Charging Utility Maximization in Wireless Rechargeable Sensor Networks by Charging Multiple Sensors Simultaneously

Yu Ma, Weifa Liang, *Senior Member, IEEE*, and Wenzheng Xu, *Member, IEEE*

***Abstract*— Wireless energy charging has been regarded as a promising technology for prolonging sensor lifetime in wireless rechargeable sensor networks (WRSNs). Most existing studies focused on one-to-one charging between a mobile charger and a sensor that suffers charging scalability and efficiency issues. A new charging technique – one-to-many charging scheme that allows multiple sensors to be charged simultaneously by a single charger can well address the issues. In this paper, we investigate the use of a mobile charger to charge multiple sensors simultaneously in WRSNs under the energy capacity constraint on the mobile charger. We aim to minimize the sensor energy expiration time by formulating a novel charging utility maximization problem, where the amount of utility gain by charging a sensor is proportional to the amount of energy received by the sensor. We also consider the charging tour length minimization problem of minimizing the travel distance of the mobile charger if all requested sensors must be charged, assuming that the mobile charger has sufficient energy to support all requested sensor charging and itself travelling. Specifically, in this paper, we first devise an approximation algorithm with a constant approximation ratio for the charging utility maximization problem if the energy consumption of the mobile charger on its charging tour is negligible. Otherwise, we develop an efficient heuristic for it through a non-trivial reduction from a length-constrained utility maximization problem. We then, devise the very first approximation algorithm with a constant approximation ratio for the charging tour length minimization problem through exploiting the combinatorial property of the problem. We finally evaluate the performance of the proposed algorithms through experimental simulations. Simulation results demonstrate that the proposed algorithms are promising, and outperform the other heuristics in various settings.**

***Index Terms*— Wireless energy transfer, multi-node energy charging, approximation algorithms, mobile chargers, charging tour scheduling, maximal independent set, energy optimization, wireless rechargeable sensor networks.**

I. Introduction

Wireless Sensor Networks (WSNs) have been widely applied in various industries, from military surveillance and disaster forecasting to cutting-edge smart cities and homes [30], [32]. They all rely on ubiquitous sensors to capture multi-dimensional data from surrounding objects for various purposes. However, each sensor is usually powered by an on-board battery with limited energy capacity, sensor lifetime prolongation remains a critical issue [15]. Although energy harvesting technologies [7], [19], [20] have been proposed to accumulate energy from ambience such as solar and wind energy, these methods are sensitive to environments, and cannot provide stable energy supplies to sensors. Wireless energy charging was proposed to wirelessly replenish sensor energy perpetually through a mobile charger [10], [17], [22]. This technology possesses many advantages as it does not require direct contact between the mobile charger and the sensor, or even it does not require line-of-sight (LOS) as long as the charging sensor is within the wireless energy transmission range of the mobile charger. Also, compared to renewable energy harvesting, wireless energy transfer can provide stable energy to sensors. This charging process can be applied in an on-demand manner when sensory devices request to be charged. The powerfulness of wireless energy charging technology has brought wide applications [3], [18], [24].

Despite wireless energy transfer is a promising technique for sensor charging, the energy charging efficiency and charging scalability are critical issues. Fortunately, Kurs *et al.* [11] proposed a *multi-node wireless energy charging* scheme, where multiple sensors can be charged simultaneously by proper tuning operation frequencies of the sender and the receiver coils, enabling high energy transfer efficiency and larger energy transmission range. However, adopting a mobile charger to charge multiple sensors simultaneously in a WRSN poses several challenges. First, the energy capacity of the mobile charger may be insufficient to charge all sensors in the network before running out of its energy. How to utilize the limited energy of the mobile charger to charge those most needed sensors to minimize the number of sensor failures? how to strive for a non-trivial tradeoff between the amount of energy consumed for its mechanical movement and the amount of energy for sensor charging of the mobile charger? and how to minimize the energy consumption on its travelling if the mobile charger has sufficient energy to charge all requested sensors? In this paper, we will address these challenges and develop efficient solutions for them.

The novelty of this paper lies in the investigation of efficient charging multiple sensors simultaneously in a WRSN via wireless energy transfer. We are the first to formulate two novel scheduling problems for a mobile charger to charge multiple sensors simultaneously under its energy capacity constraint. We develop the very first approximation algorithms for the problems with performance guarantees through striving for a

Manuscript received October 3, 2017; revised March 6, 2018 and May 16, 2018; accepted May 19, 2018; approved by IEEE/ACM Transactions on Networking Editor C. F. Chiasserini. Date of publication June 8, 2018; date of current version August 16, 2018. The work of W. Xu was supported by the National Natural Science Foundation of China under Grant 61602330. *(Corresponding author: Weifa Liang.)*

Y. Ma and W. Liang are with the Research School of Computer Science, The Australian National University, Canberra, ACT 2601, Australia (e-mail: yu.ma@anu.edu.au; wliang@cs.anu.edu.au).

W. Xu is with the College of Computer Science, Sichuan University, Chengdu 610065, China (e-mail: wenzheng.xu@scu.edu.cn).

Digital Object Identifier 10.1109/TNET.2018.2841420

non-trivial tradeoff between the amount of energy allocated for the travelling of the mobile charger and the amount of energy allocated for sensor charging.

The main contributions of this paper are as follows. We first formulate two novel multi-node wireless energy charging problems under the constraint of either the energy capacity of the mobile charger or all requested sensors to be charged. We aim to find a closed charging tour including the depot of a mobile charger such that either the accumulative charging utility gain is maximized, or the travelling energy consumption of the mobile charger on its charging tour is minimized. We then devise an approximation algorithm with a constant approximation ratio for the charging utility maximization problem if the energy consumption of the mobile charger on its travelling is negligible; otherwise, we develop an efficient heuristic for the problem. Also, we devise the very first approximation algorithm with a constant approximation ratio for the charging tour length minimization problem, provided that the mobile charger has sufficient energy to support all requested sensor charging and its travelling. We finally evaluate the performance of the proposed algorithms through experimental simulations. Simulation results demonstrate that the proposed algorithms are promising and outperform mentioned benchmark algorithms.

The rest of the paper is organized as follows. Section II reviews related work. Section III introduces notions, notations, and problem definitions. The NP-hardness of the defined problems are also shown in this section. Section IV deals with the charging utility maximization problem. Section V studies the charging tour length minimization problem. Section VI evaluates the proposed algorithms empirically, and Section VII concludes the paper.

