Minimizing the Maximum Charging Delay of Multiple Mobile Chargers Under the Multi-Node Energy Charging Scheme

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Abstract—Wireless energy charging has emerged as a very promising technology for prolonging sensor lifetime in wireless rechargeable sensor networks (WRSNs). Existing studies focused mainly on the one-to-one charging scheme that a single sensor can be charged by a mobile charger at each time, this charging scheme however suffers from poor charging scalability and inefficiency. Recently, another charging scheme, the multi-node charging scheme that allows multiple sensors to be charged simultaneously by a mobile charger, becomes dominant, which can mitigate charging scalability and improve charging efficiency. However, most previous studies on this multi-node energy charging scheme focused on the use of a single mobile charger to charge multiple sensors simultaneously. For large scale WRSNs, it is insufficient to deploy only a single mobile charger to charge many lifetime-critical sensors, and consequently sensor expiration durations will increase dramatically. To charge many lifetime-critical sensors in large scale WRSNs as early as possible, it is inevitable to adopt multiple mobile chargers for sensor charging that can not only speed up sensor charging but also reduce expiration times of sensors. This however poses great challenges to fairly schedule the multiple mobile chargers such that the longest charging delay among sensors is minimized. One important constraint is that no sensor can be charged by more than one mobile charger at any time due to the fact that the sensor cannot receive any energy from either of the chargers or the overcharging will damage the recharging battery of the sensor. Thus, finding a closed charge tour for each of the multiple chargers such that the longest charging delay is minimized is crucial. In this paper we address the challenge by formulating a novel longest charging delay minimization problem. We first show that the problem is NP-hard. We then devise the very first approximation algorithm with a provable approximation ratio for the problem. We finally evaluate the performance of the proposed algorithms through experimental simulations. Experimental results demonstrate that the proposed algorithm is promising, and outperforms existing algorithms in various settings.

Index Terms—Wireless rechargeable sensor networks, multi-node energy charging, multiple mobile chargers, multiple charging tour scheduling, charging delay minimization, approximation algorithms, maximal independent sets, wireless energy transfer

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1 INTRODUCTION

WIRELESS Sensor Networks (WSNs) have been widely applied in various industries, from environmental monitoring, smart grid network monitoring, disaster forecasting to cutting-edge Smart Homes, Smart Cities, and the Internet of Things (IoTs) [11], [23], [44]. They all rely on ubiquitous sensors to capture multi-dimensional data from surrounding objects for various purposes. However, a sensor is usually powered by an on-board battery with limited energy capacity, sensor lifetime prolongation remains a critical issue [20], [21]. Although renewable energy harvesting technologies [13], [33] have been proposed to accumulate energy from ambience, such as solar and wind energy, these methods are sensitive to surrounding environments, thus they cannot provide stable energy to sensors.

Wireless energy charging was proposed to address stable energy provisioning issues in WSNs [15], [24], [32], [40], [42]. It can be achieved by charging a nearby sensor with a *Mobile Charging Vehicle (MCV)*. This technology possesses many advantages as it does not require direct contact between the mobile charger and the sensor, or even it does not require line-of-sight (LOS) as long as the charging device is within the wireless energy transmission range of the charger. Also, compared to renewable energy harvesting, wireless energy transfer can provide stable energy to sensors. This charging process can be applied in an on-demand manner when devices request to be charged. The powerfulness of wireless energy charging technology brings about broad commercial applications [3], [27], [35], [43].

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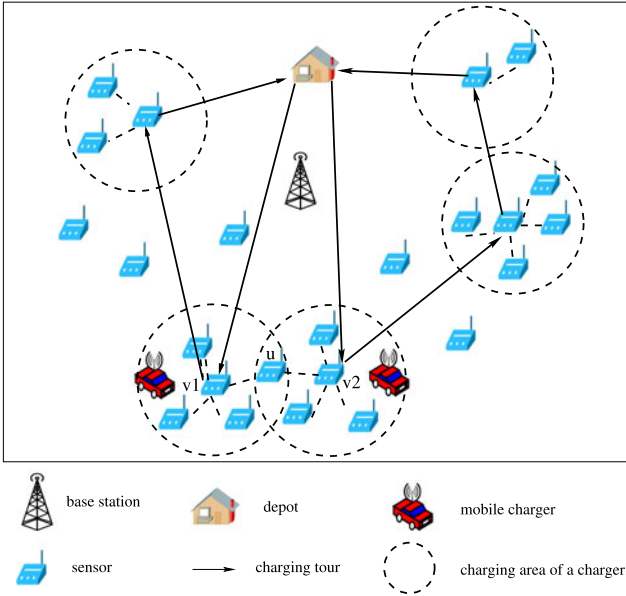
Fig. 1. An example of multi-node energy charging with two mobile chargers, where sensor u will be charged by the two chargers simultaneously if they stay at locations v_1 and v_2, respectively, at the same time.

Despite wireless energy transfer is a promising technique to prolong sensor lifetime, its energy charging efficiency and scalability has been explored in the past. For example, Kurs *et al.* [16] proposed a *multi-node energy charging* scheme, where multiple sensors can be charged simultaneously by properly tuning operation frequencies of both the sender and the receiver coils, enabling high energy transfer efficiency and longer energy transmission range.

Most existing studies on the multi-node energy charging scheme focused on the use of a single mobile charger to charge multiple sensors simultaneously [14], [25], [28], [38]. However, for a large scale WRSN, it is insufficient to deploy only a single mobile charger to charge many lifetime-critical sensors, as it takes a long time (e.g., 30-90 minutes) to fully charge a sensor, and consequently sensor expiration durations will increase dramatically [34], [42]. Instead, to charge many lifetime-critical sensors as early as possible, the use of multiple mobile chargers can speed up sensor charging and reduce their expiration durations, thereby improving the monitoring quality of the sensor network.

The adoption of multiple mobile chargers with each charging multiple sensors simultaneously in WRSNs poses great challenges. There are several fundamental questions to be addressed including:

(i) How to schedule the charging tours of multiple mobile chargers to ensure that all lifetime-critical sensors can be charged as soon as possible?

(ii) Since a sensor may be in the charging coverage ranges of multiple chargers at different charging locations, how to ensure that the sensor will not be charged by more than one charger at any time? For example, Fig. 1 shows that sensor u will be charged by two chargers simultaneously if they stay at locations v_1 and v_2, respectively. However, sensor u cannot be charged by the two mobile chargers at the same time due to the facts that (1) the sensor cannot receive any energy from either of the two chargers. Naderi *et al.* [26] showed that radio interference occurs when multiple chargers charge one sensor simultaneously and such interference may result in a lower energy transfer efficiency than that with a single charger; and (2) the aggregated electromagnetic radiation from the two chargers is too high, and thus threatens human health [6].

(iii) What is the charging duration of each charger at each its charging location to ensure that all sensors in its charging coverage range will be fully charged? As a sensor could be in the charging coverage ranges of multiple chargers at different charging locations, some sensors have already been charged when a charger moves to a location in which the sensor is in its charging coverage range. In this paper, we will address the aforementioned challenges by developing efficient solutions for them.

The novelty of this paper is that we study the efficient scheduling of multiple mobile chargers to charge lifetime-critical sensors, where each charger is able to replenish multiple sensors simultaneously in its charging range. To the best of our knowledge, we are the first to formulate a novel scheduling problem - the longest charging delay minimization problem, we aim to charge multiple sensors simultaneously, by deploying $K \geq 1$ mobile chargers and finding a closed charging tour for each of the K mobile chargers while no sensor can be charged by more than one mobile charger at any time. We develop the very first approximation algorithm for the problem through exploring the combinatorial property of the problem. The design and analysis techniques in the development of the approximation algorithm may have independent interest in other approximation algorithms developments.

The main contributions of this paper are as follows. We first formulate a novel longest charging delay minimization problem by adopting $K \geq 1$ mobile chargers with each enabling to charge multiple sensors simultaneously while no sensor can be charged by more than one charger at any time. We aim to minimize the longest charging delay among the K chargers, by finding a charging tour for each of the K mobile chargers, where the total delay of a charger per tour is the sum of the charging duration at each location and the travel delay in its charging tour. We then devise an approximation algorithm with a constant approximation ratio for the problem. We finally evaluate the performance of the proposed algorithm through experimental simulations. Simulation results show that the proposed algorithm is promising. Especially, the longest charging delay among the K chargers delivered by the proposed algorithm is around 50 percent shorter than those by existing algorithms.

The rest of the paper is organized as follows. Section 2 reviews related work. Section 3 introduces notions, notations, and the problem definition. The NP-hardness of the problem is also shown in this section. Section 4 deals with the longest charging delay minimization problem. Section 5 analyzes the proposed algorithm. Section 6 evaluates the performance of the proposed approximation algorithm empirically, and Section 7 concludes the paper.

2 RELATED WORK

Wireless energy transfer technology based on strongly magnetic resonances [15] has been regarded as a breakthrough

technology for lifetime prolongation of sensors in wireless rechargeable sensor networks (WRSNs) [2], [25], [32], [37]. Several studies on wireless energy charging have been conducted, by applying a mobile charger to charge sensors in WRSNs [12], [22], [25], [34]. For example, Shi *et al.* [32] theoretically studied applying this technique to charge sensors in WSNs by dispatching a mobile charging vehicle periodically such that the network can operate perpetually. Liang and Luo [19] studied multiple mobile chargers for sensor charging under the one-to-one charging scheme, for which they proposed a heuristic by a reduction to a series of minimum weight maximum matching problems. Their algorithm however cannot be extended for the problem under the multi-node charging scheme, as the constraint that a sensor cannot be charged by more than one MCV at any time does not exist in the one-to-one charging scheme. Furthermore, there is not any guarantee of their solution from the optimal one. Wu *et al.* [36] formulated a cooperative charging problem, by using multiple mobile chargers to charge sensors such that none of sensors will run out of energy. They aimed to minimize the energy consumption of mobile chargers by adopting genetic algorithms. Xu *et al.* [41] considered sensor charging by employing multiple mobile chargers. They proposed an approximation algorithm for finding a charging tour for each of the mobile chargers such that all sensors are charged and the total expiration duration of all sensors is minimized, assuming that different sensors have different energy depletion rates. Liang *et al.* [20], [21] considered a problem of minimizing the number of mobile chargers to charge a set of sensors, assuming that the energy capacity of each mobile charger is capacitated. They developed an approximation algorithm for the problem. Also, Liang *et al.* [22] studied the charging utility maximization problem by deploying one mobile charger, for which they proposed efficient approximation algorithms under both full charging and partial charging models, respectively.

