function $w : E \mapsto \mathbb{R}^+$, a monitoring period T, and a maximum charging cycle function $\tau : V \mapsto \mathbb{R}^+$, assume that the location coordinates $(x_i, y_i) \in (X, Y)$ of each sensor $v_i \in V$ are given. *The service cost minimization problem with fixed maximum charging cycles* in G is to find a series of charging schedulings $(\mathcal{C}_1, t_1), (\mathcal{C}_2, t_2), \ldots, (\mathcal{C}_p, t_p)$ of the q mobile chargers such that the total length of all closed tours (or the service cost) is minimized, where p is a positive integer to be determined by the algorithm. Specifically, the problem is formulated as follows:

$$\text{minimize} \quad \sum_{j=1}^{p} w(\mathcal{C}_j) = \sum_{j=1}^{p} \sum_{l=1}^{q} w(C_{j,l}) \tag{4}$$

subject to that, for each sensor $v_i \in V$,

1) the time gap between its any two consecutive charging schedulings $(\mathcal{C}_{j_1}, t_{j_1})$ and $(\mathcal{C}_{j_2}, t_{j_2})$ is no more than its maximum charging cycle τ_i (assuming that $t_{j_1} < t_{j_2}$), i.e., $t_{j_2} - t_{j_1} \le \tau_i$, where sensor v_i is contained in both charging schedulings $\mathcal{C}_{j_1}$ and $\mathcal{C}_{j_2}$ and there is no charging scheduling $(\mathcal{C}_j, t_j)$ such that sensor v_i is contained in $\mathcal{C}_j$ and $t_{j_1} \le t_j \le t_{j_2}$;
2) the duration from its last charging to the end of period T is no more than τ_i,

where $\mathcal{C}_j = \{C_{j,1}, C_{j,2}, \ldots, C_{j,q}\}$, $C_{j,l}$ is the charging tour of mobile charger l located at depot r_l, $1 \le l \le q$, and $0 < t_1 < t_2 < \cdots < t_p < T$.

In this problem, we not only need to determine the number of rounds p to schedule mobile chargers for sensor charging, but also to decide which sensors to be charged in which rounds and by which chargers. Intuitively, during the period of T, if more rounds are scheduled, then there are less number of sensors to-be-charged in each round. On the other hand, if less number of rounds is scheduled, there are more sensors to-be-charged in each round. Our objective is to minimize the total traveling distance of the q mobile chargers for the p charging rounds. The challenge of this optimization problem is to determine both p and the set of to-be-charged sensors in each round in order to minimize the total traveling distance of q mobile chargers.

So far, we have assumed that the maximum charging cycle of each sensor $v_i \in V$ in the entire period T is fixed. However, in reality, it may experience significant changes over time, since the data rates of different sensors usually depend on the specific application of a WSN, some sensors may be required to increase their data rates for better monitoring the area of these sensors at some time while the others may be required to reduced their data rates for saving their energy. For this general setting, we define *the service cost minimization problem with variable maximum charging cycles* as follows. Given a wireless sensor network G, a period T, q mobile chargers located at q depots, the maximum charging cycle $\tau_i(t)$ of each sensor v_i that varies with time t, the problem is to find a series of charging schedulings of the q mobile chargers such that the service cost of them is minimized, subject to that none of the sensors runs out of energy for the period of T.

We finally define a q-rooted TSP problem, which will be used as a subroutine for the problems of concern in this paper. Assume that there is a set of to-be-charged-sensors $V^c \subseteq V$ at some time point. Given a subgraph $G^c = (V^c \cup R, E^c; w)$ of G with $|R| = q \geq 1$ and q mobile chargers, the problem is to find q closed tours $C_1, C_2, \ldots, C_q$ in G^c such that the total length of the q tours, $\sum_{l=1}^{q} w(C_l)$, is minimized, subject to that these q tours cover all sensors in V^c, i.e., $V^c \subseteq \bigcup_{l=1}^{q} V(C_l)$, and each of the q tours contains a distinct depot in R. The q-rooted TSP problem is NP-hard as the classical TSP problem is a special case of it when $q = 1$.

E. NP-Hardness

Theorem 1: The service cost minimization problem with fixed maximum charging cycles is NP-hard.

Proof: We show the NP-hardness of the problem by a reduction from the classical NP-hard problem—travelling Salesman Problem (TSP). We reduce the TSP problem in a metric complete graph $H = (N, A)$ to the service cost minimization problem in a graph $G = (V \cup \{r\}, E)$ with a monitoring period $T = 2s$ as follows. We choose an arbitrary node $v_0 \in N$ as the depot r, i.e., $q = 1$. Let $V = N \setminus \{v_0\}$ and $E = A$. For each sensor $v_i \in V$, let $\tau_i = 1s$ be the maximum charging cycle of sensor v_i. Initially, we assume that each sensor at time 0 has a full energy capacity B_i. Thus, each sensor must be charged at least once within $T = 2s$; otherwise it will be dead.

Assume that $(C_1, t_1), \ldots, (C_p, t_p)$ are the optimal charging schedulings for the service cost minimization problem in G with $0 < t_j \leq T$ and $1 \leq j \leq p$. Let C^* be the optimal solution to the TSP problem in H. We show that $w(C^*) = \sum_{j=1}^{p} w(C_j)$ as follows. On one hand, we can transform the closed tour C^* into a feasible solution to the service cost minimization problem in the constructed rechargeable sensor network by charging all sensors in tour C^* at time $t = 1s$. Thus, $\sum_{j=1}^{p} w(C_j) \leq w(C^*)$. On the other hand, since each sensor $v \in V$ must appear at least once in the p closed tours with each tour containing the depot $r(= v_0)$, we can construct a Eulerian circuit C' that visits all the edges of the p tours. The weighted sum of the edges in C' thus is equal to the total length of the p tours, i.e., $w(C') = \sum_{j=1}^{p} w(C_j)$. We then obtain a less cost closed tour C from C' by removing the multiple appearances of each node in C'. As the weights of the edges in C' follow the triangle inequality, then $w(C^*) \leq w(C) \leq w(C') \leq \sum_{j=1}^{p} w(C_j)$. Therefore, $w(C^*) = w(C) = w(C') = \sum_{j=1}^{p} w(C_j)$, and the closed tour C derived from the union of $(C_1, t_1), \ldots, (C_p, t_p)$ is an optimal solution to the TSP problem in graph H. ■

IV. Algorithm for the q-Rooted TSP Problem

In this section, we propose a 2-approximation algorithm for the q-rooted TSP problem, which will serve as a subroutine of the approximation algorithm for the service cost minimization problem.

The basic idea of the algorithm for the q-rooted TSP problem is that we first find q-rooted trees with the minimum total cost, and we then show that the total cost of the q-rooted trees is a lower bound on the optimal cost of the q-rooted TSP problem. We finally convert each of the trees into a closed tour with the cost of the tour no more than twice the cost of the tree.

We start with the q-rooted minimum spanning forest (q-rooted MSF) problem: given a graph $G^c = (V^c \cup R, E^c; w)$, $q = |R|$, and $w : E^c \mapsto \mathbb{R}^+$, the problem is to find q trees $T_1, T_2, \ldots, T_q$ spanning all nodes in V^c with each tree containing a distinct depot in R such that the total cost of the q trees, $\sum_{l=1}^{q} w(T_l)$, is minimized.

For the q-rooted MSF problem, an exact algorithm is given as follows. We start by constructing an auxiliary graph $G_r = (V^c \cup \{r\}, E_r; w_r)$ from $G^c = (V^c \cup R, E^c; w)$ by contracting the q depots in R into a single root r: (i) remove the q depots in R and introduce a new node r;

(ii) for each $r_l \in R$, introduce an edge $(v, r) \in E_r$ for each edge $(v, r_l) \in E^c$, where $v \in V^c$; (iii) $w_r(v, r) = \min_l \{w(v, r_l)\}$. We then find an MST T of G_r. We finally break T into q disjoint trees $T_1, T_2, \ldots, T_q$ by un-contracting the roots in R. This un-contraction means that an edge (v, r) is mapped to an edge (v, r_l), where $w_r(v, r) = w(v, r_l)$. Note that each tree T_l roots at depot r_l. The detailed algorithm is presented in `Algorithm` 1.