II. Related Work

With the advance in the wireless energy transfer technology based on strongly magnetic resonances [10], wireless energy replenishments have been adopted for the lifetime prolongation of WSNs in literature [13], [22], [26], [31]. There are two types of wireless energy replenishments: energy radiations [4] and magnetic resonant coupling. In this paper we focus on wireless energy charging by magnetic resonant coupling, which has been regarded as a breakthrough technology for lifetime prolongation of sensors in wireless sensor networks [2]. Although this technology is still in its early stage, there have been several studies of its applications by deploying a mobile charger to charge sensors in WRSNs [6], [16], [23]. For example, Shi *et al.* [22] theoretically studied applying this technique to charge sensors in WSNs by periodically dispatching a mobile charging vehicle such that the network can operate perpetually. Li *et al.* [13] argued that existing charging schemes only passively replenish sensors that are deficient in energy supply, and cannot fully leverage the strength of wireless energy transfer technology. They instead proposed a 'charging-aware' routing protocol (J-RoC) that incorporates dynamic energy consumption rates of sensors in the design of data collection routing protocols. Although this scheme can proactively guide the routing activities and charge energy to sensors, this makes the design and management of routing protocols more complicated. For example, the deployed routing protocols in a sensor network sometimes are required to be updated periodically due to security concerns of sensing data. Liang *et al.* [14], [15] considered an optimization problem of minimizing the number of mobile chargers to charge a set of energy-critical sensors, subject to the energy capacity of each mobile charger, where there are sufficient numbers of chargers available at a depot, while in this paper we assume that there is only one mobile charger. Xu *et al.* [29] argued that charging a sensor to its full energy capacity may take a long time, they instead minimize the depletion period of each sensor by charging each sensor with an amount of energy to its 'satisfied' energy level. Thus, a mobile charger can charge as many sensors as possible. Ye and Liang [31] recently studied a charging utility maximization problem under a one-to-one charging scheme where each sensor has a survival time window and there is not any energy capacity constraint on the mobile charger, and provided a pseudo-approximation algorithm with an approximation ratio of $O(\log OPT)$ for the problem, where the OPT is the value of the optimal solution to the problem. Note that this approximation ratio depends on the value OPT, which is not constant. This paper deals with the charging utility maximization problem under a one-to-many charging model, subject to the energy capacity constraint on the mobile charger. This is a different utility maximization problem from the one in [31] as they adopt different charging models. The charging model in [31] is a one-to-one charging scheme: the mobile charger only charges one sensor at each its stopping location, and the utility gain by charging the sensor is inversely proportional to the residual energy of the sensor. In other words, the utility gain of charging a sensor is given and fixed, which does not change in the finding of a charging tour. The charging model adopted in this paper is a one-to-many charging scheme: the mobile charger can charge multiple sensors simultaneously within its energy charging range at each its stopping location. The utility gain by charging these multiple sensors at each its stopping location is dynamically changing, depending on that how many of the sensors in its charging range have not yet been charged. In other words, the utility gain at each its stopping location is determined by the number of uncharged sensors (see Eq. (2)). The technique and method for the problem in [31] thus are not applicable to the problem in this paper.

In addition to single-node wireless energy charging, Xie *et al.* [27] were the first to study multi-node wireless energy charging in WRSNs by periodically dispatching a mobile charger. Their approach relied on partitioning a 2-dimensional plane into adjacent hexagonal cells and finding a set of stopping points which are centers of these hexagonal cells. Through the scheduling of a mobile charger at these centers, all sensors in the network can be charged. They aimed to minimize the energy consumption of the mobile charger by minimizing the sojourn time at each stopping point. Xie *et al.* [28] later further considered the use of a mobile charger for both sensor charging and data collection with the aim of minimizing the energy consumption of the whole network under the constraints that none of the sensors will run out of energy and all collected data can be relayed to the base station. For both mentioned studies, they assumed that the travelling path of the mobile charger is fixed and given in advance. However, finding a path that includes chosen sojourn locations for the mobile charger is non-trivial in the multi-node charging scenario. In addition, Khelladi *et al.* [8] investigated an on-demand multi-node charging problem. They aimed at minimizing the number of stopping points and the energy consumption of a mobile charger in a charging tour. They applied a threshold based charging strategy to group requested sensors by leveraging a clique partitioning technique. Wu *et al.* [25] formulated a cooperative multi-charging problem by deploying multiple mobile chargers to charge energy-critical sensors with the aim to minimize the energy consumption of mobile chargers, and they provide a solution by applying genetic algorithms. All of these mentioned studies are based on an assumption that the mobile charger has enough energy to charge all sensors. However, in reality, the number of sensors deployed in a WRSN is quite large. It is unrealistic to charge all sensors by a mobile charger within a single tour. Thus, it leaves us a challenging question, can we find a closed charging tour for the mobile charger such that the accumulative utility gain obtained by charging sensors per tour is maximized? where the utility gain achieved by charging a sensor is inversely proportional to its residual energy and such a utility function can be expressed as a submodular function to capture the urgency of a sensor to be charged.

To tackle the problems in this paper, we will deal with a closely related problem – *the Travelling Salesman Problem with Neighborhoods* (TSPN) [5], which is defined as follows. Given a collection of regions (neighborhoods), each region is represented by a disk covering a certain number of points, the question is to find a shortest path tour that passes through each region. The TSPN is NP-hard as it is a generalization of the classical Euclidean TSP. There is an approximation algorithm [5] for TSPN with disjoint unit disk neighborhoods. However, this algorithm for the TSPN cannot be applied to one of the problems that will be addressed in this paper – the charging tour length minimization problem. As each region formed by the mobile charger within its energy transmission range γ is a disk, each disk with the radius γ covers some sensor nodes, and some of these disks are overlapping with each other. It is challenging to find a charging tour for a mobile charger such that all sensors can be charged when the mobile charger only stops at the locations in the tour. Thus, a new algorithm and its analysis techniques for the problem are desperately needed.

III. Preliminaries

In this section, we first introduce the system model, notions and notations. We then define the problems precisely.

A. Network Model

We consider a Wireless Rechargeable Sensor Network $G_s = (V_s, E_s)$ that consists of a set V_s of stationary sensors distributed over a two-dimensional region, and a set E_s of edges (links). There is an edge between two sensors if they are within the transmission range of each other. There is a fixed base station, which is the sink node for data collection. Each sensor $v \in V_s$ is powered by an on-board rechargeable battery with energy capacity C_v, which consumes energy on sensing, processing, and data transmission and reception. Denote by RE_v the residual energy of sensor v at a moment. Without loss of generality, we assume that there is a sufficient energy supply to the base station, it thus has no energy constraint.

B. Multi-Node Wireless Energy Charging, A Mobile Charger And Its Charging Tour

The technique of wireless energy transfer to multiple sensors simultaneously was invented by Kurs *et al.* [11]. They showed that the overall energy efficiency can be significantly improved by proper tuning of the coupled resonators when multiple receivers instead of a single receiver are charged simultaneously. This multi-node wireless energy charging technique is a promising technique that can address both charging efficiency and scalability for large-scale wireless rechargeable sensor networks. In this paper, we will adopt this *multi-node energy charging* scheme, where multiple sensors can be charged simultaneously if they are within the energy transmission range of a mobile charger.