All the aforementioned studies so far are under the one-to-one charging scheme, i.e., each mobile charger can charge one sensor only. However, this charging scheme is neither energy-efficient nor scalable for large scale sensor networks. Fortunately, there is another charging scheme, which is refereed to as *the multi-node charge scheme*, or the one-to-many charge scheme, where a single mobile charger can charge all sensors within its energy charging range simultaneously. Under this scheme, Xie *et al.* [38] were the first to study multi-node wireless energy charging in WRSNs with the aim to minimize energy consumption of the mobile charger, by periodically dispatching a mobile charger for sensor charging and minimizing the sojourn time at each charging location. They [39] later considered the use of a mobile charger for both sensor charging and data collection with the aim of minimizing the energy consumption of the whole network under the constraints that none of the sensors will run out of energy and all collected data must be relayed to the base station. For both mentioned studies, they assumed that the traveling path of the mobile charger is given in advance. However, planning a charging path and choosing sojourn locations in the path for the mobile charger are non-trivial in multi-node charging scenarios. Ma *et al.* [25] recently considered the multi-node charging scheme for a single mobile charger, by proposing a framework to measure the charging utility gain of each charged sensor. They proposed heuristic and approximation algorithms for maximizing the accumulative charging utility gain, subject to the energy capacity of the mobile charger. Khelladi *et al.* [14] investigated an on-demand multi-node charging problem. They aimed at minimizing the number of stopping points and energy consumption of a mobile charger in its charging tour by adopting a threshold-based charging strategy and grouping requested sensors through clique partitioning. Their heuristic however is unscalable. In reality, the number of sensors deployed in a WRSN usually is quite large, it is not realistic that all sensors can be charged by a mobile charger within a single tour. To shorten sensor expiration times and to explore the charging scalability, multiple mobile chargers should be employed for large-scale WRSNs, and novel approximation algorithms for the longest charging delay minimization problem of multi-node charging with multiple mobile chargers are desperately needed.

There are several related studies in sensory data collection with data mules [4], [10], [17]. For example, Levin *et al.* [17] considered the use of a single data mule to collect data from sensors, while taking into account the data redundancy of nearby sensors. They studied the problem of finding a tour for the data mule such that both the traveling distance of the data mule and the information uncertainty of the data collected from nearby sensors are minimized. Crowcroft *et al.* [4] studied the problem of efficient data recovery with data mules, by dispatching them to collect data from the sensors that are disconnected from the base station. They devised approximation algorithms for minimizing the sum of tour lengths of the data mules. On the other hand, Hermelin *et al.* [10] investigated the problem of scheduling data mules to replace failed sensors, such that both the downtime durations of sensors and the traveling distances of the data mules are minimized.

It is also noticed that the problem considered in this paper is related to the TSP problem with neighborhoods (TSPN) too that is to find a shortest closed tour to visit at least a point in each of n given regions in the euclidean plane [5], [29]. Dumitrescu *et al.* [5] proposed a constant approximation algorithm for the TSPN. However, their proposed algorithm for TSPNs is not applicable to the problem in this paper, as we consider minimizing the longest charging tour duration among the charging tours of multiple mobile chargers, rather than a single charging tour for a single charging tour for a single mobile charger only. Moreover, there is an important constraint on our problem, that is, no sensor can be charged by more than one mobile charger simultaneously at any time.

3 PRELIMINARIES

In this section, we first introduce the system model, notions and notations. We then define the problem precisely.

3.1 Network Model

We consider a Wireless Rechargeable Sensor Network $G_s = (V, E)$ consisting of a set V of stationary sensors distributed over a two-dimensional space, E is the set of edges, and there is an edge between two sensors if they are within

the transmission range of each other. There is a fixed base station, which is the sink node for sensor data collection. Assume that there is a routing protocol for data collection that relays sensing data generated from sensors to the base station through multi-hop relays. For example, each sensor sends its sensing data to the base station via the path with the minimum energy consumption [30].

Each sensor $v \in V$ is powered by an on-board rechargeable battery with an energy capacity B_v, sensors consume energy on sensing, data processing, and data transmission. Denote by RE_v the residual energy of sensor v when it requests for charging. Without loss of generality, we assume that there is a sufficient energy supply to the base station, it thus has no energy constraint.

We assume that there is a depot v_0 for *Mobile Charging Vehicles (MCVs)*, which may or may not be co-located with the base station, and there are K (≥ 1) MCVs located at the depot initially. Each MCV travels at a constant speed s for sensor charging.

Each sensor sends a charging request to the base station via its routing path in the network G_s when its residual energy falls below a given threshold. The base station then identifies a set $V_s \subseteq V$ of lifetime-critical sensors for energy charging [42], and schedules the MCVs to charge these lifetime-critical sensors.

3.2 Multi-Node Wireless Energy Charging, Mobile Chargers and Their Charging Tours

The technique of wireless energy transfer to multiple sensors simultaneously was invented by Kurs *et al.* [16]. They showed that the overall energy efficiency can be significantly improved by proper tuning of the coupled resonators when multiple receivers instead of a single receiver are charged simultaneously. This multi-node wireless energy charging technique is a promising technique that can address both charging efficiency and scalability for large-scale wireless rechargeable sensor networks. Especially, with the wide applications of the Internet of Things (IoTs), smart cities, and smart homes in the near future, it is expected that large-scale sensor networks with thousands of sensors will become a norm in most real WSN deployments. For example, there are over 1,000 sensors in the each of the two real sensor networks *GreenOrbs* and *CitySee* for forest surveillance and urban CO_2 monitoring, respectively [9]. The multi-node energy charging technique can significantly improve charging efficiency even for a sensor network containing only a few hundreds of sensors, due to the following two reasons: (i) Multiple sensors usually are deployed to monitor a critical Point of Interest (PoI) in a sensor network in case one of the sensors fails. These sensors may be co-located or located close to each other [31]; (ii) To save the human cost of sensor deployment, different types of sensors (e.g., a temperature sensor and a humility sensor) can be deployed at the same location, while the cost of each of them (smart dust) is inexpensive. Then, it does make sense to charge these sensors simultaneously.

In this paper, we will adopt this *multi-node energy charging* scheme, where multiple sensors can be charged simultaneously if they are within the energy transmission range of a mobile charger.

To maintain long-term operations of WRSNs and to minimize the expiration durations of sensors, multiple MCVs usually are employed to charge the sensors in V_s, where each MCV is equipped with a wireless charger that can charge multiple sensors simultaneously. We assume that all MCVs are located at depot v_0 initially. The MCVs are dispatched from depot v_0 by the base station for sensor charging through travelling along the scheduled closed tours for them. Ideally, an MCV can stop at any location in the monitoring area for sensor charging, and each stop location is referred to as a *sojourn location* of the mobile charger. However, this introduces infinite numbers of potential sojourn locations for MCVs. For the sake of problem tractability, we assume that MCVs can only stop at the locations co-located with sensors. Such an assumption has been adopted by the work in [14], [25], too.

As we consider multi-node energy charging simultaneously, once an MCV stops at a location, it can charge all sensors within its energy charging range. However, one very critical constraint in the scheduling of MCVs for their charging tours is that if there is a sensor within the charging ranges of more than one MCV, the sensor cannot be charged by any of these MCVs at the same time. Fig. 1 is an illustrative example of multi-node energy charging by two mobile chargers. It can be seen that sensor u will be charged by the two MCVs simultaneously, if the two MCVs replenish sensor energy at v_1 and v_2 at the same time. This constraint makes the charging tour scheduling of multiple MCVs very difficult.

When an MCV stops at a sensor node $v \in V_s$, sensor v and its neighbors in $N_c(v)$ within its energy charging range γ can be simultaneously charged, where $N_c(v) = \{u \mid d(u,v) \leq \gamma, u \in V_s, u \neq v\}$, $d(u,v)$ is the euclidean distance between sensor nodes u and v, and γ is the charging radius of the MCV, e.g., $\gamma = 2.7\ m$ [15]. Denote by $N_c^+(v) = \{v\} \cup N_c(v)$, only all sensors in $N_c^+(v)$ have been fully charged, the MCV can move to the next sojourn location for its sensor charging.

Denote by P the charging output power of each MCV. Assume that the charger stops at a sensor v. Following the seminal work of Kurs *et al.* [15], the *energy transfer efficiency* μ_{uv} for charging a sensor u in $N_c^+(v)$ decreases with the increase of the distance $d(u,v)$, where $d(u,v)$ is the euclidean distance between u and v. Xie *et al.* [38] showed that

$$\mu_{uv} = -0.0958 \cdot d^2(u,v) - 0.0377 \cdot d(u,v) + 1, \qquad (1)$$

where $0 \leq d(u,v) \leq \gamma = 2.7\ m$. It can be seen that $\mu_{uv} = 0.2$ if $d(u,v) = \gamma = 2.7\ m$. Then, $0.2 \leq \mu_{uv} \leq 1$. Let $\mu_{min} = 0.2$ and $\mu_{max} = 1$. By taking into account the loss of energy to heat, the charging rate P_{uv} for sensor u from the MCV at v can be calculated as

$$P_{uv} = \mu_{uv} \cdot \eta \cdot P, \qquad (2)$$

where η is the battery efficiency, e.g., $\eta = 90\%$ for Li-ion batteries [1].

We construct a charging graph $G_c = (V_s, E)$, where V_s is the set of to-be-charged sensors, and there is an edge in E between two sensors if their distance is no greater than the charging range γ. For the sake of convenience, we assume that energy leaking of sensors during this multi-node energy charging process is negligible, as their energy consumption rates are much lower than the charging rate of an MCV and there are multiple MCVs [6], [12], [18], [41], [42].

We also assume that an MCV has sufficient energy for traveling and sensor charging per charging tour as we employ sufficient numbers of MCVs for sensor charging as needed. The base station serves as not only the data collector of the network but also the scheduler of MCVs. When an MCV finishes its charging tour, it returns to depot v_0 to recharge itself for its next charging tour. Each charging tour C_k of MCV k is a *closed tour* including the depot.

3.3 Problem Definition

In this paper, we formulate the following multiple mobile chargers tour scheduling problem, by leveraging the multi-node energy charging technique. Given a set V_s of on-demand charging sensors, each sensor $u \in V_s$ has an energy capacity B_u and its current residual energy RE_u, let t_u be the charging duration for charging sensor u to its full capacity by an MCV at location v. Then, t_u is defined as

$$t_u = \frac{B_u - RE_u}{P_{uv}}, \tag{3}$$

where P_{uv} is the charging rate to sensor u, see Eq. (2).

Recall that an MCV located at v can charge all sensors within its charging range. To ensure that all sensors in the range will be fully charged, the longest charging duration of the MCV at v is upper bounded by

$$\tau(v) = \max_{u \in N_c^+(v)} \{t_u\}. \tag{4}$$

We assume that there are K MCVs located at depot v_0 initially. Each MCV at its sojourn location can charge multiple sensors simultaneously as long as the sensors are within its charging radius γ.

Let $V(C_k) = \{v_{k,i_j} \mid 1 \le j \le k\}$ be the set of sojourn locations in the closed tour C_k of MCV k with $1 \le k \le K$. Then, $\cup_{k=1}^{K} V(C_k) \subseteq V_s$, $\cup_{k=1}^{K} \cup_{v \in V(C_k)} N_c^+(v) = V_s$ and $V(C_i) \cap V(C_j) = \{v_{k,i_0}\}$ if $i \ne j$, where $v_{1,i_0} = v_{2,i_0} = \cdots = v_{K,i_0}$ is the depot of the K MCVs.