Algorithm 1 q-`rooted MSF`

Input: $G^c = (V^c \cup R, E^c; w)$, $w : E^c \mapsto \mathbb{R}^+$, and $q = |R|$.
Output: a solution for the q-rooted MSF problem
1: Construct a graph $G_r = (V^c \cup \{r\}, E_r; w_r)$ from G^c by contracting the q depots in R into a single root r;
2: Find an MST T in G_r;
3: Decompose the MST T into q disjoint rooted trees $T_1, T_2, \ldots, T_q$ by un-contracting depots in R.

Lemma 1: There is an algorithm for the q-rooted MSF problem, which delivers an optimal solution and takes $O(n^2)$ time, where $n = |V^c \cup R|$.

Proof: Assume that trees $T_1^*, T_2^*, \ldots, T_q^*$ form an optimal solution to the q-rooted MSF problem. We show that the solution consisting of trees $T_1, T_2, \ldots, T_q$, delivered by `Algorithm` 1, is optimal. On one hand, since the q trees $T_1, T_2, \ldots, T_q$ form a feasible solution, then $\sum_{l=1}^{q} w(T_l^*) \leq \sum_{l=1}^{q} w(T_l)$. On the other hand, as each tree T_l^* contains a depot $r_l \in R$, we can construct a spanning tree T' in graph G_r by contracting the q depots into a single root r, and $w(T') = \sum_{l=1}^{q} w(T_l^*)$. As the MST T is the minimum one, we have $w(T) \leq w(T')$. Since $\sum_{l=1}^{q} w(T_l) = w(T)$, $\sum_{l=1}^{q} w(T_l) = w(T) \leq w(T') \leq \sum_{l=1}^{q} w(T_l^*)$. Therefore, $\sum_{l=1}^{q} w(T_l) = \sum_{l=1}^{q} w(T_l^*)$, i.e., the found trees $T_1, T_2, \ldots, T_q$ form an optimal solution to the problem. The time complexity of `Algorithm` 1 is analyzed as follows. Constructing graph G_r takes time $O(E^c) = O(n^2)$. Finding the MST T in G_r takes $O(n^2)$ time, while un-contracting the MST T also takes time $O(E^c) = O(n^2)$. `Algorithm` 1 thus runs in $O(n^2)$ time. ■

With the help of the exact algorithm for the q-rooted MSF problem, we now devise a 2-approximation algorithm for the q-rooted TSP problem in `Algorithm 2`.

We show that `Algorithm 2` delivers a 2-approximate solution by the following theorem.

Theorem 2: There is a 2-approximation algorithm for the q-rooted TSP problem, which takes time $O(|V^c \cup R|^2)$.

Proof: Assume that closed tours $C_1^*, C_2^*, \ldots, C_q^*$ form an optimal solution to the q-rooted TSP problem in G^c.

Algorithm 2 q-`rooted TSP`

Input: $G^c = (V^c \cup R, E^c; w)$, $w : E^c \mapsto \mathbb{R}^+$, and $q = |R|$.
Output: A solution $\mathcal{C}$ for the q-rooted TSP problem
1: Find q optimal trees $T_1, T_2, \ldots, T_q$ for the q-rooted MSF problem in G^c by calling `Algorithm 1`;
2: For each tree T_l, double the edges in T_l, find a Eulerian tour C_l', and obtain a less cost closed tour C_l by short-cutting repeated nodes in C_l'. Let $\mathcal{C} = \{C_1, C_2, \ldots, C_q\}$.

For each tour C_l^*, we can obtain a tree T_l' by removing any edge in C_l^*. Then, $w(T_l') \le w(C_l^*)$, $1 \le l \le q$. It is obvious that trees $T_1', T_2', \ldots, T_q'$ form a feasible solution to the q-rooted MSF problem. As trees $T_1, T_2, \ldots, T_q$ form the optimal solution by Lemma 1, $\sum_{l=1}^{q} w(T_l) \le \sum_{l=1}^{q} w(T_l') \le \sum_{l=1}^{q} w(C_l^*)$. Also, we can see that the total cost of each found tour C_l is no more than twice the total cost of tree T_l, i.e., $w(C_l) \le 2w(T_l)$. Therefore, $\sum_{l=1}^{q} w(C_l) \le \sum_{l=1}^{q} 2w(T_l) \le 2\sum_{l=1}^{q} w(C_l^*)$. The time complexity analysis is straightforward, omitted. ∎

V. Approximation Algorithm With Fixed Maximum Charging Cycles

In this section, we devise an approximation algorithm for the service cost minimization problem, assuming that each sensor has a fixed maximum charging cycle. We start with the basic idea behind the algorithm. We then present the approximation algorithm, and we finally analyze the approximation ratio of the proposed approximation algorithm.

A. Overview of the Approximation Algorithm

Given a maximum charging cycle function: $\tau : V \mapsto R^+$ and a monitoring period T, if there is a series of mobile charger schedulings for T such that no sensor depletes its energy, then we say that these schedulings form *a feasible solution* to the service cost minimization problem, i.e., for each sensor $v_i \in V$, the maximum duration between its any two consecutive chargings is no more than τ_i. *A series of feasible charging schedulings* of the q mobile chargers is *an optimal solution* if the service cost of the solution is the minimum one.

The basic idea behind the proposed approximation algorithm is to construct another charging cycle function $\tau'(\cdot)$ for the sensors based on the maximum charging cycle function $\tau(\cdot)$, by exploring the combinatorial property of the problem. We construct a very special charging cycle function $\tau'(\cdot)$ such that charging cycles of the n sensors will form a geometric sequence as follows.

Let $\tau_1, \tau_2, \ldots, \tau_n$ be the maximum charging cycles of sensors $v_1, v_2, \ldots, v_n$ in the network. Assume that $\tau_1 \le \tau_2 \le \cdots \le \tau_n$. Let $\tau_1', \tau_2', \ldots, \tau_n'$ be the charging cycles of the sensors and $\tau_i' \le \tau_j'$ if $\tau_i \le \tau_j$. We construct $\tau'(\cdot)$ as follows. We partition the set V of the sensors into $K+1$ disjoint subsets $V_0, V_1, \ldots, V_K$, where $K = \lfloor \log_2(\tau_n/\tau_1) \rfloor$, and sensor $v_i \in V$ with its maximum charging cycle τ_i is contained in V_k if $2^k\tau_1 \le \tau_i < 2^{k+1}\tau_1$. Then, $k = \lfloor \log_2(\tau_i/\tau_1) \rfloor$. Let $\tau_i' = 2^k\tau_1$. We assign each sensor in V_k with the identical charging cycle $2^k\tau_1' = 2^k\tau_1$. Consequently, the charging cycles of sensors in $V_0, V_1, \ldots, V_K$ are $\tau_1, 2\tau_1, \ldots, 2^K\tau_1$, respectively. We can see that the assigned charging cycle τ_i' of sensor v_i is no less than the half its maximum charging cycle τ_i, since

$$\tau_i' = 2^{\lfloor \log_2 \frac{\tau_i}{\tau_1} \rfloor}\tau_1 > 2^{\log_2 \frac{\tau_i}{\tau_1} - 1}\tau_1 = \frac{\tau_i}{2}, \ \forall v_i \in V. \quad (5)$$

B. Approximation Algorithm

Given the charging cycle function $\tau'(\cdot)$, we can see that τ_j' is divisible by τ_i' for any two sensors v_i and v_j if $\tau_i \le \tau_j$ and $1 \le i < j \le n$. For simplicity, assume that the monitoring period T is divisible by the maximum assigned charging cycle τ_n', let $T = 2m\tau_n' = 2m2^K\tau_1$, where m is a positive integer. Furthermore, we assume that each sensor is fully charged at time $t = 0$. The solution delivered by the proposed algorithm consists of a series of schedulings of the q mobile chargers. Specifically, we first find a sequence of schedulings of the q mobile chargers for a period τ_n'. Then, we repeat the found schedulings for the next time period of τ_n', and so on. We repeat these scheduling sequence for the period of T no more than $\lfloor T/\tau_n' \rfloor - 1 = 2m - 1$ times.