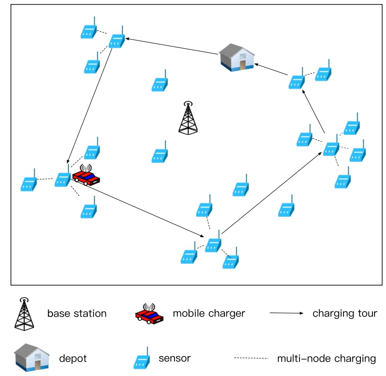


Fig. 1. An example of multi-node energy charging by a mobile charger.

To maintain long-term operations of a WRSN, a mobile charger is employed to charge the sensors in V_s, which is a mobile vehicle equipped with a wireless charger that can charge multiple sensors simultaneously. The mobile charger is dispatched from its depot v_0 which may or may not be co-located with the base station, and travels along a closed tour and sojourns at each stopping location in the tour to charge multiple sensors simultaneously. Ideally, the mobile charger can stop at any location in the monitoring area. However, this introduces infinite numbers of potential stopping locations for the mobile charger. For the sake of problem tractability, we assume that the mobile charger can only stop at the locations that are co-located with sensors.

Denote by IE the mobile charger's energy capacity. When the mobile charger stops at a sensor node $v \in V_s$, sensor v and its neighbors in $N_c(v)$ within its energy charging range γ can be simultaneously charged, where $N_c(v) = \{u \mid d(u, v) \leq \gamma,\ u \in V_s, u \neq v\}$, $d(u, v)$ is the Euclidean distance between nodes u and v, and γ is the mobile charger's charging radius. Denote by $N_c^+(v) = \{v\} \cup N_c(v)$. Only all sensors in $N_c^+(v)$ have been fully charged, the mobile charger can move to the next stopping location for sensor charging. For the sake of convenience, we assume that there is no energy loss of sensors or such energy loss is negligible during this multi-node charging process. We also assume that the mobile charger consumes the amount of energy e_l when travelling per unit distance.

The base station serves as not only the data collector of the network but also the scheduler of the mobile charger. When the mobile charger finishes its charging tour, it will return to its depot to replenish energy for its next charging tour. Each charging tour C of the mobile charger is a *closed tour* including the depot. Note that the mobile charger consumes energy on both its mechanical movement and sensor charging, its total energy consumption per tour thus is upper bounded by its energy capacity IE. Figure 1 is an illustrative example of multi-node energy charging by a mobile charger.

C. Charging Cost and Charging Utility Gain

Intuitively, we should charge sensors with less residual energy first. Also, we should charge as many sensors as possible per tour to minimize the number of dead sensors. To this end, we model the energy charging to a sensor by a utility function $f(\cdot)$, which is a non-increasing submodular function on the residual energy of the sensor as a diminishing return property (i.e., $f(x+\Delta) - f(x) \geq f(y+\Delta) - f(y)$ if $x \leq y$, and $\Delta > 0$, where x is the residual energy of the sensor, if it is charged with the amount of energy Δ, assuming that both $x+\Delta \leq C_v$ and $y+\Delta \leq C_v$). In other words, if the same amount of energy Δ will be charged to a sensor, then the sensor with less residual energy will have a larger utility gain. The submodular property of $f(\cdot)$ favors charging sensors with less residual energy to prolong sensor lifetimes. Define the *charging cost* $\Delta_v = C_v - RE_v$ of charging a single sensor $v \in V_s$ as the amount of energy charged to it. The *charging utility gain* g_v of sensor $v \in V_s$ is the marginal gain of its utility, i.e., $g_v = f(C_v) - f(RE_v)$. In the multi-node wireless energy charging scheme, when the mobile charger stops at a sensor node v, any sensor in $N_c^+(v)$ will be charged to its full capacity if the sensor has not been fully charged.

Denote by S the set of sensors visited (or stopped) by the mobile charger so far prior to visiting sensor v. The accumulative utility gain $g'(v)$ of the mobile charger at the location of sensor v by charging sensor v and its uncharged neighbors in $N_c^+(v)$ thus is

$$g'(v) = \sum_{u \in N_c^+(v) \setminus \cup_{s \in S} N_c^+(s)} (f(C_u) - f(RE_u)), \tag{1}$$

while the total amount of charging energy consumed $\Delta'(v)$ of the mobile charger at sensor v is

$$\Delta'(v) = \sum_{u \in N_c^+(v) \setminus \cup_{s \in S} N_c^+(s)} \frac{C_u - RE_u}{\eta}, \tag{2}$$

where η is the charging energy efficiency rate with $0 < \eta < 1$, e.g., it was reported that $\eta \approx 0.68$ in [11]. As the charging range of the mobile charger γ usually is small (e.g., $\gamma \approx 2.7$ meters in [11]), we here do not distinguish the charging energy difference among the sensors in the energy charging range of the mobile charger, in terms of energy efficiency and their distance from the mobile charger location. If their residual energy are drastically unbalanced, we may not charge all of them to their full capacities, instead we charge them according to a modified utility function. That is, we will stop charging after a certain period at which the amount of energy charged further will not lead to the maximum utility gain per unit energy charged and move to the next stopping location. Meanwhile, if the distance between a sensor and the mobile charger needs to be taken into account, the proposed algorithms can deal with such a case as well, the only modification is in the calculation of the utility gain of charging a sensor in Eq. (2), by incorporating the distance of the sensor to the mobile charger. The proposed algorithms are still applicable to this general setting, and neither its performance nor its time complexity will be affected.

D. Problem Definitions

In this paper, we formulate the following three novel multi-sensor charging optimization problems for the mobile charger tour scheduling, by leveraging the multi-node charging technique.

Definition 1: Given a wireless rechargeable sensor network $G_s = (V_s, E_s)$, assume that the residual energy RE_v of each sensor $v \in V_s$ is given. There is a mobile charger with energy capacity IE and energy charging radius γ, which is initially located at a depot v_0. The *charging utility maximization problem* in G_s is to find a closed charging tour $C = \langle v_0, v_1, v_2, \ldots, v_k \rangle$ for the mobile charger including its depot v_0 such that the accumulative charging utility gain $g(C) = \sum_{v_i \in C} g'(v_i)$ by the mobile charger is maximized, subject to the energy capacity constraint IE of the mobile charger, where the energy of the mobile charger is consumed for both itself travelling along the tour C and sensor charging.

To solve the charging utility maximization problem, we define *the length-constrained utility maximization problem* under the constraints on both the tour length and the energy capacity of a mobile charger for sensor charging as follows. The charging utility maximization problem then can be reduced to the length-constrained utility maximization problem, and a solution to the latter will return a solution to the former.