Let tour $C_k = \langle v_{k,i_0}, v_{k,i_1}, v_{k,i_2}, \ldots, v_{k,i_{n_k}}, v_{k,i_0} \rangle$, where n_k is the number of sojourn locations in tour C_k. Also, let $N_c'(v_{k,i_l})$ be the set of sensors that have not been charged by any MCV and the sensors are within the charging range of sojourn location v_{k,i_l}, when MCV k arrives at v_{k,i_l}, where $1 \le l \le n_k$. It can be seen that $N_c'(v_{k,i_l}) \subseteq N_c^+(v_{k,i_l})$, where $N_c^+(v_{k,i_l})$ is the set of sensors within the charging range of v_{k,i_l}. Denote by $\tau'(v_{k,i_l})$ the actual charging time of MCV k at sojourn location v_{k,i_l} in C_k. Then

$$\tau'(v_{k,i_l}) = \max_{u \in N_c'(v_{k,i_l})} \{t_u\}. \tag{5}$$

It can be seen that $\tau'(v_{k,i_l}) \le \tau(v_{k,i_l})$, as some sensors in the charging range of MCV k at its sojourn location v_{k,i_l} may have already been charged by the mobile charger itself or other mobile chargers prior to its arrival at the current sojourn location v_{k,i_l}. The *charge delay* of MCV k along C_k thus is

$$T'(k) = \sum_{l=0}^{i_k-1} (\tau'(v_{k,l}) + d(v_{k,l}, v_{k,l+1})/s) + d(v_{k,i_k}, v_{k,0})/s, \tag{6}$$

Let $T(k)$ be the upper bound on the delay $T'(k)$ of MCV k on its charging tour $C_k = \langle v_{k,i_0}, v_{k,i_1}, v_{k,i_2}, \ldots, v_{k,i_{n_k}}, v_{k,i_0} \rangle$. Then

$$T(k) = \sum_{l=0}^{i_k-1} (\tau(v_{k,i_l}) + d(v_{k,i_l}, v_{k,i_{l+1}})/s) + d(v_{k,i_k}, v_{k,0})/s, \tag{7}$$

where $d(v_{k,l}, v_{k,i_{l+1}})/s$ is the travel time of MCV k from its current sojourn location v_{k,i_l} to its next sojourn location $v_{k,i_{l+1}}$ with a constant speed s. Clearly, $T'(k) \le T(k)$.

The longest charging delay minimization problem then is to find K node-disjoint closed tours for the K MCVs (all sojourn locations form the set of nodes for k-node-disjoint closed tours) to cover all sensor nodes $v \in V_s$ such that the longest charging delay $\max_{1 \le k \le K} \{T'(k)\}$ among the K closed tours is minimized, subject to that no sensor can be charged by more than one MCV at the same time. In other words, let u and v be the sojourn locations of two MCVs currently, assuming that they arrive at their sojourn locations u and v at time points s_u and s_v, then their charging finish times are $f_u = s_u + \tau'(u)$ and $f_v = s_v + \tau'(v)$, respectively. We say that these two MCVs are *overlapping* with each other at u and v if there is a sensor $w \in N_c^+(u) \cap N_c^+(v)$ in their charging overlapping area and their charging time intervals $[s_u, f_u]$ and $[s_v, f_v]$ overlap with each other, i.e., $[s_u, f_u] \cap [s_v, f_v] \ne \emptyset$, or sensor s will be charged by both of them at any time point in $[s_u, f_u] \cap [s_v, f_v]$. This implies that it is prohibited that two MCVs at u and v can charge sensors at the mentioned time intervals.

Definition 1. *Given a set of sensors V_s to be charged with each sensor $v \in V_s$ having its residual energy RE_v, there are K mobile chargers (MCVs) to charge the sensors, the* longest charging delay minimization problem *in the wireless sensor network $G = (V_s, E_s)$ is to find a closed charging tour $C_k = \langle v_{k,i_0}, v_{k,i_1}, v_{k,i_2}, \ldots, v_{k,i_{n_k}}, v_{k,i_0} \rangle$ including depot v_{k,i_0} for each mobile charger k such that the longest charging delay among the K MCVs is minimized, subject to that no sensor can be charged by more than one MCV at any time and no common node in each closed charging tour except the depot, assuming that v_{k,i_0} is depot v_0 with $1 \le k \le K$.*

The rationale behind the problem definition is that we aim to make each requested sensor to be charged as soon as possible to reduce its potential expiration time.

Theorem 1. *The longest charging delay minimization problem is NP-hard.*

Proof. We consider a very special case of the problem where $K = 1$ and the energy charging range is $\gamma = 0$. It can be seen that this special case of the problem is equivalent to the well-known Traveling Salesman Problem, which is NP-hard. Therefore, the longest charging delay minimization problem is NP-hard too. □

4 APPROXIMATION ALGORITHM FOR THE LONGEST CHARGING DELAY MINIMIZATION PROBLEM

In this section we assume that K MCVs are employed, and we aim to devise an approximation algorithm for the longest charging delay minimization problem. We start with the overview of the proposed algorithm. We then consider a special

case of the problem where to-be-charged sensors can be partitioned into multiple groups, and there is no overlapping of charging ranges of the MCVs for sensors in different groups. For this special case, we devise an approximation algorithm, and we then extend this approximate solution for the original problem where the charging ranges of the K MCVs are allowed to be overlapping with each other, by modifying the solution to include extra sojourn locations to cover uncovered sensors. We finally show the correctness of the solution and analyze the approximation ratio and time complexity of the proposed approximation algorithm.

4.1 Overview of the Proposed Algorithm

The basic idea behind the proposed algorithm is as follows. We first choose a subset of sensors in V_s such that the sensors in the subset can be partitioned into multiple disjoint groups and the sensors in different groups can be charged simultaneously by different MCVs at their sojourn locations. We then find an approximate solution to the problem of concern in this subset. It can be seen that this approximate solution is part of the solution to the original problem, we then extend this approximate solution by including the other sojourn locations of the MCVs that cover the sensors that are not in the subset, and modify the approximate solution through expanding its K closed tours to include the identified sojourn locations. We finally show that the modified solution is an approximate solution to the longest charging delay minimization problem in set V_s.

4.2 An Approximate Solution Without Overlapping of Charging Ranges of MCVs

As mentioned, we here identify a subset of V_s such that the sensors in the subset can be partitioned into multiple groups and each group can be charged by an MCV at one of its sensor locations.

To this end, we first construct a charging graph $G_c = (V_s, E)$, where V_s is the set of to-be-charged sensors, and there is an edge in E between two sensors if their distance is no greater than the charging range γ. We find a maximal independent set (MIS) S_I in the charging graph G_c, $S_I \subseteq V_s$. Then, the distance between any nodes u and v in S_I is strictly larger than the charging range γ. Also, each node in S_I is a sojourn location of one MCV among the K MCVs for charging. In other words, if the K MCVs are located only at locations in S_I, all sensors in V_s will be charged by the MCVs. For example, Fig. 2a illustrates the sensors in set S_I in a sensor network, where each dotted circle centered at a sensor represents the charging area of an MCV, such that the MCV can charge the sensor at any location in the area.

We then construct another graph $H = (S_I, E_H)$, where S_I is the set of nodes in the MIS of G_c, and there is an edge $(u, v) \in E_H$ between two nodes u and v if there are some sensors in their overlapping coverage area, i.e., $N_c^+(u) \cap N_c^+(v) \neq \emptyset$. Thus, each edge in E_H indicates that the distance between its two endpoints is strictly larger than γ but less than 2γ. Let V_H' be a maximal independent set of graph H, then we observe that for any two nodes in V_H', there will be no overlapping between any MCVs at these two locations at any time. For example, Fig. 2b shows the sensors in set V_H', where each non-dotted circle centered at a sensor in V_H' represents

the charging area of an MCV for the sensor. Also, it can be seen that only sensors u_1 and u_2 are not in V_H', i.e., both u_1 and u_2 are in $S_I \setminus V_H'$.

We finally find K node-disjoint closed tours that contain nodes only in set V_H' such that the longest charging delay among the K closed tours is minimized. However, even for this special case of the problem where there is only a subset $V_s' = \cup_{v \in V_H'} N_c^+(v)$ ($V_s' \subset V_s$ of sensors, the problem is still NP-hard. We instead find an approximate solution to this special problem.

An auxiliary complete metric graph $G_H = (V_H' \cup \{v_0\}, E_H; w_H : E_H \mapsto \mathcal{R}^{\geq 0})$ is constructed, where v_0 is the depot, and there is an edge (u, v) in E_H between any two nodes u and v in $V_H' \cup \{v_0\}$. The weight $w_H(u, v)$ of edge (u, v) is defined as

$$w_H(u, v) = \frac{d(u, v)}{s} + \frac{\tau(u) + \tau(v)}{2}, \tag{8}$$

recall that $d(u, v)$ is the euclidean distance between u and v, s is the traveling speed of an MCV, and $\tau(u)$ and $\tau(v)$ are the charging durations for sensors in $N_c^+(u)$ and $N_c^+(v)$, respectively, see Eq. (4). Particularly, $\tau(v_0) = 0$. Notice that $N_c^+(u) \cap N_c^+(v) = \emptyset$. Zhang et al. [45] showed that, for any closed tour C in G_H, the weighted sum of the edges in C is equal to the charging duration $T(C)$ of C, where $T(C)$ is defined in Eq. (6).

Following the work of Frederickson et al. [8], a 2.5-approximate solution can be delivered for the problem of finding K v_0-rooted closed tours that contain nodes only in set $V_H' \cup \{v_0\}$ such that the longest charging duration among the K closed tours is minimized. Denote by $C_1, C_2, \ldots, C_K$ the K closed tours, where $C_k = \langle v_{k,i_0}, v_{k,i_1}, v_{k,i_2}, \ldots, v_{k,i_{n_k}}, v_{k,i_0} \rangle$ and $\cup_{k=1}^k V(C_k) = V_H'$. For instance, Fig. 2b shows K ($= 2$) node-disjoint charging tours C_1 and C_2 that contain sensors in set V_H' only.

Now, given the current setting $\{C_1, C_2, \ldots, C_K\}$, the charging finish time $f(v_{k,i_l})$ of each node v_{k,i_l} in a closed tour C_k is defined as

$$f(v_{k,i_l}) = \sum_{j=0}^{l-1} (d(v_{k,i_j}, v_{k,i_{j+1}})/s + \tau'(v_{k,i_j})) + \tau'(v_{k,i_l}), \tag{9}$$

where s is the travel speed of any MCV k with $1 \leq k \leq K$.