In the following, we construct a series of schedulings for a period $\tau_n' = 2^K\tau_1$. Recall that we have partitioned the sensor set V into $K+1$ disjoint subsets $V_0, V_1, \ldots, V_K$, and the charging cycle of each sensor in V_k is $2^k\tau_1$, $0 \le k \le K$. We further partition the period τ_n' into 2^K *equal time intervals* with each interval lasting τ_1, and label them from the left to right as the 1st, 2nd, ..., and the 2^Kth time interval. Clearly, all sensors in V_0 must be charged at each of these 2^K time intervals; all sensors in V_1 must be charged at every second time interval; and all sensors in V_k must be charged at every 2^k time interval, $0 \le k \le K$. That is

At time τ_1, charge the sensors in V_0
At time $2\tau_1$, charge the sensors in $V_0 \cup V_1$
At time $3\tau_1$, charge the sensors in V_0
At time $4\tau_1$, charge the sensors in $V_0 \cup V_1 \cup V_2$

⋮

At time $j\tau_1$, charge the sensors in $\cup_{(j \mod 2^k)=0} V_k$ where $0 \le k \le K'$, $K' = \lfloor \log_2 j \rfloor$, and $1 \le j \le 2^K$

⋮

At time $2^K\tau_1$, charge the sensors in $\cup_{i=0}^{K} V_i = V$.

There are 2^K charging schedulings of the q mobile chargers and one charging scheduling is dispatched at each time interval. Let $\mathcal{C}_j = \{C_{j,1}, C_{j,2}, \ldots, C_{j,q}\}$ be the set of closed tours of the q mobile chargers at time interval j, where $1 \leq j \leq 2^K$. Furthermore, it can be seen that in the 2^K charging schedulings, there are 2^{K-1} identical charging schedulings with each only containing the sensors in V_0, there are 2^{K-2} identical charging schedulings with each containing the sensors only in $V_0 \cup V_1$. In general, there are 2^{K-1-k} identical charging schedulings with each containing the sensors only in $V_0 \cup V_1 \cdots \cup V_k$, $0 \leq k \leq K-1$. Finally, there is one charging scheduling containing the sensors in $V_0 \cup V_1 \cdots \cup V_K = V$. Denote by $\mathcal{D}_k = \{D_{k,1}, D_{k,2}, \ldots, D_{k,q}\}$ the set of q closed tours for the q-rooted TSP problem in the induced graph $G[R \cup V_0 \cdots \cup V_k]$, which is delivered by `Algorithm` 2 and $0 \leq k \leq K$.

The series of charging schedulings for a period τ'_n thus is $(\mathcal{C}_1, \tau_1), \ldots, (\mathcal{C}_j, j\tau_1), \ldots, (\mathcal{C}_{2^K}, 2^K\tau_1)$, where the 2-tuple $(\mathcal{C}_j, j\tau_1)$ represents that the q mobile chargers are dispatched at time $j\tau_1$ and the set of to-be-charged sensors is $\cup_{C_{j,i} \in \mathcal{C}_j} V(C_{j,i}) = \cup_{(j \mod 2^k)=0} V_k$, $0 \leq k \leq K'$, $K' = \lfloor \log_2 j \rfloor$, and $1 \leq j \leq 2^K$. As a result, there are $p = 2m \cdot 2^K - 1$ charging schedulings found for a period of $T = 2m\tau'_n$ as follows:

$$(\mathcal{C}_1, \tau_1), \ldots, \left(\mathcal{C}_{2^K-1}, (2^K-1)\tau_1\right),\ (\mathcal{C}_{2^K}, 2^K\tau_1)$$
$$(\mathcal{C}_1, \tau'_n+\tau_1), \ldots, \left(\mathcal{C}_{2^K-1}, \tau'_n+(2^K-1)\tau_1\right), \left(\mathcal{C}_{2^K}, \tau'_n+2^K\tau_1\right)$$
$$\vdots$$
$$(\mathcal{C}_1, (2m-1)\tau'_n+\tau_1), \ldots, \left(\mathcal{C}_{2^K-1}, (2m-1)\tau'_n+(2^K-1)\tau_1\right).$$

Note that we do not perform a charging scheduling at time $T = 2m\tau'_n$ as there is no such need in the end of period T. The proposed algorithm is described in `Algorithm` 3.

Algorithm 3 `MinDis`

Input: $G = (V \cup R, E; w)$, maximum charging cycles $\tau : V \mapsto \mathbb{R}^+$, q chargers, and a monitoring period T.
Output: A series of charging schedulings $\mathcal{C}$ for period T

```
Let τ_1, τ_2, ..., τ_n be the sorted maximum charging cycles of sensors v_1, v_2, ..., v_n in ascending order;
For each sensor v_i, let τ'_i = 2^{⌊log_2(τ_i/τ_1)⌋};
Partition sensors in V into K + 1 disjoint subsets V_0, V_1, ..., V_K, where sensor v_i ∈ V_k if 2^k τ_1 = 2^{⌊log_2(τ_i/τ_1)⌋} τ_1, 0 ≤ k ≤ K, and K = ⌊log_2(τ_n/τ_1)⌋. All sensors in V_k have the same charging cycle 2^k τ'_1.
for k ← 0 to K do
    Find q charging tours D_k = {D_{k,1}, D_{k,2}, ..., C_{j,q}} in the induced subgraph G[R ∪ V_0 ··· ∪ V_k] by applying Algorithm 2;
end for
C ← ∅; /* the solution */
/* Construct schedulings (C_1, τ_1), ..., (C_{2^K}, 2^K τ_1) */
for j ← 1 to 2^K do
    /* Find the charging scheduling C_j of the q mobile chargers at time t_j = jτ_1 */
    Let C_j = D_k, where k is the largest integer so that j mod 2^k = 0, where 0 ≤ k ≤ K' and K' = ⌊log_2 j⌋;
    C ← C ∪ {(C_j, t_j)};
end for
for m' ← 2 to ⌊T/τ'_n⌋ do
    for j ← 1 to 2^K do
        C = C ∪ {(C_j, m' · τ'_n + t_j)}
    end for
end for
return C.
```

C. Algorithm Analysis

In the following we dedicate ourselves to analyzing the approximation ratio of the proposed approximation algorithm. We start by showing that `Algorithm` 3 delivers a feasible solution to the service cost minimization problem by Lemma 2. We then provide a lower bound on the minimum cost of the problem by Lemma 3. We finally derive the approximation ratio of `Algorithm` 3 based on the lower bound, which is stated in Theorem 3.

Lemma 2: `Algorithm` 3 delivers a feasible solution to the service cost minimization problem.

Proof: It is obvious that the solution delivered by `Algorithm` 3 is feasible, as the charging cycle τ'_i of each sensor $v_i \in V$ in the solution is no more than its maximum charging cycle τ_i, i.e., $\tau'_i \leq \tau_i$. Thus, no sensors will die in the period T, the claim then follows. ∎

The following lemma provides a lower bound on the optimal service cost, which bounds the service cost of the solution delivered by `Algorithm` 3.

Lemma 3: Given the sensor set partitioning $V_0, V_1, \ldots, V_K$ based on the maximum charging cycles of sensors, each sensor in V_k is assigned with the same charging cycle $2^k\tau_1$, $0 \leq k \leq K$. Let OPT be the service cost of an optimal solution to the service cost minimization problem. Denote by $\mathcal{D}^*_k = \{D^*_{k,1}, D^*_{k,2}, \ldots, D^*_{k,q}\}$ the optimal q closed tours for the q-rooted TSP problem in the induced graph $G[R \cup V_0 \cup V_1 \cup \cdots \cup V_k]$, then $OPT \geq m2^{K-k} \cdot w(\mathcal{D}^*_k)$, assuming that $T = 2m\tau'_n$, where $w(\mathcal{D}^*_k) = \sum_{l=1}^{q} w(D^*_{k,l})$, $K = \lfloor \log_2(\tau_n/\tau_1) \rfloor$, and $0 \leq k \leq K$.

Proof: To show that $OPT \geq m2^{K-k} \cdot w(\mathcal{D}^*_k)$, we partition the entire period $T = 2m\tau'_n = 2m \cdot 2^K\tau_1$ into $m \cdot 2^{K-k}$ time intervals with each lasting time $t_k = 2^{k+1}\tau_1$. Let $(0, t_k]$, $(t_k, 2t_k], \ldots, ((j-1)t_k, jt_k], \ldots, ((m2^{K-k}-1)t_k, m2^{K-k}t_k]$ be these $m \cdot 2^{K-k}$ intervals, where time interval j is the interval $((j-1) \cdot t_k, j \cdot t_k]$, $1 \leq j \leq m \cdot 2^{K-k}$. Note that $m2^{K-k}t_k = m2^{K-k}2^{k+1}\tau_1 = T$.