Definition 2: Given a wireless rechargeable sensor network $G_s = (V_s, E_s)$ and a mobile charger with energy capacity IE for sensor charging only and energy charging radius γ, and a given tour length constraint L, assuming that the mobile charger is located at its depot v_0 initially, *the length-constrained utility maximization problem* in G_s is to find a closed tour C including the depot v_0 for the mobile charger such that the accumulative utility gain of the nodes in C is maximized, subject to the sensor charging energy capacity IE and the tour length L.

In Definition 1, we assume that the energy capacity of the mobile charger is upper bounded by IE, we aim to charge as many sensors as possible per tour such that the accumulative charging utility gain is maximized if not all sensors can be charged at each tour. However, if a subset $V' \subseteq V_s$ of sensors to be charged is given and the mobile charger has sufficient energy to charge them, then how to spend the minimum amount of energy on the charging tour of the mobile charger to charge all the sensors in V' is another challenging question, which is defined as follows.

Definition 3: Given a wireless sensor network $G_s = (V_s, E_s)$, a base station and a depot v_0, a mobile charger with energy charging range γ, assume that there is a subset $V' \subseteq V_s$ of sensors requested to be charged, the *charging tour length minimization problem* is to find such a closed charging tour C including its depot v_0 for the mobile charger such that the length $l(C)$of C is minimized while all sensors in V' will be charged during the tour, assuming that the mobile charger has sufficient energy capacity for sensor charging and itself travelling.

E. The Christofides Algorithm

Let V be a set of nodes distributed in a 2-D plane, the Travelling Salesman Problem (TSP) is to find a closed tour including all the nodes in V such that the tour length is minimized. This TSP is NP-hard. There is an efficient algorithm for it due to Christofides [1], which delivers an approximate solution with an approximation ratio of $\beta = 1.5$. In the rest of this paper, we will make use of the Christofides algorithm for a set of nodes V in a 2-D plane to find a closed tour with the minimum length. We term the Christofides algorithm as algorithm Closed-Tour(V).

F. NP Hardness of Problems

In the following we show that all the three defined optimization problems are NP-hard.

Theorem 1: The charging utility maximization problem is NP-hard.

Proof: See the proof in Appendix. □

Theorem 2: The decision version of the length-constrained charging utility maximization problem is NP-hard.

Proof: See the proof in Appendix. □

Theorem 3: The charging tour length minimization problem is NP-hard, too.

Proof: See the proof in Appendix. □

IV. Algorithm for the Charging Utility Maximization Problem

In this section, we deal with the charging utility maximization problem. We start with a special case of the length-constrained utility maximization problem where the tour length constraint of the mobile charger is not considered. We then consider the length-constrained utility maximization problem by making use of the special case as its subroutine. We thirdly solve the charging utility maximization problem by reducing it to the length-constrained utility maximization problem. We finally analyze the approximation ratio and time complexity of the proposed algorithms.

A. Approximation Algorithm for a Special Case of the Length-Constrained Utility Maximization Problem

Since the tour length constraint L of the mobile charger will not be considered in this case, the problem is then reduced to choose a subset of sensors that the mobile charger will visit them, charge them and their neighbors if their neighbors have not been charged yet. A closed charging tour is then formed by including all visited sensors and the depot v_0 of the mobile charger.

We first construct an undirected charging graph $G_c = (V_s, E_c)$ for the mobile charger as follows. All sensors in V_s are the potential stopping locations of the mobile charger during its tour. There is an edge $(u, v) \in E_c$ between two sensors u and v if they are within the energy charging range γ of the mobile charger when the charger stops at one of them. Therefore, each sensor in $N_c(v)$ – the neighbor set of v in G_c, will be charged if it has not been charged when the mobile charger is located at v.

The proposed approximation algorithm proceeds iteratively. Let $S_k = \{v_0, v_1, v_2, \ldots, v_k\}$ be the set of sensors visited by the mobile charger so far, i.e., they and their neighbors have been charged by the mobile charger.

Denote by $E(S_k)$ the total energy consumption of the mobile charger on sensor charging so far, then

$$E(S_k) = E(S_{k-1}) + \sum_{u \in N_c^+(v_k) \setminus \cup_{j=1}^{k-1} N_c^+(v_j)} \frac{C_u - RE_u}{\eta}, \quad (3)$$

assuming that $S_0 = \{v_0\}$ and $E(S_0) = 0$. The next node v_k to which the mobile charger moves is the node with the maximum ratio $\rho(v_k)$ among the nodes in $V_s \setminus S_{k-1}$ and $E(S_k) \le IE$, where $\rho(v_k)$ is defined as follows.

$$\rho(v_k) = \frac{g'(v_k)}{\Delta'(v_k)} = \frac{\sum_{u \in N_c^+(v_k) \setminus \cup_{j=1}^{k-1} N_c^+(v_j)} (f(C_u) - f(RE_u))}{\sum_{u \in N_c^+(v_k) \setminus \cup_{j=1}^{k-1} N_c^+(v_j)} \frac{C_u - RE_u}{\eta}}. \quad (4)$$

The above procedure continues until the residual energy of the mobile charger cannot support to charge any sensor. In other words, the charging tour of the mobile charger cannot be further extended. When this procedure terminates, the accumulative charging utility gain by the mobile charger will be compared to the highest charging utility gain by charging a single sensor v_{max} and its neighbors $N_c(v_{max})$. The algorithm then selects the larger one as its solution.

Since the tour length of the mobile charger is not considered, the energy consumption on travelling the closed tour is negligible. We find a closed charging tour C that connects all selected sensors S_k. To be consistent with the rest of discussions, we optimize the tour such that its length as short as possible, by invoking the Christofides algorithm Closed-Tour(S_k). The detailed algorithm is presented in Algorithm 1.

Algorithm 1 Finding a Closed Charging Tour C for the Special Length-Constrained Utility Maximization Problem

Input: A WRSN $G_s = (V_s, E_s)$, a depot v_0, a mobile charger with energy capacity IE, energy charging range γ, and the tour length constraint L is not considered.
Output: A v_0-rooted closed charging tour C such that the accumulative charging utility gain of all charged sensors is maximized.