4.3 An Approximate Solution to the Problem With Overlapping of Charging Ranges of MCVs

The rest is to modify the approximate solution, by including some potential sojourn locations in $S_I \setminus V_H'$ to the found K closed tours. That is, we need to determine where each node $u \in S_I \setminus V_H'$ should be inserted into one of the K closed tours in order to charge all sensors in V_s. We claim that at least one sensor in the neighborhood $N_c^+(u)$ of u has not been covered by the nodes in $\cup_{k=1}^K \cup_{v \in V(C_k)} N_c^+(v)$. Especially, node u (in $N_c^+(u)$) itself has not been covered yet. Otherwise (u has been covered already), the distance between u and some sojourn location in the K closed tours is no more than the charging range γ. It is noted that node u and these sojourn locations are in set S_I, which implies that the distance between u and any of the sojourn locations is strictly longer than γ. Then, the location of node u has not been covered, and node u must be
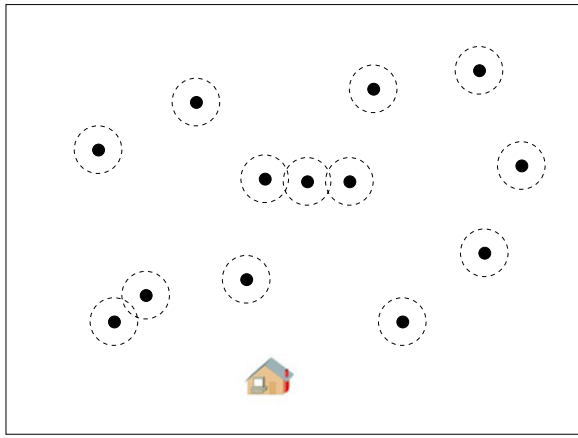
(a) A maximal independent set S_I in graph G_c
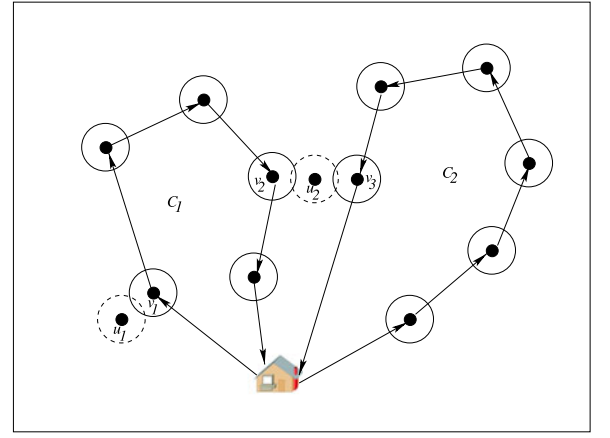
(b) A maximal independent set V'_H in graph H and $K = 2$ node-disjoint charging tours C_1 and C_2 that contain all sensors in V'_H
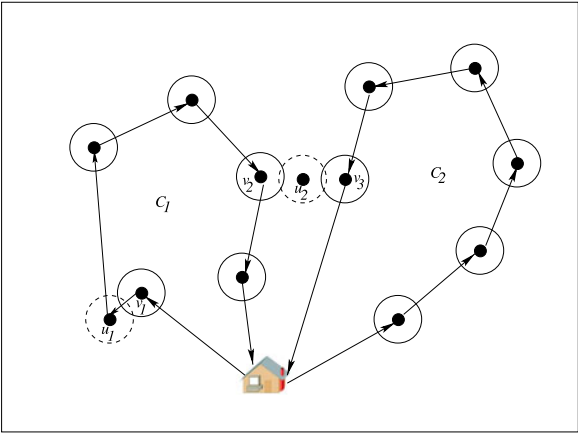
(c) Sensor u_1 is added to tour C_1 after location v_1
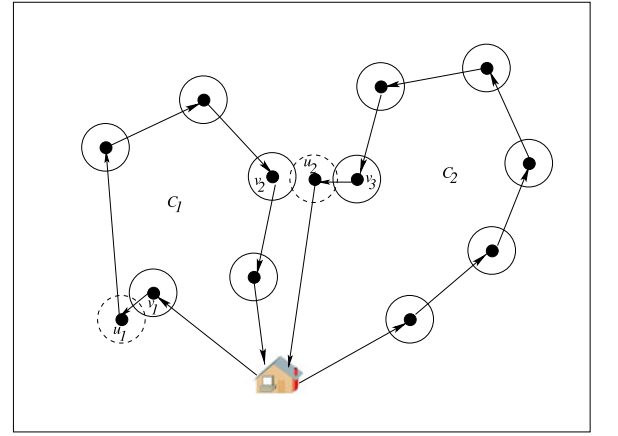
(d) Sensor u_2 is added to tour C_2 after location v_3

Fig. 2. The execution illustrations of `Algorithm 1`, where there are $K = 2$ MCVs.

inserted into one of the K closed tours. However, inserting a node to one of the closed tours is challenging as the following two issues must be addressed.

One is that node u cannot be arbitrarily inserted to one closed tour; otherwise, assuming that u is inserted to a closed tour in a position between two neighboring nodes v_1 and v_2 of the closed tour, then the traveling distance between v_1 and u and the traveling distance between u and v_2 may become very large, implying that it will take a long time for an MCV traveling from v_1 to v_2 through node u. Instead, u should be inserted to one of its neighbors v_i in H, and the distance between u and v_i is strictly less than 2γ by the construction of graph H.

The other is that the insertion of u into a closed tour should still maintain the sensor charging property for all charging tours, ensuring that the charging scheduling is feasible. That is, no sensor will be charged by more than one

MCV at any time. Otherwise, this important constraint might be violated.

In the following, we deal with the insertion of node $u \in S_I \setminus V'_H$ into a closed tour while maintaining the solution obtained is feasible.

For a node $u \in S_I \setminus V'_H$, the neighboring set of node u in graph H can be expressed as follows:

$$N_H(u) = N'_H(u) \cup N''_H(u), \qquad (10)$$

where $N'_H(u)$ and $N''_H(u)$ are the sets of neighbor nodes of u in graph H that have already been assigned and unassigned to any of the K closed tours, respectively, i.e., $N'_H(u) \subseteq \cup_{k=1}^{K} V(C_k)$, and $N''_H(u) \cap (\cup_{k=1}^{K} V(C_k)) = \emptyset$. We claim that $N'_H(u) \neq \emptyset$; otherwise, node u should have been included in V'_H already. Thus, without loss of generality, in the rest of our discussion, we assume that $N'_H(u) \neq \emptyset$.

For each sensor $u \in S_I \setminus V'_H$, we consider the latest charging finish time $f_N(u)$ of its neighbors in $N'_H(u)$, i.e.,

$$f_N(u) = \max_{v_{k,j} \in N'_H(u)} \{f(v_{k,j})\}, \qquad (11)$$

where sensor $v_{k,j}$ is charged in the closed tour C_k and $f(v_{k,j})$ is its charging finish time.

We sort the nodes in $S_I \setminus V'_H$ in increasing order by their latest neighbor charging finish times. Assume that the sorted sequence is $u_1, u_2, \ldots, u_{n_I}$, then $f_N(u_1) \leq f_N(u_2) \leq \cdots \leq f_N(u_{n_I})$, where $n_I = |S_I \setminus V'_H|$.

We deal with the insertions of nodes $u_1, u_2, \ldots, u_{n_I}$ into the K closed tours one by one. Assume that u is the next to-be inserted node. We distinguish our discussion into two cases.

Case (i). $\exists k$ such that $N'_H(u) \subseteq V(C_k)$ with $1 \leq k \leq K$, i.e., all the neighbors of u in graph H are in a single closed tour C_k.

Case (ii). The nodes in $N'_H(u)$ are in at least two or more closed tours among the K closed tours.

We consider Case (i) first. Recall that each node v_{k,i_j} in closed tour C_k has its charging finish time $f(v_{k,i_l})$ with $1 \leq k \leq K$ and $0 \leq j \leq k$. Assume that $N'_H(u) \subseteq V(C_{k_0})$, where

$$k_0, j_0 = \arg\max_{k,j}\{f(v_{k,i_j}) \mid v_{k,i_j} \in N'_H(u)\}. \qquad (12)$$

We insert node u just after node (location) $v_{k_0,i_{j_0}}$ in closed tour C_{k_0}, and calculate the charging duration of MCV k_0 at location u as follows:

$$\tau'(u) = \max_{v \in N_c^+(u) \setminus \cup_{v' \in \cup_{k=1}^{K} V(C_k)} N_c^+(v')} \left\{ \frac{B_v - RE_v}{P_{vu}} \right\}, \qquad (13)$$

where $\cup_{v' \in \cup_{k=1}^{K} V(C_k)} N_c^+(v')$ is the set of sensors that have been covered by the charging locations in the K tours before the insertion of u, and P_{vu} is the charging rate for sensor v if the MCV stops at location u. For example, Fig. 2b shows that the neighbor set $N'_H(u_1)$ of node u_1 contains only a single node v_1 in tour C_1, and u_1 then is added to tour C_1 after node v_1, see Fig. 2c.

We recalculate the charging finish time of all nodes in C_{k_0}, i.e., the charging finish time of each node in C_{k_0} will be updated, due to the insertion of node u. In other words, we only update the charging finish time of each node after node $v_{k_0,i_{j_0}}$ in tour C_{k_0}. That is, the charging finish time of the newly inserted node u in C_{k_0} is

$$f(u) = f(v_{k_0,i_{j_0}}) + d(v_{k_0,i_{j_0}}, u)/s + \tau'(u). \qquad (14)$$

For every other node v_{k_0,i_l} in C_{k_0} with $j_0 < l \leq k$, we have

$$\begin{aligned} f(v_{k_0,i_l}) = {} & f(v_{k_0,i_l}) + d(v_{k_0,i_{j_0}}, u)/s + d(u, v_{k_0,i_{j_0+1}})/s \\ & - d(v_{k_0,i_{j_0}}, v_{k_0,i_{j_0+1}})/s + \tau'(u). \end{aligned} \qquad (15)$$

The rationale behind the handling of Case (i) is that MCV k_0 at location u is only overlapping with itself at the other location $v' \in N'_H(u)$ in closed tour C_{k_0}, and some sensors in $N_c^+(u)$ have already been charged by MCV k_0 or other MCVs prior to MCV k_0 moving to location u, and the

charging duration of MCV k_0 at location u is $\tau'(u)$. Notice that location u in C_{k_0} has the largest charging finish time, compared with the charging finish time of any other neighbor $v' \in N'_H(u)$ of u in C_{k_0}. Thus, no sensor will be charged by more than one MCV at any time.

We then deal with Case (ii). Consider a being considered node u in Case (ii).

Let

$$k_0, j_0 = \arg\max_{k,j}\{f(v_{k,i_j}) \mid v_{k,i_j} \in N'_H(u)\}. \qquad (16)$$

We first insert node u to closed tour C_{k_0} just after node (the location) $v_{k_0,i_{j_0}}$ in closed tour C_{k_0}. The charging duration of MCV k_0 at location u is $\tau'(u)$ can be calculated by Eq. (14). We then recalculate the charging finish time of all nodes in closed tour C_{k_0} by Eq. (15), which is almost identical to Case (i), omitted. For example, Fig. 2c shows that the neighbor set of node u_2 in graph H is $N'_H(u_2) = \{v_2, v_3\}$, where v_2 and v_3 are contained in tours C_1 and C_2, respectively. Node u_2 then is added to tour C_2 after node v_3, since the charging finish time of v_3 is later than that of v_2, see Fig. 2d.

The rationale behind the node insertion in Eq. (16) is that if node u is inserted after a neighbor location in H such that its charging finish time is not the maximum one among the neighbors $N'_H(u)$ of u, then it is very likely that there will be overlapping between MCV k_0 at location $v_{k_0,i_{j_0}}$ and another MCV located at its neighbor in H in another closed tour. In other words, the charging intervals of these two MCVs will be overlapping, and if a sensor is within their overlapping area, it will be charged by both of them at the same time.