In the following we first show that there is at least one time interval among the $m2^{K-k}$ time intervals such that (i) the service cost of charging schedulings within the interval is no more than $1/m2^{K-k}$ of the service cost OPT in the optimal solution; (ii) each sensor in $\bigcup_{i=0}^{k} V_i$ must be charged at least once in this interval; and (iii) the service cost within this interval in the optimal solution is no less than the cost $w(\mathcal{C}_k)$ of a feasible solution $\mathcal{C}_k$ to the q-rooted TSP problem in graph

$G[R \cup V_0 \cup \cdots \cup V_k]$. Since $\mathcal{D}_k^*$ is the optimal solution to the q-rooted TSP problem, $w(\mathcal{D}_k^*) \leq w(\mathcal{C}_k) \leq (OPT/m2^{K-k})$.

Assume that an optimal solution consists of p charging schedulings $(\mathcal{C}_1^*, t_1^*), (\mathcal{C}_2^*, t_2^*), \ldots, (\mathcal{C}_p^*, t_p^*)$ with $0 < t_1^* \leq \cdots \leq t_p^* < T$. Recall that OPT is the sum of lengths of the p charging schedulings i.e., $OPT = \sum_{s=1}^p w(\mathcal{C}_s^*) = \sum_{s=1}^p \sum_{l=1}^q w(C_{s,l}^*)$. We partition the p charging schedulings into $m2^{K-k}$ disjoint groups according to their dispatching times, the charging scheduling $\mathcal{C}_s^*$ is in group j if its dispatching time t_s^* is within time interval j, i.e., $t_s^* \in ((j-1)t_k, jt_k]$, where $1 \leq s \leq p$ and $1 \leq j \leq m2^{K-k}$. Denote by $\mathcal{G}_j$ and $w(\mathcal{G}_j)$ the set of charging schedulings in group j and the cost sum of charging schedulings in $\mathcal{G}_j$, respectively, i.e., $w(\mathcal{G}_j) = \sum_{\mathcal{C}_s^* \in \mathcal{G}_j} w(\mathcal{C}_s^*)$, $1 \leq j \leq m2^{K-k}$. Then, $\sum_{j=1}^{m2^{K-k}} w(\mathcal{G}_j) = OPT$. Among the $m2^{K-k}$ groups, there must be a group $\mathcal{G}_j$ whose service cost $w(\mathcal{G}_j)$ is no more than $1/(m2^{K-k})$ of the optimal cost OPT, i.e.,

$$w(\mathcal{G}_j) \leq \frac{OPT}{m2^{K-k}}. \tag{6}$$

We then show that each sensor in $\bigcup_{i=0}^k V_i$ must be charged at least once by the charging schedulings in $\mathcal{G}_j$ by contradiction. Assume that there is a sensor $v_i \in \bigcup_{i=0}^k V_i$ which will not be charged by any charging scheduling in $\mathcal{G}_j$. Since $v_i \in \bigcup_{i=0}^k V_i$, its maximum charging cycle τ_i must be strictly less than $2 \cdot 2^k \tau_1 = 2^{k+1}\tau_1$ by inequality (5), i.e., $\tau_i < 2^{k+1}\tau_1$. On the other hand, as v_i will not be charged by any charging scheduling in $\mathcal{G}_j$ while it is still survived, this implies that its maximum charging cycle must be no less than the length t_k of the time interval, i.e., $\tau_i \geq t_k = 2^{k+1}\tau_1$, this results in a contradiction. Thus, v_i must be charged by at least one charging scheduling in $\mathcal{G}_j$.

We finally construct a feasible solution $\mathcal{C}_k = \{C_{k,1}, C_{k,2}, \ldots, C_{k,q}\}$ to the q-rooted TSP problem in graph $G[R \cup V_0 \cup \cdots \cup V_k]$ based on the charging schedulings in $\mathcal{G}_j$ such that the service cost $w(\mathcal{C}_k)$ is no more than $w(\mathcal{G}_j)$. Since each closed tour in $\mathcal{G}_j$ contains a depot $r_l \in R$, we partition the closed tours in $\mathcal{G}_j$ by the depot that each tour contains. To this end, we partition tours in $\mathcal{G}_j$ into q disjoint subgroups $\mathcal{G}_{j,1}, \mathcal{G}_{j,2}, \ldots, \mathcal{G}_{j,q}$, where subgroup $\mathcal{G}_{j,l}$ includes all closed tours in $\mathcal{G}_j$ that contains depot r_l, $1 \leq l \leq q$. For each subgroup $\mathcal{G}_{j,l}$, since each tour contains depot r_l, the union of all close tours in $\mathcal{G}_{j,l}$ forms a connected Eulerian graph. Then, we can derive a Eulerian circuit $C'_{k,l}$ from this Eulerian graph and $w(C'_{k,l}) = w(\mathcal{G}_{j,l})$. We further obtain a closed tour $C_{k,l}$ including only nodes in $R \cup V_0 \cup \cdots \cup V_k$ once from $C'_{k,l}$, by the removal of the nodes not in $R \cup V_0 \cup \cdots \cup V_k$ and the nodes with multiple appearances, and performing path short-cutting. Since edge weights satisfy the triangle inequality, we have

$$w(C_{k,l}) \leq w\left(C'_{k,l}\right) \leq w(\mathcal{G}_{j,l}),\ 1 \leq l \leq q. \tag{7}$$

As each sensor in $\bigcup_{i=0}^k V_i$ will be charged at least once by the charging schedulings in $\mathcal{G}_j$, and tour $C_{k,l}$ contains depot r_l, we have $\bigcup_{i=0}^k V_i \subseteq \bigcup_{l=1}^q V(C_{k,l})$. Then, all tours in $\mathcal{C}_k$ form a feasible solution to the q-rooted TSP problem in graph $G[R \cup V_0 \cup \cdots \cup V_k]$. Let $\mathcal{D}_k^* = \{D_{k,1}^*, D_{k,2}^*, \ldots, D_{k,q}^*\}$ be the optimal q tours. Then

$$\sum_{l=1}^q w\left(D_{k,l}^*\right) \leq \sum_{l=1}^q w(C_{k,l}). \tag{8}$$

By combining inequalities (6)–(8), the lemma then follows. ■

According to Lemmas 2 and 3, we show the approximation ratio of `Algorithm` 3 by the following theorem.

Theorem 3: There is a $2(K+2)$-approximation algorithm for the service cost minimization problem with fixed maximum charging cycles, which takes time $O(\lfloor \log(\tau_{\max}/\tau_{\min}) \rfloor n^2 + (T/\tau_{\min})n)$, where $\tau_{\max} = \max_{i=1}^n\{\tau_i\}$, $\tau_{\min} = \min_{i=1}^n\{\tau_i\}$, and $K = \lfloor \log_2(\tau_n/\tau_1) \rfloor$.

Proof: By Lemma 2, `Algorithm` 3 delivers a feasible solution. The rest is to analyze its approximation ratio. Recall that the charging schedulings delivered by `Algorithm` 3 for period $T = 2m\tau'_n$ are: $(\mathcal{C}_1, \tau_1), \ldots, (\mathcal{C}_{2^K}, 2^K\tau_1), (\mathcal{C}_1, \tau'_n + \tau_1), \ldots, (\mathcal{C}_{2^K}, \tau'_n + 2^K\tau_1), \ldots, (\mathcal{C}_1, (2m-1)\tau'_n + \tau_1), \ldots, (\mathcal{C}_{2^K-1}, (2m-1)\tau'_n + (2^K-1)\tau_1)$. The total service cost during T then is

$$(2m-1)\sum_{j=1}^{2^K} w(\mathcal{C}_j) + \sum_{j=1}^{2^K-1} w(\mathcal{C}_j) \leq 2m\sum_{j=1}^{2^K} w(\mathcal{C}_j). \tag{9}$$