Construct the charging graph $G_c = (V_s, E_c)$;
$S_0 \leftarrow \{v_0\}$; /* the set of selected sensors */
$E(S_0) \leftarrow 0$; /* the total energy consumed */
$g(S_0) \leftarrow 0$; /* the accumulative charging utility gain */
$k \leftarrow 1$; $U \leftarrow V_s$;
while $U \neq \emptyset$ **do**
  Choose a sensor node $v_k \in U$ with the maximum ratio $\rho(v_k)$ in Eq. (4);
  Calculate $\Delta'(v_k)$ by Eq. (2); /* the amount of energy consumed when stopping at v_k */
  if $E(S_{k-1}) + \Delta'(v_k) \le IE$ **then**
    Let $S_k \leftarrow S_{k-1} \cup \{v_k\}$, $E(S_k) \leftarrow E(S_{k-1}) + \Delta'(v_k)$, $g(S_k) \leftarrow g(S_{k-1}) + g'(v_k)$, and $k \leftarrow k + 1$;
  end if
  $U \leftarrow U \setminus \{s_k\}$;
end while
Identify a sensor $v_{max} \in V_s$ with the maximum charging utility gain $g'(v_{max}) = \sum_{u \in N_c^+(v_{max})} (f(C_u) - f(RE_u))$;
if $g(S_k) < g'(v_{max})$ **then**
  $S_k \leftarrow \{v_0, v_{max}\}$;
end if
Find a closed tour C, by invoking algorithm Closed-Tour(S_k) [1];
return the closed charging tour C.

B. Algorithm for the Length-Constrained Utility Maximization Problem

The proposed approximation algorithm Algorithm 1 assumed that the tour length constraint of the mobile charger L is not bounded. We now remove this assumption by taking the tour length constraint into consideration. In the following, we start with a candidate solution delivered by the proposed approximation algorithm, Algorithm 1 for the problem. If the tour length of the solution is no more than L, then the solution is a feasible solution. Otherwise, the solution will be refined by two stages: the compression stage, followed by the expansion stage.

In *the compression stage*, we reduce the tour length by removing some nodes from the tour such that the removal of a node from the tour results in the minimum reduction on the utility gain. This procedure continues until the tour length is bounded by L.

To this end, we first find a closed charging tour C for a set of selected sensors S_k to maximize the accumulative charging utility gain, by invoking Algorithm 1. We then examine the length $l(C)$ of the tour C to see whether $l(C) \leq L$. If yes, then C is a feasible charging tour; otherwise, a sensor $v \in C$ with the minimum ratio $\rho(v)$ will be removed, where $\rho(v)$ is defined as follows.

$$\rho(v) = \frac{g'(v)}{\Delta'(v)} = \frac{\sum_{u\in N_c^+(v)\setminus \cup_{s\in S_k\setminus\{v\}} N_c^+(s)} (f(C_u) - f(RE_u))}{\sum_{u\in N_c^+(v)\setminus \cup_{s\in S_k\setminus\{v\}} N_c^+(s)} \frac{C_u - RE_u}{\eta}}. \quad (5)$$

Assume that a sensor v is to be removed from the charging tour C and its two neighbors in C are v' and v'' respectively, i.e., there exist two edges (v', v) and (v, v'') in C. When node v is removed, both edges (v', v) and (v, v'') are removed, and a new edge (v', v'') is added to the tour. The length $l(C)$ of the resulting charging tour C is reduced, due to the triangle inequality property. The procedure of node removals continues until $l(C) \leq L$.

In *the expansion stage*, we expand the charging tour C by adding more sensor nodes into C if there is room for the solution improvement in terms of the tour length bound and/or the residual energy of the mobile charger, which proceeds iteratively too. That is, within each iteration, the closed tour C is expanded by adding a node $v \in V_s \setminus S_k$ such that the length of the resulting charging tour C' is no greater than L by invoking the Christofides algorithm Closed-Tour($S_k \cup \{v\}$), and the energy spent on sensor charging is still upper bounded by IE. The detailed algorithm for the length-constrained utility maximization problem is described in Algorithm 2.

Algorithm 2 Finding a Closed Charging Tour C for the Length-Constrained Utility Maximization Problem

Input: A WRSN $G_s = (V_s, E_s)$, a depot v_0, a mobile charger with energy capacity IE, energy charging range γ, and the charging tour length constraint L.

Output: A closed charging tour C including the depot v_0 such that the accumulative charging utility gain of the mobile charger is maximized, subject to the tour length L and energy capacity IE of the mobile charger.

Construct the charging graph $G_c = (V_s, E_c)$;
Find a charging tour C in G_c for a set S_k of sensors with charging cost $E(S_k) \leq IE$, by invoking Algorithm 1;
if $l(C) \leq L$ **then**
  return the closed tour C; /* the solution is feasible */
else
  /* The compression stage: */;
  $k \leftarrow |S_k| - 1$ /* the depot v_0 is not considered */;
  while $l(C) > L$ **do**
    Choose a sensor $v \in S_k$ with the minimum ratio $\rho(v)$ by Eq. (5);
    Let $S_{k-1} \leftarrow S_k \setminus \{v\}$, $E(S_{k-1}) \leftarrow E(S_k) - \Delta'(v)$, $k \leftarrow k - 1$;
    Remove sensor v from the charging tour C;
  end while
  /* The expansion stage: */;
  $U \leftarrow V_s \setminus S_k$;
  while $U \neq \emptyset$ **do**
    Choose a sensor $v \in U$ that with the maximum utility gain $g'(v)$ by Eq. (1);
    Find a closed tour C', by invoking algorithm Closed-Tour($S_k \cup \{v\}$) [1];
    Calculate $\Delta'(v)$ by Eq. (2); /* the amount of energy consumed when stopping at v */
    if $l(C') \leq L$ and $E(S_k) + \Delta'(v) \leq IE$ **then**
      Let $S_{k+1} \leftarrow S_k \cup \{v\}$, $E(S_{k+1}) \leftarrow E(S_k) + \Delta'(v)$, $C \leftarrow C'$, and $k \leftarrow k + 1$;
    end if
    $U \leftarrow U \setminus \{v\}$;
  end while
  return the closed charging tour C of the mobile charger.
end if

C. Algorithm for the Charging Utility Maximization Problem

We finally deal with the charging utility maximization problem by reducing it to the length-constrained utility maximization problem, and the solution to the latter in turn will return a solution to the former. Since the mobile charger consumes its energy on both its tour travelling and sensor charging, its total energy consumption per tour is upper bounded by its energy capacity IE.

The basic idea behind the proposed algorithm is to set a fraction of the energy capacity of the mobile charger for its mechanical movement and the rest for sensor charging. The problem then is reduced to the length-constrained utility maximization problem. Specifically, assume that the mobile charger travels at constant speed with the amount e_l of energy consumed per unit length, and further assume that a fraction α of its energy capacity will be consumed for travelling with $0 < \alpha < 1$, then its travelling tour length will be upper bounded by L with $L = \frac{\alpha \cdot IE}{e_l}$. The rest amount of energy $IE_c = (1 - \alpha) \cdot IE$ of the mobile charger will be used for sensor charging. A feasible solution to the length-constrained utility maximization problem, delivered by by Algorithm 2, will return a feasible solution to the charging utility maximization problem.