Notice that after a node u is inserted to a closed tour C_{k_0} after node $v_{k_0,i_{j_0}}$, we update not only the charging finish time of each of the nodes after node $v_{k_0,i_{j_0}}$ in tour C_{k_0}, but also the latest neighbor charging finish time of the nodes in $S_I \setminus V'_H$ that have not been inserted.

The proposed algorithm is given in Algorithm 1.

5 ALGORITHM ANALYSIS

In the following, we first show the correctness of the proposed algorithm, Algorithm 1. We then analyze the approximation ratio of the proposed algorithm.

5.1 Correctness of the Algorithm

We show that the mobile charger scheduling of the K closed tours is feasible, i.e., no sensors are charged simultaneously by two MCVs at the same time.

Lemma 1. *The charge scheduling of K MCVs delivered by Algorithm 1 is feasible for the longest charging delay minimization problem.*

Proof. We show that (1) all sensors in V_s will be charged; and (2) the solution is feasible, i.e., no sensor will be charged by more than one MCV at any time.

For any sensor $v \in V_s$, it must be covered by a location $u \in S_I$ as S_I is a maximal independent set of charging coverage. Then sensor v will be charged in the end as node u in S_I will be contained in $\cup_{k=1}^{K} V(C_k)$. Note that we also use C_k to represent the final closed tour after each node in S_I has been either inserted to one of the K

Algorithm 1. Algorithm `Appro`

Input: A set of sensors V_s to be charged, a depot v_0, K mobile charging vehicles with each having an energy charging range γ and a traveling speed s.

Output: K closed charging tours $C_1, C_2, \ldots, C_K$ with each including the depot v_0 such that the longest charging delay closed tour among the K closed tours is minimized.

1: Construct a charging graph $G_c = (V_s, E)$, where there is an edge between two vertices in E if their distance is no greater than the charging radius γ;

2: Find a Maximal Independent Set S_I in G_c. Thus, if there is an MCV at each vertex $v \in S_I$ to charge all sensors in $N_c^+(v) = \{u \mid u \in V, \& \ d(u, v) \leq \gamma\}$ for a duration $\tau'(v)$ which is defined in Eq. (5);

3: Construct another auxiliary graph $H = (S_I, E_H)$ where S_I is an MIS of G_c, and there is an edge $(u, v) \in E_H$ if there $N_c^+(v) \cap N_c^+(u) \neq \emptyset$, i.e., if there are two MCVs located at u and v, they are overlapping with each other when they charge at the same time;

4: Find an MIS V_H' of graph H;

5: Construct a graph $G_H = (V_H' \cup \{v_0\}, E_H; w_H : E_H \mapsto \mathcal{R}^{\geq 0})$ from V_H', where there is an edge (u, v) in E_H between any two nodes u and v in $V_H' \cup \{v_0\}$, and the weight of edge (u, v) is $w_H(u, v) = \frac{d(u,v)}{s} + \frac{\tau(u) + \tau(v)}{2}$;

6: Find K node-disjoint closed tours in G_H with each tour containing the depot v_0 of MCVs and the union of the nodes in these tours is V_H'. Let $C_1, C_2, \ldots, C_K$ be the K node-disjoint closed tours delivered by a 2.5-approximation algorithm for the K-optimal closed tour problem due to Frederickson *et al.* [8];

7: Calculate the charging duration at each node v in C_k; i.e., $\tau'(v) \leftarrow \tau(v)$; and its charging finish time $f(v)$ by formulas (14) or (15);

8: $U \leftarrow S_I \setminus V_H'$; /* the set of residual sojourn locations of the K mobile chargers */

9: **for** $U \neq \emptyset$ **do**

10:     Pick a node $u \in U$ with the smallest latest neighbor charging finish time,

11:     **if** all neighbors of u in H are in a single C_{k_0} for some k_0 with $1 \leq k_0 \leq K$ **then**

12:         /* Case (i) */

13:         Identify the insertion location of u in tour C_{k_0} by Eq. (12), calculate the charging duration $\tau'(u)$ of MCV k_0 at u and insert u after node $v_{k_0, i_{j_0}}$ in C_{k_0};

14:     **else**

15:         /* Case (ii): the neighbors of u in H are in at least two closed tours */

16:         Identify the insertion tour C_{k_0} and location $v_{k_0, i_{j_0}}$ of u by Eq. (16), calculate the charging duration $\tau'(u)$ of MCV k_0 at location u, and insert node u just after node $v_{k_0, i_{j_0}}$ in C_{k_0};

17:         Recalculate the charging finish time of each node in C_{k_0}, and the latest neighbor charging finishing time of nodes in $U \setminus \{u\}$;

18:     **end if**

19:     Recalculate the charging finish time of all the nodes after node $v_{k_0, i_{j_0}}$ in C_{k_0}, and the latest neighbor charging finishing time of nodes in $U \setminus \{u\}$;

20:     Let $U \leftarrow U \setminus \{u\}$;

21: **end for**

22: **return** The K charging tours $C_1, C_2, \ldots, C_K$.

closed tours or its coverage area has been covered by others and thus will not be considered any more.

The rest is to show that the MCV scheduling delivered by the proposed approximation algorithm is feasible.

We assume that sensors $u_1, u_2, \ldots, u_{n_I}$ in set $U (= S_I \setminus V_H')$ are inserted one by one in the solution delivered by `Algorithm 1`. Then, the latest neighbor charging finish times of sensors in U meet the property that $f_N(u_1) \leq f_N(u_2) \leq \cdots \leq f_N(u_{n_I})$ by the proposed algorithm.

We show that no sensor will be charged by more than one MCV at any time after the insertion of each node u_i with $1 \leq i \leq n_I$ as follows.

We prove that no sensors are charged simultaneously in the initially K closed tours built upon the locations in V_H'. We claim that for each node in V_H', no matter where it is located and in which closed tour it is contained, an MCV at the location of this node for charging does not overlap with any other MCV at other node in V_H' at any time. Clearly, this is true as their charging ranges do not overlap with each other. Thus, the sensor charging in the initial K closed tours are feasible, i.e., no sensor will be charged by multiple MCVs at the same time.

We assume that no sensors are charged by two MCVs at the same time after the first $(i - 1)$ nodes $u_1, u_2, \ldots, u_{i-1}$ have been inserted. We now consider the case after the ith node u_i is inserted.

Suppose that after the insertion of node u_i, there is a sensor w such that it is charged by two MCVs simultaneously at a location v_1 in a closed tour C_k and another location v_2 in a closed tour C_l at time t, i.e., $d(w, v_1) \leq \gamma$, $d(w, v_2) \leq \gamma$, and the charging intervals of nodes v_1 and v_2 are overlapping with each other.

Following `Algorithm 1`, node u_i is inserted after a node $v_{k_0, i_{j_0}}$ in a tour C_{k_0} such that $k_0, j_0 = \arg\max_{k, j}\{f(v_{k, i_j}) \mid v_{k, i_j} \in N_H'(u)\}$. On one hand, no sensors are charged simultaneously before the insertion of node u_i, but sensor w is charged by two MCVs at locations v_1 and v_2 at the same time after the insertion of u_i. On the other hand, it can be seen that after the insertion of u_i, only the charging intervals of the sensors after $v_{k_0, i_{j_0}}$ in closed tour C_{k_0} change, while the charging intervals of the sensors before $v_{k_0, i_{j_0}}$ in closed tour C_{k_0} and the sensors in other charging tours do not change. Then, one of the two locations v_1 and v_2 must be one of the nodes after $v_{k_0, i_{j_0}}$ in closed tour C_{k_0}, while the other location is contained in another closed tour. Without loss of generality, we assume that location v_1 is one of the nodes after $v_{k_0, i_{j_0}}$ in closed tour C_{k_0}. Also, it can be seen that sensor w must be charged by the two MCVs simultaneously at time t after the latest neighbor charging finish time $f_N(u_i) = \max_{v_{k,j} \in N_H'(u_i)}\{f(v_{k,j})\}$ of u_i, i.e., $t \geq f_N(u_i)$.

On the other hand, we claim that both nodes v_1 and v_2 are contained in the maximal independent set V_H'. Following the insertion order of nodes in U, we know that the nodes $u_{i+1}, u_{i+2}, \ldots, u_{n_I}$ with $f_N(u_{n_I}) \geq \cdots \geq f_N(u_{n_{i+2}}) \geq f_N(u_{n_{i+1}}) \geq f_N(u_i)$ have not been inserted after the insertion of u_i. Then, each node in one of the k closed tours that is charged after time $f_N(u_i)$ must be in set $V_H' \cup \{u_i\}$. However, it can be seen that location v_1 cannot be the inserted node u_i, as u_i is inserted after node $v_{k_0, i_{j_0}}$ such that

$k_0, j_0 = \arg\max_{k,j}\{f(v_{k,i_j}) \mid v_{k,i_j} \in N'_H(u)\}$. We then conclude that both nodes v_1 and v_2 are in V'_H.

From the construction of V'_H, we know that for each node in V'_H, no matter where it is located and in which closed tour it is contained, an MCV at its location for charging does not overlap with any other MCV at other node in V'_H at any time. Therefore, the charging areas of locations v_1 and v_2 do not overlap with each other. It thus is impossible that sensor w is charged by the two MCVs at locations v_1 and v_2 respectively at the same time, and the assumption is incorrect. We thus conclude that no sensors are charged by two MCVs simultaneously after the insertion of node u_i.

By combining the aforementioned discussions, the K charging tours delivered by Algorithm 1 is feasible. $\square$

5.2 Analysis of the Approximation Ratio

In this subsection, we analyze the approximation ratio of the proposed algorithm, Algorithm 1. We first estimate an important lower bound on the value of an optimal solution to the longest charging delay minimization problem. We then show that the ratio of the longest charging tour duration among the closed tours delivered by the proposed algorithm to the lower bound on the optimal solution can be upper bounded by a constant.

We now estimate a lower bound as follows. We observe that the value of an optimal solution to the longest charging delay minimization problem in any subset $V' \subseteq V_s$ of sensors is no greater than the value of the optimal solution of the longest charging delay minimization problem in a set V_s. We thus make use of the optimal solution of the problem in a special subset V'_s ($\cup_{u \in V'_H} N_c^+(u)$) of V_s as the approximate estimation on the optimal solution of the problem in set V_s, and then derive the approximation ratio of the proposed approximation algorithm for the longest charging delay minimization problem in V_s as follows.

Recall that V'_H is a maximal independent set of graph H. Each node v in V'_H thus covers a set of sensors, and the coverage areas of any two nodes in V'_H are not overlapping with each other. We define the following optimization problem.

Definition 2. *Given a set V'_H of nodes and a depot v_0 in a 2-D metric space, each node $v \in V'_H$ has a charging duration $\tau(v)$, assume that there are K MCVs at the depot initially, the travelling time of a mobile charger between two nodes u and v with constant traveling speed s is $d(u,v)/s$, i.e., each edge (u,v) has a travel delay weight $d(u,v)/s$, the K-optimal closed tour problem is to find K (≥ 1) node-disjoint closed tours except that the depot will be contained by all K closed tours such that the longest charging delay among the K closed tours is minimized, subject to that the union of nodes in the K closed tour is V'_H, where the total delay of a closed tour is the weighted sum of nodes and edges in the tour.*

Notice that the k-optimal closed tour problem is NP-hard, and there is a 2.5-approximation algorithm for the K-optimal closed tour problem due to Frederickson et al. [8]. The exact (optimal) solution to the K-optimal closed tour problem is exactly an optimal solution to the longest charging delay minimization problem in a set V'

($= \cup_{u \in V'_H} N_c^+(u)$) of sensors, and for any two nodes u and v in V'_H, $N_c^+(u) \cap N_c^+(v) = \emptyset$ and $d(u,v) \geq 2\gamma$, i.e., there is no overlapping between any two MCVs located at any two nodes in V'_H at any time.

