Supplementary Materials for “Approximation Algorithms for Charging Reward Maximization in Rechargeable Sensor Networks via A Mobile Charger”

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1 Proof of Theorem 1

Proof: We show the decision version of the fully charging reward maximization problem is NP-hard by a reduction from a well-known NP-hard problem - the Traveling Salesman Problem (TSP). Given a complete graph K_n and an integer n, assume that each edge in K_n is assigned a weight either 1 or 2, the decision version of the TSP in K_n is to determine whether there is a Hamiltonian cycle C such that the weighted sum of the edges in C is n.

We construct an instance K'_n of the fully charging reward maximization problem, where K'_n is a complete graph, too. Each of its edges is assigned a weight that is identical to the corresponding one in K_n, which is the amount of energy consumed by the mobile charger traveling along the edge. Each node in K'_n is assigned a weight of 1, corresponding to the amount of energy to charge it and the prize assigned to it. Given a node v_0 in K'_n and an integer $2n$, assume that the energy capacity of the mobile charger IE is $2n$. The decision version of the fully charging reward maximization problem in K'_n is to determine whether there is a maximum reward closed tour C starting at a node v_0 (i.e., the depot of the mobile charger) such that the total prize collected from the nodes in C is n, subject to that the total energy consumption in C is no more than $IE(=2n)$, assuming it consumes a unit of energy traveling on an edge of weight 1, and 2 units of energy traveling on an edge of weight 2. Clearly, if there is an optimal solution to the fully charging reward maximization problem, there is an optimal solution to the TSP. □

2 Proof of Lemma 1

Proof: Recall that Δ_{min} and Δ_{max} are the minimum and maximum amounts of energy charged to the sensors in V_c. Assuming that $\Delta_{min} \geq \frac{\Delta_{max}}{n^2}$, the original network G_c can then be transformed into another auxiliary undirected graph $G_1 = (V_1, E_1; \pi, l)$ which has identical topological structure as G_c, i.e., $V_1 = V_c$ and $E_1 = E_c$. Each node $v \in V_1$ is assigned a prize $\pi(v) = \lfloor \frac{B_v - RE_v}{\delta} \rfloor = \lfloor \frac{\pi_0(v)}{\delta} \rfloor$ if $RE_v \geq \epsilon$; otherwise $\pi(v) = M$, where $\delta = \frac{\Delta_{max}}{n^2}$ and $\epsilon \geq 0$ is the minimum amount of residual energy of a sensor. When the residual energy of a node is no greater than ϵ, it will be assigned the largest prize M. The prize assignment on nodes aims to encourage the mobile charger to charge sensors with less residual energy. Notice that the prize $\pi(v)$ of any $v \in V_1$ is an integer in the range between 1 and n^2. As we under-round the prize at each node, the additive prize is no more than n as $|V_c| = n$. The optimal solution to the charging utility maximization problem under the under-round reward version thus is at least n^2.

Given the original graph $G_c = (V_c, E_c; \pi_0, l)$ with $\pi_0 : V_c \mapsto R^{\geq 0}$ and $l : E_c \mapsto Z^+$, where $\pi_0(v) = B_v - RE_v$ for all $v \in V_c$, a source node s and a destination node t in V_c, and an $s-t$ path length constraint L, we assume that the length of any path $s \to v \to t$ in G_c is no longer than the path length constraint L for each node $v \in V_c$; otherwise, node v can be removed from G_c since it will not be contained in any feasible solution. In general, given a scaling factor $\delta = \frac{\Delta_{\max}}{n^k}$ and $\Delta_{max} = \max_{v \in V_c}\{B_v - RE_v\} = \max_{v \in V_c}\{\pi_0(v)\}$, the auxiliary graph $G_1 = (V_1, E_1; \pi, l)$ with $\pi : V_c \mapsto \mathcal{Z}^{\geq 0}$ is constructed from G_c by setting $\pi(v) = \lfloor \frac{\pi_0(v)}{\delta} \rfloor$ for every node $v \in V_c$ and $k \geq 1$ is a positive integer. It can be seen that the maximum prize among nodes in G_1 is n^k.

Let $P^*_{G_c}$ and $P^*_{G_1}$ be the optimal solutions to the charging utility maximization problem in G_c and G_1, respectively. Let P be a γ-approximate solution to the charging utilization maximization problem in G_1 delivered by an approximation algorithm with $\gamma \geq 1$. We assume that the prize $\pi(P)$ of path P is no less than the maximum prize n^k in G_1, since the prize of the path $s \to v_{\max} \to t$ is n^k, where $v_{\max}$ is the node with the maximum prize in G_1. In the following we analyze the approximation ratio of the solution P in G_c.

Since P is a γ-approximate solution in G_1, the ratio of the prize of the optimal solution $P^*_{G_1}$ to the prize of path P is no greater than γ, i.e., $\frac{\pi(P^*_{G_1})}{\pi(P)} \leq \gamma$. The prize of the optimal solution $P^*_{G_c}$ in G_c is

$$\begin{aligned}\pi_0(P^*_{G_c}) &= \sum_{v\in P^*_{G_c}} \pi_0(v) \le \sum_{v\in P^*_{G_c}} (\lfloor \frac{\pi_0(v)}{\delta} \rfloor + 1)\delta \\ &\le n\delta + \pi(P^*_{G_c})\cdot\delta \text{ as } \pi(v) = \lfloor \frac{\pi_0(v)}{\delta} \rfloor, \\ &\le n\delta + \pi(P^*_{G_1})\cdot\delta \text{ as } P^*_{G_1} \text{ is optimal in } G_1, \\ &\le (\gamma\cdot\pi(P)+n)\delta \text{ as } \pi(P^*_{G_1}) \le \gamma\cdot\pi(P). \quad (1)\end{aligned}$$