The key thus is to choose a proper α such that the accumulative utility gain of the closed tour C is maximized. If the value of α is too small, the closed tour is short and less numbers of sensors will be charged; otherwise, less energy

Algorithm 3 Finding a Closed Charging Tour C for the Charging Utility Maximization Problem

Input: A WRSN $G_s = (V_s, E_s)$, a depot v_0, a mobile charger with energy capacity IE and energy charging range γ.

Output: A closed charging tour C including the depot v_0 such that the accumulative charging utility gain of all charged sensors is maximized, subject to the energy capacity constraint on the mobile charger.

Construct the charging graph $G_c = (V_s, E_c)$;
$max \leftarrow 0$; /* the maximum charging utility gain */
$C \leftarrow \emptyset$; /* the closed tour for the mobile charger */
$\alpha \leftarrow \epsilon$; /* ϵ is a given value with $0 < \epsilon < 1$ */
while $\alpha < 1$ **do**
  Let $L \leftarrow \frac{\alpha \cdot IE}{e_l}$ and $IE_c \leftarrow (1-\alpha) \cdot IE$;
  Find a closed charging tour $C(L)$ with the charging utility gain $g(C(L))$ under the constraints on both the tour length L and the energy capacity IE_c for sensor charging, by invoking `Algorithm` 2;
  if $g(C(L)) > max$ **then**
    $max \leftarrow g(C(L))$;
    $C \leftarrow C(L)$;
  end if
  $\alpha \leftarrow \alpha + \epsilon$;
end while
return the closed charging tour C.

will be consumed on sensor charging. Therefore, there is a non-trivial tradeoff between the amount of energy allocated to the mechanical movement of the mobile charger and the amount of energy allocated to sensor charging. We will find an α such that the accumulative utility gain of the mobile charger is maximized. The detailed algorithm for the charging utility maximization problem is given in `Algorithm` 3.

D. Analysis on the Proposed Algorithms

The rest is to analyze the time complexities of the three proposed algorithms. We first show the approximation ratio of `Algorithm` 1, by adopting the analytical technique similar to the one in [9]. We here abuse the notation OPT that denotes both the optimal solution and the value of the optimal solution interchangeably. Recall that sensor v_k is the kth node added to the solution S_k. Let $v_{k'}$ be the first sensor in OPT that has not been added to the solution yet by `Algorithm` 1 due to the fact that if it were added, the charging energy capacity constraint IE of the mobile charger will be violated. We thus have the following lemma.

Lemma 1: Given a set S, let $g(S) = \sum_{s \in S} g'(s)$. Then, for each iteration k with $1 \leq k \leq k'$, we have that (i) $g(S_k) - g(S_{k-1}) \geq \frac{\Delta'(v_k)}{IE}(OPT - g(S_{k-1}))$; and (ii) $g(S_k) \geq OPT \cdot (1 - \prod_{j=1}^{k}(1 - \frac{\Delta'(v_j)}{IE}))$.

Proof: See the proof in Appendix. □

Lemma 2: [9] Let b and A be two positive numbers, and let $\mathbf{a} = (a_0, a_1, \cdots, a_n)$ be a sequence of positive numbers such that $\sum_{j=0}^{n} a_j \geq b \cdot A$. Then, function $\lambda(\mathbf{a}) = 1 - \prod_{j=0}^{n}(1 - \frac{a_j}{A})$ achieves its minimum when $a_j = b \cdot \frac{A}{n+1}$ for all j with $0 \leq j \leq n$, and that $\lambda(\mathbf{a}) \geq 1 - (1 - \frac{b}{n+1})^{n+1} \geq 1 - e^{-b}$.

Theorem 4: Given a wireless rechargeable sensor network $G_s = (V_s, E_s)$, a depot v_0, and a mobile charger with energy capacity IE and energy charging range γ, there is an approximation algorithm with an approximation ratio of $\frac{1}{2} \cdot (1 - \frac{1}{e})$, `Algorithm` 1, for a special case of the length-constrained utility maximization problem where the tour length of the mobile charger is not considered. The algorithm takes $O(|V_s|^3)$ time.

Proof: See the proof in Appendix. □

We then analyze the time complexity of `Algorithm` 2.

Theorem 5: Given a wireless rechargeable sensor network $G_s = (V_s, E_s)$, a depot v_0, a mobile charger with energy charging range γ and energy capacity IE for sensor charging, and the tour length constraint L, there is an algorithm, `Algorithm` 2, for the length-constrained utility maximization problem, which takes $O(|V_s|^3|)$ time to deliver a feasible solution for the problem.

Proof: The analysis of the time complexity of `Algorithm` 2 is trivial, omitted. □

We finally analyze the time complexity of `Algorithm` 3.

Theorem 6: Given a wireless rechargeable sensor network $G_s = (V_s, E_s)$, a depot v_0, and a mobile charger with energy charging range γ and energy capacity IE for both the mechanical movement of the mobile charger and sensor charging, there is an algorithm, `Algorithm` 3, for the charging utility maximization problem, which takes $O(|V_s|^3/\epsilon)$ time and delivers a feasible solution, where ϵ is a given constant with $0 < \epsilon < 1$.

Proof: The analysis of the time complexity of `Algorithm` 3 is trivial, omitted. □

V. Approximation Algorithm for the Charging Tour Length Minimization Problem

In this section, we investigate the charging tour length minimization problem by devising a novel approximation algorithm with a provable approximation ratio for it.

A. Algorithm Overview

The basic idea behind the proposed algorithm is to find a non-trivial lower bound on the optimal solution of the problem. This lower bound in fact is a lower bound on the optimal solution of the problem in a subset of sensors (i.e., a maximal independent set of sensors) of set V' with $V' \subseteq V_s$. We then find an approximate closed tour to cover all sensors centralized in the nodes in the maximal independent set. We also identify another subset of sensors for other sensors that cannot be covered by the disks with radius γ centralized at the nodes in the maximal independent set, and expand the closed tour including these identified sensors. We finally show that the expanded closed tour is also bounded. An approximate solution to the charging tour length minimization problem will be obtained eventually.

B. Approximation Algorithm

We first construct an auxiliary graph $G' = (V', E')$, where V' ($V' \subseteq V_s$) is the set of sensors requested to be charged, and there is an edge $(u, v) \in E'$ between two sensor nodes $u \in V'$ and $v \in V'$ if their Euclidean distance is no greater than 2γ, i.e., $d(u, v) \leq 2\gamma$, where γ is the wireless energy charging range of the mobile charger.

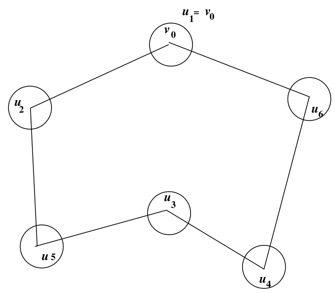


Fig. 2. An example of an optimal tour for a TSPN with disjoint disks.