Denote by L'_{OPT} and L_{OPT} the optimal solutions of the longest charging delay minimization problem in sets $\cup_{u \in V'_H} N_c^+(u)$ and V_s, respectively. It can be seen that $L'_{OPT} \leq L_{OPT}$.

Let L be the solution delivered by an approximation algorithm for the K-optimal closed tour problem in V'_H. Then

$$L \leq 2.5 \cdot L'_{OPT} \leq 2.5 \cdot L_{OPT}. \tag{17}$$

Denote by Δ_H the maximum degree of graph $H = (S_I, E')$. Let C_k be any charging closed tour obtained after considering the nodes in V'_H initially by applying the approximation algorithm due to Frederickson et al. [8]. We then add nodes $u \in S_I \setminus V'_H$ in cases (i) and (ii) to the K closed tours. We finally estimate the length (delay) of each closed tour in the end by showing that the length (delay) of each C_k is no greater than constant times of the initial delay of C_k as follows.

Consider a node v in the initial closed tour of C_k, then the cardinality of its neighborhood in H is $|N_H(v)| \leq \Delta_H$ and the distance of each its neighbor from v is strictly less than 2γ; otherwise, there will be no overlapping between their coverage areas, following the definition of graph H.

The analysis on the length of the final tour C_k, compared with its initial length L_k^0 is given by the following lemma.

Lemma 2. *The length of each final tour C_k is upper bounded by $(2\Delta_H + 1) \cdot L_k^0$, where Δ_H is the maximum degree of graph H, and L_K^0 is the initial length of C_k before the insertion of any nodes in $S_I \setminus V'_H$.*

Proof. As the initial length L_k^0 of C_k is no less than $2\gamma \cdot |V(C_k)|$, the traveling time of MCV k on C_k is no less than $2\gamma \cdot |V(C_k)|/s$. The traveling length of MCV k on the final closed tour C_k after inserting nodes in cases (i) and (ii) to the closed tour thus is upper bounded by

$$L_k^0 + \sum_{v \in V(C_k)} 4\gamma \cdot |N_H(v)| \tag{18}$$

$$\leq L_k^0 + |V(C_k)| \cdot 4\gamma \cdot \Delta_H, \text{ as } |N_H(v)| \leq \Delta_H$$
$$\leq L_k^0 + 2\Delta_H L_k^0, \text{ as } L_k^0 \geq 2\gamma \cdot |V(C_k)| \tag{19}$$
$$= (1 + 2\Delta_H) \cdot L_k^0,$$

where Eq. (18) holds since the insertion of each node will increase the travel length no more than $2 \cdot 2\gamma = 4\gamma$, see Fig. 2c. Then, the traveling time of MCV k along its tour C_k is $(1 + 2\Delta_H)$ times the traveling time on the initial closed tour C_k. $\square$

We then analyze the total charging time in C_k. The initial total charging time of MCV k in C_k is $TC_k^0 = \sum_{v \in V(C_k)} \tau(v)$, as the coverage area by each node in C_k are not overlapping with each other.

The total charging time of MCV k after inserting nodes of cases (i) and (ii) in the closed tour C_k is no more than

$$\sum_{v \in V(C_k)} \tau(v) + \sum_{v \in V(C_k)} \sum_{u \in N_H(v)} \tau'(u)$$

$$\leq \sum_{v \in V(C_k)} \tau(v) + |V(C_k)| \cdot \Delta_H \cdot \tau_{max}$$

$$\leq TC_k^0 + TC_k^0 \cdot \Delta_H \cdot \frac{\tau_{max}}{\tau_{min}}, \quad \text{since } TC_k^0 \geq |V(C_k)| \cdot \tau_{min} \tag{20}$$

$$\leq TC_k^0 \cdot \left(1 + \Delta_H \cdot \frac{\tau_{max}}{\tau_{min}}\right),$$

where $\tau_{max} = \max_{v \in V_s}\{\tau(v)\}$ and $\tau_{min} = \min_{v \in V_s}\{\tau(v)\}$ are the longest and shortest charging durations of any MCV at any sojourn locations.

By combining Ineqs. (19) and (20), the total charging delay of MCV k along the closed tour C_k after the inserting nodes in $S_I \setminus V_H'$ thus is

$$\left(1 + \Delta_H \cdot \frac{\tau_{max}}{\tau_{min}}\right)TC_k^0 + (1 + 2\Delta_H) \cdot L_k^0/s. \tag{21}$$

Let D_k^0 be the longest charging delay in the initial K closed tour C_k. Then

$$TC_k^0 + L_k^0/s \leq D_k^0, \tag{22}$$

where TC_k^0 is the total charging time of the initial tour, L_k^0 is the tour length, and s is the traveling speed of each MCV.

Recall that L'_{OPT} and L_{OPT} are the optimal solutions to the longest charging delay minimization problem in sets V_s and $\cup_{u \in V_H'} N_c^+(u)$, respectively. Then, $D_k^0 \leq 2.5 \cdot L'_{OPT}$ by the approximation algorithm for the K-optimal closed tour problem [8]. We also know that $L'_{OPT} \leq L_{OPT}$. Thus

$$D_k^0 \leq 2.5 \cdot L'_{OPT} \leq 2.5 \cdot L_{OPT}. \tag{23}$$

The rest is to show that Δ_H is a constant. Thus, the approximation ratio ρ of the proposed approximation algorithm is constant, by the following lemma.

Lemma 3. $\Delta_H \leq 18$.

Proof. Following the construction of graph H, the distance between any neighbor $v \in N_H(u)$ of node $u \in S_I$ in H and node u is no less than $(1 + \epsilon)\gamma$ but less than 2γ, where ϵ is a value no less than zero. On the other hand, it can be seen that the distance between any two nodes v_i and v_j in $N_H(u)$ is no less than γ, as they are all in set S_I.

Following the work in [7], we know that it is impossible to have 20 points in a circle of radius 2γ such that one of the points is at the center and the distance between any two of the points is at least γ, i.e., $|N_H(u)| + 1 < 20$. Then, $|N_H(u)| + 1 \leq 19$, as $|N_H(u)|$ is a nonnegative integer. Therefore, $|N_H(u)| \leq 18$.

Notice that the number $|N_H(v)|$ of neighbors of a sensor v in graph H usually is much smaller than the maximum number of neighbors Δ_H and $\Delta_H \leq 18$. For example, consider a sensor network with 1,000 sensors in a $100\ m \times 100\ m$ square, assume that the charging range of each charger is $\gamma = 2.7\ m$ [15]. Then, the average number Δ_{avg} of neighboring sensors so that the euclidean distance between each neighbor sensor u and v is between γ and 2γ is no more than $\frac{\pi(2\gamma)^2 - \pi\gamma^2}{100 \times 100} \cdot 1,000 = 6.9 \ll 18$. □

Theorem 2. *Given a wireless rechargeable sensor network in a plane and a set V_s of sensors required to be charged, assume that each sensor $v \in V_s$ with energy capacity B_v and residual energy RE_v when the sensor sent out its charging request. There are K homogeneous mobile charging vehicles with $K \geq 1$ with constant speed s, each mobile charger has a wireless energy transmission range γ with charging output power P, and charge all sensors within his energy transmission range. It is assumed that no sensor can be charged by more than one MCV at any time. There is an approximation algorithm with a constant approximation ratio ρ for the longest charging delay minimization problem, and the algorithm takes $O(|V_s|^3)$ time, where $\rho = 2.5 + 225 \cdot \frac{e_{max}}{e_{min}} = O(1)$, e_{max} and e_{min} are the maximum and minimum amounts of energy charged to a sensor, respectively, i.e., $e_{max} = \max_{v \in V_s}\{B_v - RE_v\}$ and $e_{min} = \min_{v \in V_s}\{B_v - RE_v\}$.*

Proof. Following Lemma 1, the solution delivered by the approximation algorithm is feasible. In the following, we analyze the approximation ratio.

Notice that

$$\tau_{max} \leq \frac{e_{max}}{P \cdot \mu_{min} \cdot \eta}, \tag{24}$$

where P is the charging output power of an MCV, and μ_{min} is the minimum charging efficiency with $\mu_{min} = 0.2$. Similarly

$$\tau_{min} \geq \frac{e_{min}}{P \cdot \mu_{max} \cdot \eta}, \tag{25}$$

where μ_{max} is the maximum charging efficiency with $\mu_{max} = 1$. Then

$$\frac{\tau_{max}}{\tau_{min}} \leq \frac{\mu_{max}}{\mu_{min}} \cdot \frac{e_{max}}{e_{min}} = 5 \cdot \frac{e_{max}}{e_{min}}. \tag{26}$$

Following Eq. (21), the longest charging delay of each closed tour is no more than

$$\left(1 + \Delta_H \cdot \frac{\tau_{max}}{\tau_{min}}\right)TC_k^0 + (1 + 2\Delta_H) \cdot L_k^0/s$$

$$\leq \left(1 + 18 \cdot \frac{\tau_{max}}{\tau_{min}}\right)TC_k^0 + (1 + 36) \cdot L_k^0/s, \text{ as } \Delta_H \leq 18$$

$$\leq \left(1 + 90 \cdot \frac{e_{max}}{e_{min}}\right)TC_k^0 + 37 \cdot L_k^0/s, \text{ by Ineq. (26)}$$

$$\leq \left(1 + 90 \cdot \frac{e_{max}}{e_{min}}\right)(TC_k^0 + L_k^0/s), \text{ as } e_{max} \geq e_{min}$$

$$\leq \left(1 + 90 \cdot \frac{e_{max}}{e_{min}}\right)D_k^0, \text{ by Ineq. (22)}$$

$$\leq \left(1 + 90 \cdot \frac{e_{max}}{e_{min}}\right)2.5 \cdot L_{OPT}, \text{ by Ineq. (23)}$$

$$= \left(2.5 + 225 \cdot \frac{e_{max}}{e_{min}}\right) \cdot L_{OPT}, \tag{27}$$

where e_{max} and e_{min} are the maximum and minimum amounts of energy charged to a sensor, respectively.

It can be seen that the approximation ratio is a constant if the ratio of the maximum amount e_{max} of energy charged to a sensor to the minimum amount e_{min} of energy charged to a sensor is a constant, this assumption is true in a real setting where each to-be-charged sensor has consumed a significant amount of its energy. For example, assume that

each sensor sends a charging request if its residual energy falls below 20 percent of its energy capacity [21]. Then, $e_{max} \leq B$ and $e_{min} \geq (1 - 0.2)B = 0.8B$, where B is the energy capacity of a sensor. The ratio of e_{max} to e_{min} thus is no more than $\frac{B}{0.8B} = 1.25$. In this case, the approximation ratio is about $225 \cdot \frac{e_{max}}{e_{min}} + 2.5 \leq 284$.