Recall that $\mathcal{D}_k = \{D_{k,1}, D_{k,2}, \ldots, D_{k,q}\}$ is the set of q closed tours for the q-rooted TSP problem in graph $G[R \cup V_0 \cup \cdots \cup V_k]$ delivered by `Algorithm 2`. Let $\mathcal{C}(\tau'_n) = \{(\mathcal{C}_1, \tau'_1), (\mathcal{C}_2, \tau'_2), \ldots, (\mathcal{C}_{2^K}, \tau'_n)\}$. From the construction of $\mathcal{C}(\tau'_n)$, we can see that there are 2^{K-1-k} identical charging schedulings in $\mathcal{C}(\tau'_n)$ with each only containing the nodes in $R \cup V_0 \cup V_1 \cdots \cup V_k$. Denote by $w(\mathcal{D}_k)$ the cost of the charging scheduling $\mathcal{D}_k$, where $0 \leq k \leq K-1$. And there is one charging scheduling in $\mathcal{C}(\tau'_n)$ containing the nodes in $R \cup V_0 \cup \cdots \cup V_K = R \cup V$, denote by $w(\mathcal{D}_K)$ the cost of the charging scheduling $\mathcal{D}_K$. We then rewrite the upper bound on the service cost in Inequality (9) as

$$2m\sum_{j=1}^{2^K} w(\mathcal{C}_j) = 2m(w(\mathcal{D}_K) + \sum_{k=0}^{K-1} 2^{K-1-k} w(\mathcal{D}_k)). \tag{10}$$

Denote by $\mathcal{D}_k^* = \{D_{k,1}^*, D_{k,2}^*, \ldots, D_{k,q}^*\}$ the set of the optimal q closed tours for the q-rooted TSP problem in graph $G[R \cup V_0 \cup \cdots \cup V_k]$. Since $\mathcal{D}_k$ is an approximate solution by Theorem 2, $w(\mathcal{D}_k) \leq 2w(\mathcal{D}_k^*)$, $0 \leq k \leq K$. Also, by Lemma 3, $w(\mathcal{D}_k^*) \leq \frac{OPT}{m2^{K-k}}$. We have

$$\begin{aligned} &2m(w(\mathcal{D}_K) + \sum_{k=0}^{K-1} 2^{K-1-k} w(\mathcal{D}_k)) \\ &\quad \leq 4m(\frac{OPT}{m} + \sum_{k=0}^{K-1} 2^{K-1-k}\frac{OPT}{m2^{K-k}}) = 2(K+2)OPT. \end{aligned} \tag{11}$$

The time complexity of `Algorithm 3` is analyzed as follows. Partitioning the sensor set V into $K+1$ disjoint subsets $V_0, V_1, \ldots, V_K$ takes $O(n)$ time, based on the assigned charging cycles for the n sensors. Given the sensor set $V(\mathcal{D}_k) = R \cup V_0 \cdots \cup V_k$, it takes $O(|V(\mathcal{D}_k)|^2)$ time to find a 2-approximate solution to the q-rooted TSP problem by Theorem 2, where $0 \leq k \leq K$. Since $V(\mathcal{D}_k) \subseteq V \cup R$, it takes no more than $(K+1)O(n^2) = O(Kn^2)$ time to construct $\mathcal{D}_0, \mathcal{D}_1, \ldots, \mathcal{D}_K$. Then, $\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_{2^K}$ are obtained from $\mathcal{D}_0, \mathcal{D}_1, \ldots, \mathcal{D}_K$ by only replication, and it takes $O(2^K n)$ time. Having these 2^K charging schedulings, the rest ones can be obtained by repeatedly copying these 2^K charging schedulings but assigning them different dispatching times. Therefore, the running time of `Algorithm 3` is $O(Kn^2) + O(2^K n) + O((2m-1)2^K n) = O(Kn^2 + 2m \cdot 2^K n) = O(Kn^2 + (T/\tau_{\min})n)$. ■

VI. Heuristic Algorithm With Variable Maximum Charging Cycles

So far we have developed an approximation algorithm for the service cost minimization problem, assuming that the maximum charging cycle of each sensor is fixed for the given monitoring period. This assumption however sometimes may be restrictive and unrealistic in some applications. In this section we devise a novel heuristic algorithm by removing this assumption.

A. Dynamic Maximum Charging Cycles of Sensors

Within the period T, the energy consumption rates of sensors may dynamically change over time, resulting in the changes of sensor maximum charging cycles eventually. To respond to such a variation, each sensor monitors its own energy information, including its residual energy and its energy consumption rate periodically. Based on the energy information, it predicts its residual lifetime with existing prediction techniques. For example, a sensor can use an Exponentially Weighted Moving-Average prediction technique as follows [4]. Let $\hat{\rho}_i(t+1)$ be the predicted energy consumption rate of sensor v_i at time $t+1$, $\hat{\rho}_i(t+1) = \omega \cdot \rho_i(t) + (1-\omega) \cdot \hat{\rho}_i(t)$, where ω is a constant with $0 < \omega < 1$, $\hat{\rho}_i(t)$ and $\rho_i(t)$ are the predicted and monitored energy consumption rates of sensor v_i at time t, respectively.

Assume that the residual energy of sensor v_i at time t is $re_i(t)$, sensor v_i estimates its residual lifetime $l_i(t)$ and maximum charging cycle $\tau_i(t)$ by $\hat{l}_i(t) = re_i(t)/\hat{\rho}_i(t+1)$ and $\hat{\tau}_i(t) = B_i/\hat{\rho}_i(t+1)$, respectively. Meanwhile, the base station maintains the updated energy information of each sensor. We assume that there is a variation threshold of the maximum charging cycle at each sensor. If the variation is under the predefined threshold, nothing is to be done. Otherwise, the sensor sends an updating request of its energy information to the base station and the base station takes proper actions.

B. Heuristic Algorithm

Assume that the base station receives the maximum charging cycle updatings from some sensors at time t, this implies that the charging schedulings based on the previous maximum charging cycles of these sensors may not be applicable any more, otherwise these sensors will deplete their energy prior to their next chargings. For example, assume that a sensor has changed its maximum charging cycle from a longer one to a shorter one, it might be dead if the sensor is still charged according to its previous longer charging cycle since the sensor now can last for only a shorter cycle once it is fully charged.

The basic idea of the heuristic algorithm is as follows. When the base station receives maximum charging cycle updatings, it checks whether the previous schedulings are still applicable for these updated maximum charging cycles. If so, nothing needs to be done. Otherwise, it re-computes a new series of schedulings, by first applying the approximation algorithm based on the updated maximum charging cycles, followed by modifications to the solution delivered by the approximation algorithm.

Assume that the previous maximum charging cycle of sensor v_i is $\hat{\tau}_i(t-1)$ and it was charged at a charging cycle $\hat{\tau}'_i(t-1)$ in the previous series of schedulings. At time t, the base station receives the maximum charging cycle updating of sensor v_i, which changes from $\hat{\tau}_i(t-1)$ to $\hat{\tau}_i(t)$. The base station then checks the feasibility of the previous schedulings as follows. If $\hat{\tau}'_i(t-1) \leq \hat{\tau}_i(t) < 2\hat{\tau}'_i(t-1)$, the previous schedulings are still feasible as sensor v_i will be charged with a charging cycle $\hat{\tau}'_i(t-1)$ no more than its current maximum charging cycle $\hat{\tau}_i(t)$. Otherwise ($\hat{\tau}_i(t) < \hat{\tau}'_i(t-1)$ or $\hat{\tau}_i(t) \geq 2\hat{\tau}'_i(t-1)$), we re-compute a new series of schedulings based on the updated maximum charging cycles since the previous schedulings are not feasible any more (i.e., $\hat{\tau}_i(t) < \hat{\tau}'_i(t-1)$), or though the schedulings still are feasible, they are not optimal in terms of the service cost (i.e., $\hat{\tau}_i(t) \geq 2\hat{\tau}'_i(t-1)$). In the following, we re-compute a new series of schedulings.