We then find a maximal independent set MIS in G' including the depot node v_0. Denote by $\{u_1, u_2, \ldots, u_p\}$ the set MIS, thus p is the cardinality of set MIS. Denote by $R(u_i, \gamma)$ the disk centered at u_i with radius γ, which is a disk that sensor u_i can be charged by the mobile charger co-located at any sensor in the disk, i.e., any sensor in $N_c^+(u_i)$. It can be seen that these p disks are disjoint with each other, and no two nodes in set MIS can be simultaneously charged if the mobile charger stops only at any location of the sensors in set MIS, which will be shown later.

Consider the Travelling Salesman Problem with disjoint Neighborhoods (TSPN) for visiting the p disjoint disks centering at the nodes in set MIS [5]. Denote by L_{OPT} the length of an optimal solution for the TSPN (e.g., see Fig. 2). It can be seen that L_{OPT} is a lower bound on value of the optimal solution for the charging tour length minimization problem for charging nodes in V', as set MIS is only a subset of V'.

Let V_1' be the set of sensors covered (charged) by the p disjoint disks, then $V_1' = \cup_{u \in MIS} N_c^+(u)$. Let $V_2' = V' \setminus V_1'$. We then need to identify sensors that cannot be covered by these p disks, i.e., sensors in set $V_2' = V' \setminus \cup_{u \in MIS} N_c^+(u)$. We later show that all the sensors in V_2' will be covered by concentric disks $R(u_i, 2\gamma)$ of disks $R(u_i, \gamma)$ with radius 2γ, i.e., all sensors in V' will be covered by p disks with radius 2γ that are concentric with the p disks centralized at the nodes in set MIS. Denote by $VR(u, 2\gamma)$ the set of sensors covered by disk $R(u, 2\gamma)$ but $R(u, \gamma)$, i.e., $VR(u, 2\gamma) = V(u, 2\gamma) \setminus V(u, \gamma)$, where $V(u, \gamma)$ is the set of sensors covered by disk $R(u, \gamma)$. Thus, $V_2' = \cup_{u \in MIS} VR(u, 2\gamma)$.

We now show how to charge the sensors in V_2'. Specifically, the sensors in $VR(u, 2\gamma)$ for each $u \in MIS$ can be covered, by identifying a subset of sensors in $VR(u, 2\gamma)$ as the stopping locations of the mobile charger. We choose one sensor $v \in VR(u, 2\gamma)$ that covers the maximum number of uncovered sensors in it, a disk centered at v with radius γ then is found, the sensors in V_2' covered by this disk will be removed from $VR(u, 2\gamma) \setminus V(u, \gamma)$. We then choose the next one from the remaining sensors, and this process continues until all sensors in $VR(u, 2\gamma)$ are covered. Denote by V_u the set of chosen sensors from set $V(u, 2\gamma) \setminus V(u, \gamma)$.

Having identified set V_u for each node u in set MIS, a closed tour C that consists of the nodes in $MIS \cup \bigcup_{u \in MIS} V_u$ then can be found, by applying an approximation algorithm for the TSP problem [1]. The tour C is an approximate solution to the charging tour length minimization problem, and the detailed algorithm for the problem thus is described in `Algorithm` 4.

C. Algorithm Analysis

In the following, we analyze the approximation ratio and time complexity of `Algorithm` 4. We first show that the charging tour C obtained is feasible, i.e., every sensor in V' will be charged by the mobile charger when traversing along tour C. We then show a non-trivial lower bound on the length of an optimal solution (tour).

Algorithm 4 Finding a Closed Charging Tour C for the Charging Tour Length Minimization Problem

Input: A WRSN $G_s = (V_s, E_s)$ with a subset of sensors $V' \subseteq V_s$ requested to be charged, a depot v_0, a mobile charger with sufficient energy capacity and energy charging range γ.

Output: A closed tour C including the depot v_0 such that the length of the closed tour C is minimized while all sensors in V' are charged.

Construct an auxiliary graph $G' = (V', E')$;
Find a maximal independent set MIS including the depot v_0 of the mobile charger in G';
for each node $u \in MIS$ **do**
  Identify a subset V_u of sensors in $VR(u, 2\gamma)$;
end for;
Find a closed tour C consisting of nodes in $MIS \cup \cup_{u \in MIS} V_u$, by invoking algorithm `Closed-Tour`($MIS \cup \cup_{u \in MIS} V_u$);
return the closed tour C.

Lemma 3: All sensors in V' will be covered (charged) by the coverage union of disks $\bigcup_{u \in MIS} R(u, 2\gamma)$. Then, all sensors in V' are covered by tour C.

Proof: See the proof in Appendix. □

Given an optimal tour C^* to the charging tour length minimization problem with length $|C^*|$, the rest is to analyze the approximation ratio of `Algorithm` 4, i.e., the ratio of the length $|C|$ of tour C to the length $|C^*|$ of tour C^*.

Let $|C'|$ be the length of an optimal tour C' of the TSP problem for visiting the p nodes in set MIS, and let $L(u)$ be the length of an optimal tour of the TSP problem for visiting nodes in V_u for each u in set MIS. Then, the length $|C|$ of tour C is bounded by

$$|C| \leq (|C'| + \sum_{u \in MIS} L(u)) \cdot \beta, \tag{6}$$

where β is the best approximation ratio of the TSP problem so far, i.e., $\beta = 1.5$ [1]. We then bound the lengths $|C'|$ and $\sum_{u \in MIS} L(u)$ in Ineq. (6) as follows.

We first bound the tour length $|C'|$. There are p disjoint disks with each centered at a node in set MIS with radius γ. And no two sensors in set MIS can be simultaneously charged if the mobile charger stops only at any one of their locations, since the maximum charging range of the mobile charger is γ, see Lemma 5.

Lemma 4: Given a maximal independent set $MIS = \{u_1, u_2, \ldots, u_p\}$ of graph $G' = (V', E')$ that contains the depot node v_0, let $R(u_i, \gamma)$ be the disk with radius γ and centered at u_i that contains all sensors in $N_c^+(u_i)$. Then, $N_c^+(u_i) \cap N_c^+(u_j) = \emptyset$ if $i \neq j$ with $1 \leq i, j \leq p$.

Proof: See the proof in Appendix. □

Lemma 5: To charge all sensors in V', the mobile charger stops at least $p = |MIS|$ locations that are co-located with the sensors in V', assuming that the depot v_0 is included in set MIS. In other words, no two sensors in set MIS can be simultaneously charged when the mobile charger stops only at any one of the p locations.