The time complexity of the proposed algorithm is analyzed as follows. The constructions of auxiliary graph G_c and one of its maximal independent sets S_I take $O(|V_s|^2)$ time. The construction of graph H and a maximal independent set V'_H on the graph take $O(|S_I|^2) = O(|V_s|^2)$ time, too. The construction of the initial K closed tours in a complete graph $G_K(V'_H, V'_H \times V'_H; \tau, d)$ where $\tau : V'_H \mapsto \mathcal{R}^+$ and $d(\cdot, \cdot) : (V'_H \times V'_H) \mapsto \mathcal{R}^+$, takes $O(|V'_H|^3)$ time, by applying the approximation algorithm for the K-optimal closed tour problem due to Frederickson et al. [8].

The insertion of a node $u \in S_I \setminus V'_H$ to one of the K closed tours takes no more than $O(|S_I|)$ time since $V'_H \subseteq S_I$. Also, it takes $O(|S_I|)$ time to recalculate the charging finish time of each node in the closed tour. Thus, the total amount of time for the insertions of nodes in cases (i) and (ii) takes $O(\sum_{v \in V(C_k)} \sum_{u \in N_H(v)} |V(C_k)|) = O(\Delta_H \cdot \max_{1 \leq k \leq K}\{|V(C_k)|^2\}) = O(\max_{1 \leq k \leq K}\{|V(C_k)|^2\}) = O(|S_I|^2) = O(|V_s|^2)$ time. The approximation algorithm thus takes $O(|V_s|^3)$ time in total. □

6 PERFORMANCE EVALUATION

In this section, we evaluate the performance of the proposed algorithm through experimental simulations. We also study the impact of important parameters on the algorithm performance including network size, data rates of sensors, and the number of MCVs and their charging rate.

6.1 Experimental Environment Settings

We consider a wireless rechargeable sensor network with size from 200 to 1,200 sensors randomly distributed in a $100 \times 100\ m^2$ square. Assume that the base station and the depot of MCVs are co-located at the center of the monitoring area. The energy capacity of each sensor is set as $10.8\ kJ$ [32]. The data sensing rate b_i of each sensor v_i is randomly chosen from an interval $[b_{min}, b_{max}]$ with $b_{min} = 1\ kbps$ and $b_{max} = 50\ kbps$, respectively. We adopt a real sensor energy consumption model from [18]. Specifically, each sensor consumes its energy on sensing, data transmission, and data reception, and their energy consumptions for these three components are modelled in Eqs. (28), (29), and (30), respectively [18]

$$P_{sense} = \lambda \times b_i, \tag{28}$$

$$P_{Tx} = (\beta_1 + \beta_2 d_{ij}^2) \times b_i^{Tx}, \tag{29}$$

$$P_{Rx} = \alpha \times b_i^{Rx}, \tag{30}$$

where b_i is the data rate of v_i, b_i^{Tx} and b_i^{Rx} are the data transmission rate and reception rate of v_i, respectively, d_{ij} is the euclidean distance between v_i and v_j, $\lambda = 60 \times 10^{-9}\ J/b$, $\beta_1 = 45 \times 10^{-9}\ J/b$, $\beta_2 = 10 \times 10^{-12}\ J/b/m^2$, and $\alpha = 135 \times 10^{-9}\ J/b$. Each sensor sends its data to the base station via the routing path with the minimum energy consumption [30].

TABLE 1
Parameters Table

Parameters	Values
number of sensors n	200 to 1,200
sensor battery capacity B	$10.8\ kJ$
sensor data rate b_i	$[b_{min}, b_{max}] = [1\ kbps,\ 50\ kbps]$
number of MCVs K	1 to 5
charging range γ	2.7 m
charging output power P	2 W
MCV travel speed s	1 m/s
Monitoring period T_M	one year

The wireless energy transfer range γ of each MCV is set at $2.7\ m$ [14]. Then, the average number of sensors within the charging range of an MCV at each sojourn location varies from $\frac{\pi 2.7^2}{100 \times 100} \cdot 200 = 0.46$ to $\frac{\pi 2.7^2}{100 \times 100} \cdot 1,200 = 2.75$. An MCV supporting multi-node energy charging can charge such a small number of sensors simultaneously. The number of MCVs K in the network is set from 1 to 5. Each MCV travels at a speed of $s = 1\ m/s$ and its charging output power P is set at 2 W. The charging duration of an energy-empty sensor then is $\frac{10.8\ kJ}{2\ W} = \frac{10.8\ kJ}{2\ J/s} = 1.5$ hours, if the MCV is located at the sensor. We consider the monitoring of the sensor network for a period T_M of one year. The simulation environment is implemented with the programming language C++. Table 1 lists the parameters used in the experiments. Unless otherwise specified, these parameters will be adopted in the default setting.

To evaluate the performance of the proposed algorithm Appro for the longest charging delay minimization problem, we adopt the following five benchmarks.

(i) In algorithm Earliest Deadline First with K MCVs (K-EDF), it first sorts to-be-charged sensors by their residual lifetimes in increasing order, then partitions the sensors into multiple groups with each group containing K sensors (except that the last group may contain less than K sensors), finally assigns the K sensors in each group to the K MCVs such that the sum of the traveling distances of the K MCVs from their current locations to the K sensors are minimized.

(ii) In algorithm NETWRAP [34], each MCV selects the next to-be-charged sensor that has the minimum weighted sum of the travel time from the MCV to the sensor and the residual lifetime of the sensor, a tie is broken arbitrarily if a sensor is selected by multiple MCVs.

(iii) In algorithm K-minMax [21], it finds K node-disjoint closed tours that contain to-be-charged sensors, such that the longest charging delay among the K tours is minimized. Algorithm K-minMax delivers a 5-approximate solution.

(iv) In algorithm AA [34], it first partitions the to-be-charged sensors into K groups, by applying the K-means algorithm, and each MCV charges the sensors in one group only. Each MCV charges a proportion of sensors in its assigned group before their energy expirations, so as to maximize the total amount of energy charged to sensors minus the total traveling energy cost of the charger.

(v) In algorithm LB_optimal, it finds a lower bound on the optimal solution to the longest charging delay

(a) Average longest tour duration   (b) Average dead duration per sensor in T_M (c) Longest dead duration per sensor in T_M (d) Running time
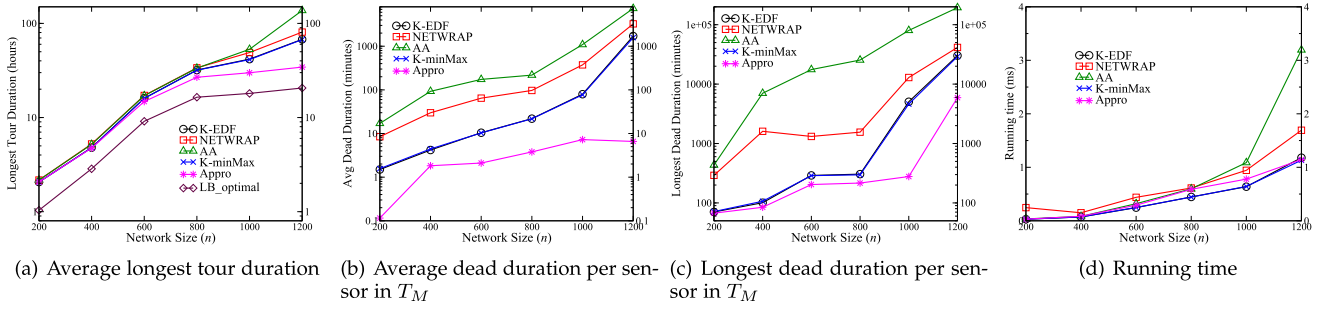
Fig. 3. Performance of different algorithms by varying the network size n from 200 to 1,200 with $K = 2$ chargers.

minimization problem. Specifically, given a set V_s of to-be-charged sensors, it first finds the set S_I of charging locations of MCVs, following Step 1 and Step 2 of Algorithm 1. It then constructs a complete metric graph $G' = (S_I \cup \{v_0\}, E'; w' : E' \mapsto \mathcal{R})$, where v_0 is the depot, there is an edge (v_i, v_j) in E' between any two nodes v_i and v_j in $S_I \cup \{v_0\}$, and the edge weight $w'(v_i, v_j)$ is the traveling time of an MCV between nodes v_i and v_j. Consider a minimum spanning tree T in G', the weighted sum $w'(T)$ of the edges in T', i.e., $w'(T) = \sum_{e' \in T} w'(e')$, is no more than the sum of the traveling times of the K MCVs in the optimal solution, as each of the K optimal tours contains depot v_0. The lower bound thus is

$$LB\_optimal = \frac{w'(T) + h(T)}{K}, \qquad (31)$$

where $h(T)$ is a lower bound on the total charging time for MCVs at the charging locations in set S_I, which is calculated as follows.

Denote by t_i the charging time at a location $u_i \in S_I$. Let $N(u_i)$ be the set of sensors within the charging range of a location u_i and $N_C(v_j)$ be the set of charging locations within the charging range of a sensor $v_j \in V_s$. We use a binary variable x_{ij} to indicate whether a sensor v_j is charged by an MCV at location u_i, i.e., $x_{ij} = 1$ if sensor v_j is charged by an MCV at u_i; otherwise, $x_{ij} = 0$. The minimum total charging time for MCVs at locations in S_I can be calculated by the following integer program:

$$\min \sum_{u_i \in S_I} t_i, \qquad (32)$$

subject to the following constraints:

$$\sum_{u_i \in N_C(v_j)} x_{ij} \geq 1, \qquad \forall v_j \in V_s \qquad (33)$$

$$t_i \geq \frac{x_{ij} \cdot (B_j - RE_j)}{P_{ij}}, \qquad \forall u_i \in S_I, \ \forall v_j \in N(u_i) \qquad (34)$$

$$x_{i,j} \in \{0, 1\}, \quad 1 \leq i \leq |S_I|, 1 \leq j \leq |V_s|, \qquad (35)$$

where Constraint (33) ensures that each sensor must be charged by an MCV. Constraint (34) calculates the charging time t_i at a location u_i, $B_j - RE_j$ is the amount of energy needed to charge sensor v_j, and P_{ij} is the charging rate.

The lower bound $h(T)$ on the total charging time can be obtained by solving a linear programming relaxation to the integer program, by relaxing Constraint (35) to $0 \leq x_{i,j} \leq 1$.

Notice that the existing algorithms K-EDF, NETWRAP, K-minMax, AA have been slightly modified so that they are fitted into the multi-node energy charging scheme. Specifically, when an MCV arrives at a sensor along its charging tour, if the sensor has been fully charged by any MCVs at other locations, the MCV moves to the next to-be-charged sensor in its tour. In case a sensor is within the overlapping charging area of multiple MCVs, only one MCV is allowed to perform charging at any time.

The value in each figure is the mean of the results out of 100 WRSN instances with the same network size. The running time of each algorithm is obtained based on a server with a 3.6 GHz Intel i7 CPU and an 8 GB RAM.

6.2 Performance Evaluation of Different Algorithms

We first evaluate the performance of algorithms Appro, K-EDF, NETWRAP, AA, and K-minMax, by varying the network size n from 200 to 1,200 and there are $K = 2$ mobile chargers deployed.