We first invoke the proposed approximation algorithm based on the updated maximum charging cycles. Let $\hat{\tau}_1(t), \hat{\tau}_2(t), \ldots, \hat{\tau}_n(t)$ be the updated maximum charging cycles of the n sensors. Assume that residual lifetimes of the n sensors are $\hat{l}_1(t), \hat{l}_2(t), \ldots, \hat{l}_n(t)$, respectively. We further assume that the solution delivered by the approximation algorithm based on the updated maximum charging cycles consists of

$$\begin{aligned}
&(\mathcal{C}_1, t+\hat{\tau}_1(t))\,,\ (\mathcal{C}_2, t+2\hat{\tau}_1(t))\,, \ldots, \left(\mathcal{C}_{2^K}, t+2^K\hat{\tau}_1(t)\right)\\
&(\mathcal{C}_1, t+\hat{\tau}'_n(t)+\hat{\tau}_1(t))\,,\ (\mathcal{C}_2, t+\hat{\tau}'_n(t)+2\hat{\tau}_1(t))\,, \ldots,\\
&\quad \left(\mathcal{C}_{2^K}, t+\hat{\tau}'_n(t)+2^K\hat{\tau}_1(t)\right)\\
&\quad \vdots\\
&(\mathcal{C}_1, t+x\hat{\tau}'_n(t)+\hat{\tau}_1(t))\,, \ldots, (\mathcal{C}_y, t+x\hat{\tau}'_n(t)+y\hat{\tau}_1(t)
\end{aligned}$$

where $t + x\hat{\tau}'_n(t) + y\hat{\tau}_1(t) < T$, $t + x\hat{\tau}'_n(t) + (y+1)\hat{\tau}_1(t) \geq T$, and x and y are positive integers. The most updated charging cycles of the n sensors in the solution are $\hat{\tau}'_1(t), \hat{\tau}'_2(t), \ldots, \hat{\tau}'_n(t)$, where $\hat{\tau}'_i(t) = 2^{\lfloor \log_2(\hat{\tau}_i(t)/\hat{\tau}_1(t)) \rfloor}\hat{\tau}_1(t)$.

Note that the solution delivered may not be feasible as different sensors may have different amounts of residual energy. This violates the condition of applying the approximation algorithm, that is, all sensors must be fully charged initially. The residual energy in some sensor v_i may not support its operation until its next charging time $t + \hat{\tau}'_i(t)$, i.e., $\hat{l}_i(t) < \hat{\tau}'_i(t)$. Denote by V^a the set of sensors with $\hat{l}_i(t) < \hat{\tau}'_i(t)$. We then schedule the mobile chargers to replenish sensors in V^a to avoid their energy depletion, through adding a new charging scheduling $(\mathcal{C}_0', t)$ and modifying the first 2^K schedulings from $(\mathcal{C}_1, t+\hat{\tau}_1(t)), (\mathcal{C}_2, t+2\hat{\tau}_1(t)), \ldots, (\mathcal{C}_{2^K}, t+2^K\hat{\tau}_1(t))$ to $(\mathcal{C}_1', t+\hat{\tau}_1(t)), (\mathcal{C}_2', t+2\hat{\tau}_1(t)), \ldots, (\mathcal{C}_{2^K}', t+2^K\hat{\tau}_1(t))$. Also, the charging schedulings delivered by the heuristic algorithm after the first 2^K schedulings are the same as them delivered by the approximation algorithm. The rest is to construct the first 2^K+1 charging schedulings.

Let $V_t^a = \{v_i | v_i \in V^a \ \& \ \hat{l}_i(t) < \hat{\tau}_1(t)\}$, which implies that the residual lifetime of each sensor in V_t^a is less than $\hat{\tau}_1(t)$ and $V_t^a \subseteq V^a$. We construct a scheduling $(\mathcal{C}_0, t)$, in which all sensors in V_t^a will be charged at time t. We then, like the node set partition in the approximation algorithm, partition the set $V^a \setminus V_t^a$ into $K+1$ disjoint sets $V_0^a, V_1^a, \ldots, V_K^a$ according to their residual lifetimes, where $K = \lfloor \log_2(\hat{\tau}_n(t)/\hat{\tau}_1(t)) \rfloor$ and a sensor $v_i \in V^a \setminus V_t^a$ is contained in V_k^a if $2^k\hat{\tau}_1(t) \le \hat{l}_i(t) < 2^{k+1}\hat{\tau}_1(t)$. Note that the residual lifetime $\hat{l}_i(t)$ of each sensor v_i in V_k^a at time t is no less than $2^k\hat{\tau}_1(t)$ but no greater than its charging cycle $\hat{\tau}_i'(t)$, i.e., $2^k\hat{\tau}_1(t) \le \hat{l}_i(t) < \hat{\tau}_i'(t)$. To avoid the energy depletion of sensor v_i, we can add it into any one of the schedulings: $\{(\mathcal{C}_0, t), (\mathcal{C}_1, t+\hat{\tau}_1(t)), (\mathcal{C}_2, t+2\hat{\tau}_1(t)), \ldots, (\mathcal{C}_{2^k}, t+2^k\hat{\tau}_1(t))\}$. However, to minimize the service cost, we add sensor v_i into a nearest scheduling $\mathcal{C}_j$, The detailed construction of the 2^K+1 schedulings is as follows.

We construct the 2^K+1 schedulings by iteratively invoking `Algorithm 1` for the q-rooted minimum spanning forest problem. Denote by $V(\mathcal{C}_j^{(k)})$ and $V(\mathcal{C}_j^{(k+1)})$ the constructed node sets of scheduling $\mathcal{C}_j'$ before and after iteration k, respectively, where $0 \le k \le K$. Note that $\mathcal{C}_j^{(k)} = \{C_{j,1}^{(k)}, \ldots, C_{j,q}^{(k)}\}$ and $V(\mathcal{C}_j^{(k)}) = \bigcup_{l=1}^q V(C_{j,l}^{(k)})$. After $K+1$ iterations, we let $V(\mathcal{C}_j') = V(\mathcal{C}_j^{(K+1)})$. We finally obtain scheduling $\mathcal{C}_j'$ by applying `Algorithm 2` for the q-rooted TSP problem in the induced graph $G[V(\mathcal{C}_j')]$. Consequently, each sensor in $V_t^a \cup V_0^a \cup \cdots \cup V_K^a = V^a$ will be charged in time. Initially, let $V(\mathcal{C}_0^{(0)}) = V_t^a \cup R$ and $V(\mathcal{C}_j^{(0)}) = V(\mathcal{C}_j)$, where $1 \le j \le 2^K$. At iteration k $(0 \le k \le K)$, we first construct an auxiliary graph $G^{(k)} = (V_k^a \cup R^{(k)}, E^{(k)}; w^{(k)})$ based on node sets V_k^a and $V(\mathcal{C}_0^{(k)}), V(\mathcal{C}_1^{(k)}), \ldots, V(\mathcal{C}_{2^k}^{(k)})$, where there is a root $r_j^{(k)}$ in $R^{(k)}$ representing node set $V(\mathcal{C}_j^{(k)})$, $0 \le j \le 2^k$, and $E^{(k)} = V_k^a \times V_k^a \cup V_k^a \times R^{(k)}$. Then, $|R^{(k)}| = 2^k+1$. For each edge $(u,v) \in V_k^a \times V_k^a$, $w^{(k)}(u,v)$ is the Euclidean distance between nodes u and v. For each edge $(u, r_j^{(k)}) \in V_k^a \times R^{(k)}$, $w^{(k)}(u, r_j^{(k)})$ is the smallest Euclidean distance between node u and nodes in $V(\mathcal{C}_j^{(k)})$. We then obtain 2^k+1 minimum cost rooted trees $T_0^{(k)}, T_1^{(k)}, \ldots, T_{2^k}^{(k)}$, by invoking `Algorithm 1` on $G^{(k)}$, where tree $T_j^{(k)}$ contains root $r_j^{(k)}$ and $0 \le j \le 2^k$. Note that each sensor in V_k^a is contained in a tree $T_j^{(k)}$ and $V_k^a = V(T_0^{(k)}) \cup V(T_1^{(k)}) \cup \cdots \cup V(T_{2^k}^{(k)}) - R^{(k)}$. Then, the sensors in tree $T_j^{(k)}$ will be charged in scheduling $(\mathcal{C}_j', t + j\hat{\tau}_1(t))$. To this end, we let $V(\mathcal{C}_j^{(k+1)}) = V(\mathcal{C}_j^{(k)}) \cup V(T_j^{(k)}) - \{r_j^{(k)}\}$ if $0 \le j \le 2^k$, otherwise $(2^k + 1 \le j \le 2^K)$, $V(\mathcal{C}_j^{(k+1)}) = V(\mathcal{C}_j^{(k)})$. We refer to this heuristic algorithm as *MinDis-var*.

Theorem 4: There is a heuristic algorithm for the service cost minimization problem with variable maximum charging cycles, which takes $O((\tau_{\max}/\tau_{\min})n^2 + (T/\tau_{\min})n + (\tau_{\max}^2/\tau_{\min}^2))$ time, where $n = |V|$, $\tau_{\max} = \max_{i=1}^n\{\tau_i\}$, and $\tau_{\min} = \min_{i=1}^n\{\tau_i\}$.