On the other hand, the prize of path P in G_c is $\pi_0(P) = \sum_{v\in P}\pi_0(v) \ge \sum_{v\in P}\lfloor\frac{\pi_0(v)}{\delta}\rfloor\delta = \pi(P)\cdot\delta$. The approximation ratio of path P in G_c then is

$$\frac{\pi_0(P^*_{G_c})}{\pi_0(P)} \le \frac{(\gamma\cdot\pi(P)+n)\delta}{\pi(P)\cdot\delta} \le \gamma + \frac{1}{n^{k-1}} \text{ as } \pi(P) \ge n^k.$$

It can be seen that the larger the value of k is, the smaller the approximation ratio $\gamma + \frac{1}{n^{k-1}}$ is. For example, the approximation ratio is $\gamma + 1$ when $k = 1$, and is $\gamma + \frac{1}{n}$ when $k = 2$. Following the approximation algorithm by Bansal *et al.* [1], $\gamma = 3$. The approximation ratio of `Algorithm` 1 is $3 + \frac{1}{n}$ (≤ 4), since $k = 2$ is adopted. □

3 Proof of Theorem 2

Proof: Assume that path $P_{r_0,r_0'}$ found at Step 6 by the proposed algorithm, `Algorithm` 2, is a path in G'' from r_0 to another node r_0', where r_0' is the last node of the path which in fact is the location of depot r. The prize collected from each sensor $v \in C'$, from which v_0 is derived, is $\pi(v)$. Since the degree of each intermediate node v_0 in $P_{r_0,r_0'}$ is 2, this implies that both edges (v_1, v_0) and (v_0, v_2) are contained in the path. In other words, node v will be fully charged by the mobile charger during the tour.

It can be seen that the total energy consumption of the closed tour C by the mobile charger on its traveling and sensor charging is no more than IE, which is justified as follows. Let C be the closed tour delivered by `Algorithm` 2, and let $n_e(C)$ be the number of edges in C. Clearly, $n_e(C) = 3p$ if there are p sensor nodes (the mobile charger can be treated as a sensor) contained in C as there are three nodes in G'' for each sensor or the mobile charger, and there are two edges connecting these three nodes into a line. Furthermore, there is an edge in C to the corresponding edge in E_c between every two neighboring sensors in C. Specifically, let v be a sensor in C, then there are three nodes v_0, v_1, v_2 in G' and G''. Assume that the amount of energy it will be charged is A. Following the construction of G', there are two edges (v_1, v_0) and (v_2, v_0) in G' that are related to energy charging to sensor v, and each of them is assigned a weight is $A/2$, i.e., $w(v_1, v_0) = A/2$ and $w(v_2, v_0) = A/2$. These edge weights then are converted into integer edge weights in G'', i.e., $l(v_1, v_0) = \lceil\frac{A/2}{\delta}\rceil$ and $l(v_2, v_0) = \lceil\frac{A/2}{\delta}\rceil$ where $\delta = \frac{\Delta_{\max}}{n^2}$. Thus, the amount of energy assigned to sensor v (via these two edges) may be larger than its actual demand A ($= B_v - RE_v$). However, it can be seen that the difference between the actual demand and the extra amount of energy assigned to sensor v is no greater than 2δ. For each edge $(u, v) \in E_c$ derived from the closed tour C, its energy consumption was round up too, the amount of energy assigned for the mobile charger passing through the edge may also be larger than its actual need, and the amount of extra energy assigned to it however is no more than δ too. As a result, the total amount of extra energy assigned to the sensor nodes and traveling edges in C is no more than $3(p-1)\delta$ due to the fact that the mobile charger node will not be charged by itself. Thus, there is at most the amount of extra energy $3(n-1)\delta$ assigned to the sensors and edges in any C as there are at most n sensor nodes to be charged. In other words, the total unused amount of energy of the mobile charger per tour is no more than $3(n-1)\delta$.

Following Step 5 of `Algorithm` 2, let IE' be the actual amount of energy IE' ($\le IE$) used by the mobile charger for charging the sensors and traversing the edges in the closed tour C. Then, it can be seen that $IE' \ge (IE - 3(p-1)\delta) \ge (IE - 3(n-1)\delta) = IE - \frac{3(n-1)\Delta_{\max}}{n^2} > IE - \frac{3\Delta_{\max}}{n}$, where $\Delta_{\max}$ is the maximum amount of energy to charge a sensor. When the number of requested charging sensors n ($= |V_c|$) in the network is quite large, the term $\frac{3\Delta_{\max}}{n}$ approaches zero, i.e., there is almost no any energy left when the mobile charger returns its depot.

The solution delivered by `Algorithm` 2 thus is a feasible solution to the problem. There is a corresponding closed tour C' that is derived from C for the mobile charger. In case a node may be visited multiple times, then the prize is collected from the node only once, i.e., the prize will be collected in the first visit to the node. Following Lemma 1, the approximation ratio of the solution delivered by `Algorithm` 2 is 4. Since both auxiliary graphs G' and G'' contain $|V'| = 3|V_c|$ nodes and $|E'| = 2|V_c| + 2|E_c| = 2(|V_c| + |E_c|)$ edges, the time complexity of `Algorithm` 2 is $T_{ort}(3|V_c|, 2(|V_c| + |E_c|))$. □