Fig. 3a shows that the longest charging tour duration among the $K = 2$ charging tours delivered by the proposed algorithm Appro is much shorter than those delivered by the four existing algorithms K-EDF, NETWRAP, AA, and K-min-Max respectively. For example, when the network size is $n = 1,200$ sensors, the longest charging tour durations of algorithms Appro, K-EDF, NETWRAP, AA, and K-minMax are around 34, 68, 80, 137, 67 hours, respectively, while the longest charging tour duration by algorithm Appro is about $1 - \frac{34}{67} \approx 50\%$ shorter than those by the four comparison algorithms. Fig. 3a also demonstrates that the empirical approximation ratio of the solution by algorithm Appro to that by algorithm LB_optimal varies between 1.62 and 2, which is much smaller than the analytical approximation ratio $2.5 + 225 \cdot \frac{e_{max}}{e_{min}}$ in Theorem 2, which clearly indicates that the analytical approximation ratio is very conservative.

Fig. 3b plots the average dead duration per sensor by different algorithms for a given monitoring period T_M (e.g., one year) when the network size n increases from 200 to 1,200. It can be seen from Fig. 3b that the average sensor dead duration delivered by algorithm Appro is no more than 10 minutes when there are $n = 1,200$ sensors, while the average sensor dead durations by algorithms K-EDF, NETWRAP, AA, and K-minMax are 1,700, 3,200, 7,300, and 1,500 minutes, respectively, which are significantly higher than that by the proposed algorithm. On the other hand,
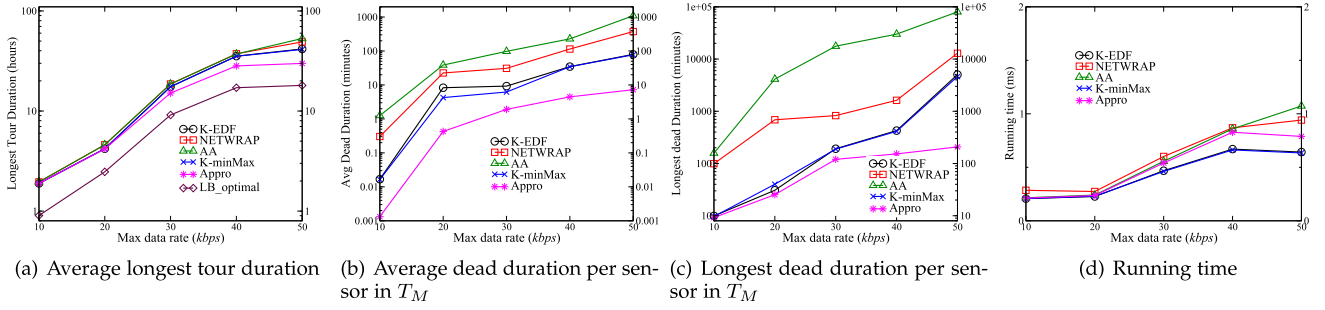
(a) Average longest tour duration   (b) Average dead duration per sensor in T_M (c) Longest dead duration per sensor in T_M (d) Running time

Fig. 4. Performance of different algorithms by varying the maximum data rate b_{max} from 10 $kbps$ to 50 $kbps$ in a network with $n = 1,000$ sensors and $K = 2$ mobile chargers, while $b_{min} = 1$ $kbps$.

Fig. 3c demonstrates the longest dead duration per sensor by different algorithms, from which it can be seen that the longest dead duration by algorithm Appro is much shorter than those by the other four comparison algorithms.

Fig. 3d depicts that the running time of each mentioned algorithm increases with the growth of network size, as there are more to-be-charged sensors in each charging tour, but the gap of running times among different algorithms is insignificant, which is no more than 5 ms.

6.3 Impact of Different Parameters on the Performance of Different Algorithms

The rest is to study the impact of several important parameters on the performance of different algorithms.

We first investigate the impact of the maximum data rate b_{max} on the performance of different algorithms, by varying its value from 10 $kbps$ to 50 $kbps$ in a network with $n = 1,000$ sensors and $K = 2$ mobile chargers, while $b_{min} = 1$ $kbps$. It can be seen that sensor energy consumption rates grow with a larger data rate, and thus there will be more to-be-charged sensors in each charging tour. Fig. 4a shows that the longest charging tour duration by algorithm Appro is no more than 29 hours, while the longest charging tour durations by the other four algorithms K-EDF, NETWRAP, AA, and K-minMax are at least 40 hours when $b_{max} = 50$ $kbps$. In addition, the actual approximation ratio of the solution by algorithm Appro to that by algorithm LB_optimal is no more than 2.1. Fig. 4b demonstrates that the average sensor dead duration by algorithm Appro is only 7 minutes, whereas the average sensor dead durations by algorithms K-EDF, NETWRAP, AA, and K-minMax are 80, 370, $1,100$, and 77 minutes, respectively, when $b_{max} = 50$ $kbps$. Fig. 4c demonstrates the longest sensor dead duration by different algorithms. Fig. 4d plots that the running times of different algorithms are about 1 ms.

We then study the impact of the number of mobile chargers K on the performance of different algorithms, by increasing the number of mobile chargers K from 1 to 5, in a network with $n = 1,000$ sensors. Fig. 5a demonstrates that the longest charging tour duration by each of the algorithms decreases significantly when K increases from 1 to 2, then becomes flat with more mobile chargers. It can also be seen from Fig. 5 that the longest charging tour duration and average/longest sensor dead duration by algorithm Appro are much shorter than those by the existing algorithms K-EDF, NETWRAP, AA, and K-minMax. Moreover, Fig. 5a shows that the approximation ratio of the solution delivered by algorithm Appro to that by algorithm LB_optimal varies between 1.45 and 3. Fig. 5d indicates the running time of each algorithm decreases with the growth of numbers of mobile chargers, as there are less numbers of to-be-charged sensors in each charging tour when more mobile chargers are deployed.

We finally consider the impact of the charging output power P of mobile chargers on the performance of different algorithms, by varying the value of P from 1 $Watt$ to 5 $Watts$, in a network with $n = 1,000$ sensors and $K = 2$ chargers. It can be seen that it takes from $\frac{10.8\ kJ}{5\ W} = 36$ minutes to $\frac{10.8\ kJ}{1\ W} = 3$ hours to fully charge an energy-empty sensor. Fig. 6 plots that the longest tour duration and average/longest sensor dead duration curves delivered by different algorithms. It can be seen that algorithm Appro significantly outperforms algorithms K-EDF, NETWRAP, AA, and K-minMax, respectively. Also, Fig. 6a plots that the empirical approximation ratio of the solution by algorithm Appro to that by algorithm LB_optimal varies between 1.5 and 3.5.

7 CONCLUSION

In this paper we studied the use of multiple mobile chargers, instead of only a single charger, to charge sensors in a WRSN,
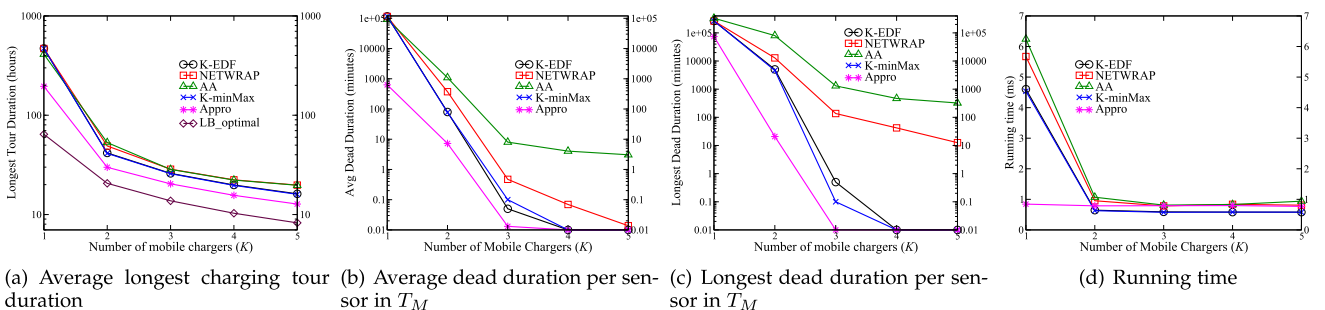

(a) Average longest charging tour duration   (b) Average dead duration per sensor in T_M (c) Longest dead duration per sensor in T_M (d) Running time

Fig. 5. Performance of different algorithms by varying the number of mobile chargers K from 1 to 5 in a network with $n = 1,000$ sensors.

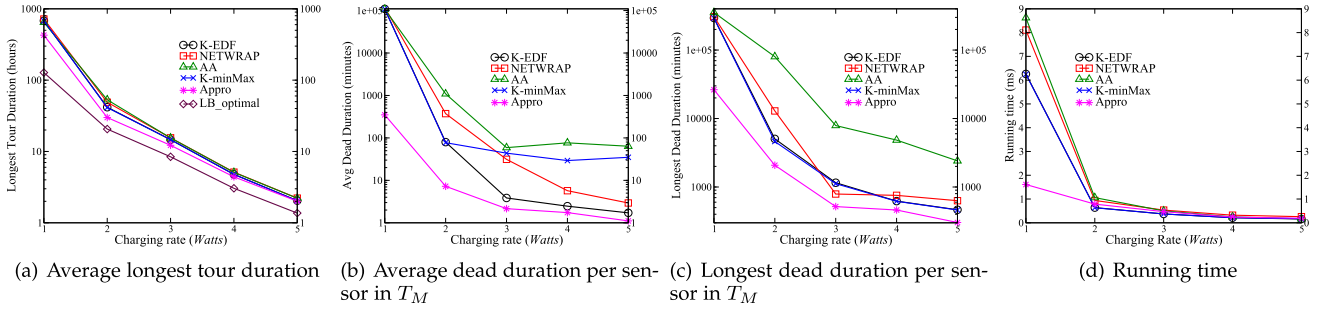(a) Average longest tour duration (b) Average dead duration per sensor in T_M (c) Longest dead duration per sensor in T_M (d) Running time

Fig. 6. Performance of different algorithms by varying the charging output power P from 1 $Watt$ to 5 $Watts$, in a network with $n = 1,000$ sensors and $K = 2$ chargers.

thereby speeding up sensor charging and reducing their expiration durations, where each mobile charger can replenish multiple sensors simultaneously within its energy charging range. We formulated a novel longest charging delay minimization problem, through finding charging tours for K given mobile charging vehicles such that the longest charging duration among the charging tours is minimized, subject to that no sensor can be charged by more than one MCV at any time. Since the problem is NP-hard, we then devised the very first approximation algorithm with a provable approximation ratio for it. We finally evaluated the performance of the proposed algorithm through experimental simulations. Simulation results demonstrate that the proposed algorithm is very promising, and outperforms the other heuristics in various settings.

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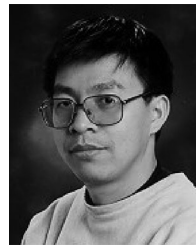
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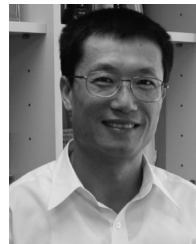
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