Proof: We fist show that the heuristic algorithm delivers a feasible solution. To this end, it is sufficient to show that each sensor $v_i \in V^a$, whose residual lifetime $\hat{l}_i(t)$ at time t is less than its next charging cycle $\hat{\tau}_i'(t)$, will be charged between current time t and time $t + \hat{l}_i(t)$. First, it is obvious that each sensor in V_t^a will be charged at time t. Then, the sensor set $V^a - V_t^a$ is partitioned into $K+1$ subsets $V_0^a, V_1^a, \ldots, V_K^a$, where sensor v_i is contained in V_k^a if $2^k\hat{\tau}_1(t) \le \hat{l}_i(t) < 2^{k+1}\hat{\tau}_1(t)$, $0 \le k \le K$. Following the heuristic algorithm, each sensor v_i in V_k^a will be charged in some scheduling in $\{(\mathcal{C}_0', t), (\mathcal{C}_1', t+\hat{\tau}_1(t)), \ldots, (\mathcal{C}_{2^k}', t+2^k\hat{\tau}_1(t))\}$, i.e., sensor v_i will be charged at some time between t and time $t + 2^k\hat{\tau}_1(t) \le t + \hat{l}_i(t)$, as $2^k\hat{\tau}_1(t) \le \hat{l}_i(t)$.

We then analyze the time complexity of the heuristic algorithm as follows. The invoking of `Algorithm 3` takes time $O(Kn^2 + (T/\tau_{\min})n)$ by Theorem 3. Then, the heuristic algorithm constructs $K+1$ auxiliary graphs $G^{(0)}, G^{(1)}, \ldots, G^{(K)}$ and finds 2^k+1 trees in each graph. For each graph $G^{(k)}$, the time of constructing graph $G^{(k)}$ is: $|V_k^a| + |R^{(k)}| + |V_k^a| \cdot |V_k^a| + |V_k^a| \cdot |R^{(k)}| \cdot |V| = O(n) + O(2^k) + O(n^2) + O(2^k n^2) = O(2^k n^2)$. Also, the time of finding the 2^k+1 trees in graph $G^{(k)}$ is $O(|V_k^a \cup R^{(k)}|^2) = O(n^2) + O(2^k n) + O(2^{2k})$ by Lemma 1. Therefore, the running time of the heuristic algorithm is $O(Kn^2 + (T/\tau_{\min})n) + \sum_{k=0}^K (O(2^k n^2) + O(n^2) + O(2^k n) + O(2^{2k})) = O(Kn^2 + (T/\tau_{\min})n) + O(2^K n^2) + O(2^{2K}) = O(\tau_{\max}/\tau_{\min})n^2 + (T/\tau_{\min})n + (\tau_{\max}^2/\tau_{\min}^2))$ as $\lfloor \tau_{\max}/\tau_{\min} \rfloor = 2^K$. ■

VII. Performance Evaluation

In this section, we evaluate the performance of the proposed algorithms through experimental simulations. We also study the impact of important parameters on the algorithm performance, including network size, data aggregation, and the ratio of the maximum data generation rate to the minimum data generation rate.

A. Simulation Environment

We consider a WSN consisting of from 100 to 500 sensors in a 1000 m × 1000 m square area, in which sensors are randomly deployed. The base station is located at the center of the square. The battery capacity B_i of each sensor v_i is 10.8 kJ [25]. The data sensing rate b_i of each sensor v_i is randomly chosen from an interval $[b_{\min}, b_{\max}]$, where $b_{\min} = 1$ kbps and $b_{\max} = 10$ kbps [25]. The coefficient α in (2) is 2. Furthermore, we assume that each sensor v_i performs data aggregations on both pass-by traffic and self-sensed data with a data aggregation factor θ, i.e., the data transmission rate b_i^{Tx} of sensor v_i is $b_i^{Tx} = \theta \cdot (b_i^{Rx} + b_i)$, where b_i^{Rx} and b_i are the data reception rate and data sensing rate of sensor v_i, respectively, and θ is constant with $0 < \theta \leq 1$ [17]. The default value of θ is 1.

There are 5 depots in the WSN (i.e., $q = 5$) and there is a mobile charger at each depot. To reduce the total travelling distance of the q mobile chargers, one depot is co-located with the base station, as the most energy-consuming sensors in a WSN usually are close to the base station for relaying data from other remote sensors. The rest of $q-1$ depots are randomly distributed in the area. The entire monitoring period T is one year, which is partitioned into equal time slots with each lasting ΔT (ΔT typically is much shorter than T, e.g., ΔT is one month). We assume that the data sensing rate b_i of each sensor $v_i \in V$ does not change within each time slot ΔT. Even if it does change within the time slot, the difference can be neglected.

To evaluate the performance of the proposed algorithms `MinDis` and `MinDis-var` against the state-of-the-art algorithms, we implement three benchmark algorithms of sensor charging `Periodic` [25], [33]–[35], `OnDemand`, and `Partition` of [29], [41], which are described as follows. In algorithm `Periodic`, the base station periodically dispatches the q mobile chargers to charge every sensor in the network with charging period being $\tau_{\min}$. The charging tours of the q chargers will be found by applying `Algorithm 2`. In algorithm `OnDemand`, each sensor sends a charging request to the base station when its residual energy is below a given energy threshold. Having received a set of such requests, the base station then dispatches the q mobile chargers to charge the sensors whose estimated residual lifetimes are less than a given threshold Δl with $\Delta l = \tau_{\min}$. The charging tours of the q mobile chargers are finally obtained by applying `Algorithm 2` for the q-rooted TSP problem in the induced graph of the to-be-charged sensors. Finally, in algorithm `Partition`, the monitoring region is divided into q subregions, in other words, the sensors in the network are first partitioned into q disjoint sets $V_1, V_2, \ldots, V_q$ with each set corresponding to the sensors in its subregion, where a sensor v_i is contained in set V_j if depot r_j is its nearest depot among the q depots. Then, the sensors in V_j will be charged by only the mobile charger located at depot r_j, where $1 \leq j \leq q$. Each sensor $v_i \in V_j$ sends a charging request to the base station when it will deplete its energy soon. Once receiving the request, the base station dispatches the mobile charger at depot r_j to charge a subset V'_j of sensors of V_j with the residual lifetime of each sensor in V'_j being less than a given threshold Δl_j, i.e., $V'_j = \{v_i | v_i \in V_j, l_i < \Delta l_j\}$, and the charging tour of the charger is a shortest closed tour visiting the sensors in V'_j and depot r_j, where $\Delta l_j = \tau^j_{\min}$ and $\tau^j_{\min}$ is the shortest maximum charging cycle of sensors in set V_j, i.e., $\tau^j_{\min} = \min_{v_i \in V_j}\{\tau_i\}$.

It must be mentioned that each value in all figures is the average of the results by applying each mentioned algorithm to 100 different network topologies with the same network size.

B. Performance With Fixed Maximum Charging Cycles

We first evaluate the performance of the proposed approximation algorithm `MinDis` against algorithms `OnDemand`, `Partition`, and `Periodic` by varying network size n, assuming that maximum charging cycles within T are fixed. Fig. 1 shows that the service costs delivered by algorithms `MinDis`, `OnDemand`, and `Partition` are much less than that by algorithm `Periodic`. For example, Fig. 1 demonstrates that the service cost by algorithm `MinDis` only about from 15% to 25% of the cost by algorithm `Periodic`, and the costs by algorithms `OnDemand` and `Partition` are from 19% to 28% of that by algorithm `Periodic`. Also, it can be seen from Fig. 1 that the proposed algorithm `MinDis` delivers a solution with the least service cost of mobile chargers, while the service costs delivered by algorithms `OnDemand` and `Partition` are almost identical and the one by algorithm `OnDemand` is only marginal better than that by algorithm `Partition`, ranging from 0.3% to 1.5% improvement. Fig. 2 plots the ratio of the service cost by algorithm `MinDis` to that by algorithm `OnDemand` with a confidence interval of 95%, from which it can be seen that the service cost by algorithm `MinDis` is about from 79% to 93% of the service cost by algorithm `OnDemand` and the ratio becomes smaller with the increase of the network size. In the following, we only compare the performance of algorithms `MinDis`, `OnDemand`, and `Partition`, and omit the performance of algorithm `Periodic`, since the service cost delivered by the algorithm is much higher than that by the three algorithms.

We then examine the impact of the data aggregation factor θ on the performance of the three algorithms, by decreasing θ from 1.0 to 0.1. Fig. 3 clearly presents that the service costs by algorithms `MinDis`, `OnDemand`, and `Partition` decrease when θ becomes smaller and the service costs by the three

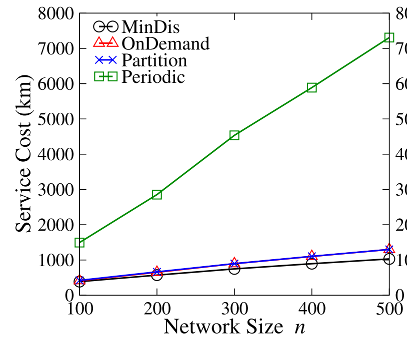

Fig. 1. Performance of algorithms `MinDis`, `OnDemand`, `Partition`, and `Periodic` by varying the network size from 100 to 500 sensors.

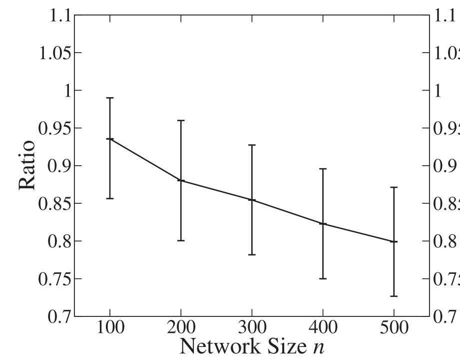

Fig. 2. Ratio of the service cost by algorithm `MinDis` to that by algorithm `OnDemand` with confidence intervals of 95%.

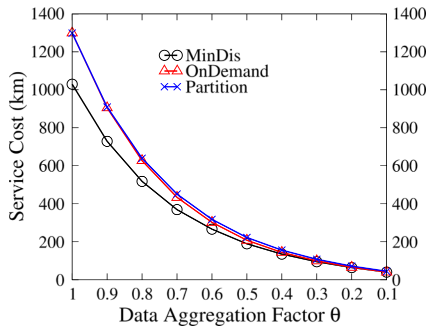

Fig. 3. Performance of algorithms `MinDis`, `OnDemand`, and `Partition` by decreasing the data aggregation factor θ from 1.0 to 0.1 when $n = 500$.

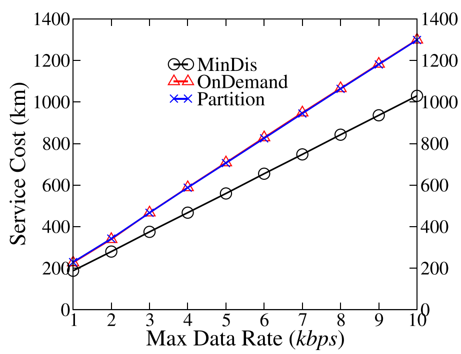

Fig. 4. Performance of algorithms `MinDis`, `OnDemand`, and `Partition` by varying the maximum data rate $b_{\max}$ from 1 kbps to 10 kbps when $b_{\min} = 1$ kbps and $n = 500$.

algorithms are almost identical when $\theta = 0.1$. The rationale behind the phenomenon is that the data transmission rates of sensors can be greatly reduced by a small data aggregation factor θ while the sensor energy consumption on data transmission is usually the dominant one [17]. As a result, the maximum charging cycles of sensors becomes longer with a smaller value of θ and the service cost of mobile chargers thus is significantly reduced.

We finally study the impact of the maximum data rate $b_{\max}$ on the algorithm performance, by varying $b_{\max}$ from 1 kbps to 10 kbps when $b_{\min} = 1$ kbps. Fig. 4 demonstrates that the service cost by algorithm `MinDis` is only from 79% to 82% of the service cost by algorithm `OnDemand` and their performance gap increases when $b_{\max}$ becomes larger. Furthermore, Fig. 4 clearly shows that the service costs by the three algorithms increase with the increase of $b_{\max}$. This is because that the energy consumption rates of sensors becomes higher when the maximum data rates of sensors $b_{\max}$ increases. As a result, sensors must be charged more frequently, which incurs more service cost of the mobile chargers.

In the following, we omit the performance of algorithm `Partition`, since the service costs by algorithms `OnDemand` and `Partition` are almost identical, which have already been shown in Figs. 1, 3, and 4.

C. Performance With Variable Maximum Charging Cycles

In this subsection, we first investigate the performance of the proposed heuristic algorithm `MinDis-var` against algorithm `OnDemand` with variable maximum charging cycles.

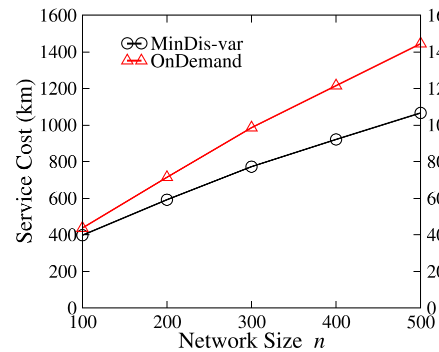

Fig. 5. Performance of algorithms `MinDis-var` and `OnDemand` by varying the network size when ΔT is one month.

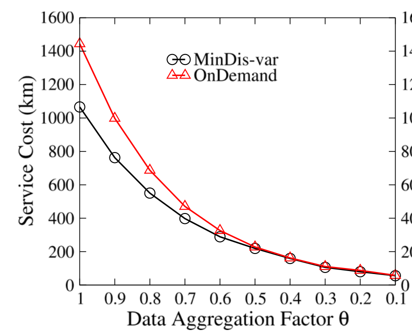

Fig. 6. Performance of algorithms `MinDis-var` and `OnDemand` by decreasing the data aggregation factor θ from 1.0 to 0.1 when ΔT is one month and $n = 500$.

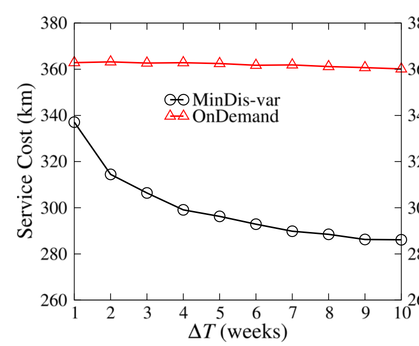

Fig. 7. Performance of algorithms `MinDis-var` and `OnDemand` by varying ΔT from 1 week to 10 weeks when $b_{\min} = 1$ kbps, $b_{\max} = 2$ kbps, and $n = 500$.

Figs. 5 and 6 illustrate the performance of both algorithms, by varying network size n and the data aggregation factor θ, respectively. It can be seen that algorithm `MinDis-var` is still very competitive as it did under fixed maximum charging cycles.

We finally study the impact of the dynamics of maximum charging cycles on the algorithm performance, by varying parameter ΔT from 1 week (i.e., extremely dynamic) to 10 weeks (i.e., rather stable). Fig. 7 shows that the service cost by algorithm `MinDis-var` decreases with the increase of the stability of the sensor maximum charging cycles (a larger ΔT), while the service cost by algorithm `OnDemand` almost does not change with the increase of ΔT. We also note that algorithm `MinDis-var` significantly outperforms algorithm

OnDemand even when the maximum charging cycles are stable only in a short time slot ΔT (e.g., $\Delta T=$ one week), which indicates that algorithm MinDis-var can quickly adapt to the changes of maximum charging cycles.

VIII. Conclusion

In this paper, we studied the use of multiple mobile chargers to charge sensors in a wireless sensor network so that none of the sensors runs out of energy for a given monitoring period, for which we first formulated a novel service cost minimization problem of finding a series of charging schedulings of the mobile chargers to maintain the perpetual operations of sensors so that the total travelling distance of the mobile chargers for the period is minimized. As this optimization problem is NP-hard, we then devised an approximation algorithm with a provable approximation ratio if the maximum charging cycle of each sensor is fixed in the given monitoring period. Otherwise, we developed a novel heuristic solution through modifications to the approximate solution. We finally evaluated the performance of the proposed algorithms through extensive experimental simulations and experimental results showed that the proposed algorithms are very promising.

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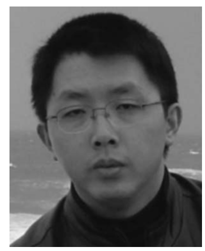

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