Proof: The claim is directly derived from Lemma 4, omitted. □

Consider the TSPN with the p disjoint disks centered at locations in set MIS. Let L_{OPT} be the length of an optimal tour for the TSPN, which is a lower bound on the optimal solution to the charging tour length minimization problem. By adopting a proof technique due to Dumitrescu and Mitchell [5], the length of tour C' is bounded as follows.

Lemma 6: The length $|C'|$ of tour C' is no more than $(1 + 8/\pi)|C^*| + 8\gamma$.

Proof: See the proof in Appendix. □

Recall that L_{OPT} is the length of an optimal charging tour for the TSPN with disjoint disks. Clearly, L_{OPT} is a lower bound on the optimal solution to the charging tour length minimization problem as the mobile charger travelling along this charging tour can only charge some but not all sensors in V'. Following Ineq. (14) in the proof body of Lemma 6, the lower bound of the optimal solution is no less than L_{OPT}, where

$$L_{OPT} \geq \frac{|C'| - 8 \cdot \gamma}{1 + 8/\pi} \geq \frac{|C|/\beta - 8 \cdot \gamma}{1 + 8/\pi}. \quad (7)$$

The value of L_{OPT} will serve as the optimal performance benchmark for the proposed approximation algorithm in later experimental evaluation, where C is an approximation of C' with $|C| \leq \beta|C'|$ by the Christofides algorithm [1].

We then bound the length sum of tour segments $\sum_{u \in MIS} L(u)$ by the following lemma.

Lemma 7: The length sum of tour segments $\sum_{u \in MIS} L(u)$ is no more than $4\sqrt{2}\pi p\gamma$, where $p = |MIS|$.

Proof: See the proof in Appendix. □

We now analyze the approximation ratio of `Algorithm` 4. By combining Eq. (6) and lemmas 7 and 8, the length $|C|$ of tour C delivered by `Algorithm` 4 is bounded as follows.

Lemma 8: The length of the closed tour C covering all sensors in V', delivered by `Algorithm` 4, is no more than $((1+16\sqrt{2}+\frac{8}{\pi})|C^*|+(8+16\sqrt{2})\gamma)\cdot\beta$, where $|C^*|$ is the length of an optimal tour to the charging tour length minimization problem and $\beta = 1.5$.

Proof: See the proof in Appendix. □

We finally have the following theorem.

Theorem 7: Given a wireless rechargeable sensor network $G_s = (V_s, E_s)$ with a subset of sensors $V' \subseteq V_s$ requested to be charged, a depot v_0, and a mobile charger with sufficient energy capacity, there is an approximation algorithm with an approximation ratio 40, `Algorithm` 4, for the charging tour length minimization problem, assuming that the energy charging range γ is far less than the length of an optimal tour $|C^*|$ (i.e., $\gamma \ll |C^*|$). The algorithm takes $O(|V'|^3)$ time.

Proof: See the proof in Appendix. □

VI. Performance Evaluation

In this section, we evaluate the performance of the proposed algorithms for the charging utility maximization problem and the charging tour length minimization problem through experimental simulations. We also investigate the impact of important parameters on the algorithm performance.

A. Experimental Environment Settings

We consider a WRSN consisting of sensors with a moderate size from 200 to 1,200 randomly distributed in a 100×100 square meters. We assume that the base station and the depot are co-located at the center of the monitoring area. The energy capacity of the battery of each sensor is set at 10.8 kJ [22]. The residual energy of each sensor is randomly drawn from $(0, 10.8]$ kJ. The energy capacity IE of the mobile charger is set at $4,000$ kJ and $40,000$ kJ for the moderate and large size monitoring areas respectively. The energy charging range of the mobile charger is set at 2.7 meters [8], and its charging energy efficiency rate η is set at 0.68 [11]. The energy consumption of the mobile charger per unit distance (1 meter) is set at 600 J [21]. The submodular utility function $f(x)$ used for charging a sensor is $\log(x + 1)$ [16]. The value in each figure is the mean of the results out of 50 WRSN instances of the same size. The running time of an algorithm is obtained based on a machine with 3.4 GHz Intel i7 Quad-core CPU and 16 GB RAM. Unless otherwise specified, these parameters will be adopted in the default setting.

To evaluate the performance of `Algorithm` 3 for the charging utility maximization problem, we here introduce an iterative benchmark heuristic `K-Lookahead` as follows. It starts with a charging closed tour C at the depot of the mobile charger. It then expands the tour iteratively, by adding up to K sensors to the tour at each iteration until the energy capacity constraint IE of the mobile charger is run out. Instead of searching each potential node individually as the next node of the tour, it explores a group of K (≥ 1) nodes within each iteration and adds the group of K nodes with the maximum ratio of their accumulative charging utility gain to their charging energy cost into the tour C. This procedure continues until no more sensors can be added to the tour without violating the energy capacity constraint of the mobile charger.

We evaluate the performance of `Algorithm` 3 not only under a synthetic sensor energy consumption model but also a real sensor energy consumption model [12]. The amounts P_{sense}, P_{Rx}, and P_{Tx} of energy consumed of each sensor v on data sensing, data reception and transmission in the real sensor energy consumption model are $P_{sense} = \lambda \times b_v$, $P_{Rx} = \gamma \times b_v^{Rx}$, and $P_{Tx} = (\beta_1 + \beta_2\, d_{uv}^{\alpha}) \times b_v^{Tx}$, respectively, where b_v (in bps) is the data sensing rate of sensor v that is randomly drawn from $[1, 10]$ $kbps$ [22], b_v^{Rx} and b_v^{Tx} are its data reception and transmission rates respectively, d_{uv} is the Euclidean distance between sensors u and v, and α is a constant that is equal to 2 or 4. In this experiment, we set α as 2. We assume each sensor v performs data aggregations on both pass-by traffic and self-sensed data. Thus, $b_v^{Tx} = b_v^{Rx} + b_v$, where $\lambda = 60 \times 10^{-9} J/b$, $\beta_1 = 45 \times 10^{-9} J/b$, $\beta_2 = 10 \times 10^{-12} J/b/m^2$ when $\alpha = 2$, and $\gamma = 135 \times 10^{-9} J/b$ [12].

To evaluate the performance of `Algorithm` 4 for the charging tour length minimization problem, we propose an efficient heuristic `MIS` based on a Maximal Independent Set (MIS) in the charging graph G_c. The construction of set MIS is as follows. We first construct the charging graph G_c for set $V' \subseteq V_s$ of requested sensors. We then find a maximal independent set $V_{mis} \subseteq V'$ in G_c. A closed tour C can then be obtained by applying the Christofides algorithm `Closed-Tour`$(V_{mis} \cup \{v_0\})$ [1]. The nodes in V_{mis} will be visited by the mobile charger along the closed tour. Due to the property of the maximal independent set, all requested sensors in V' will be either on the closed tour C or one hop from a node in C.

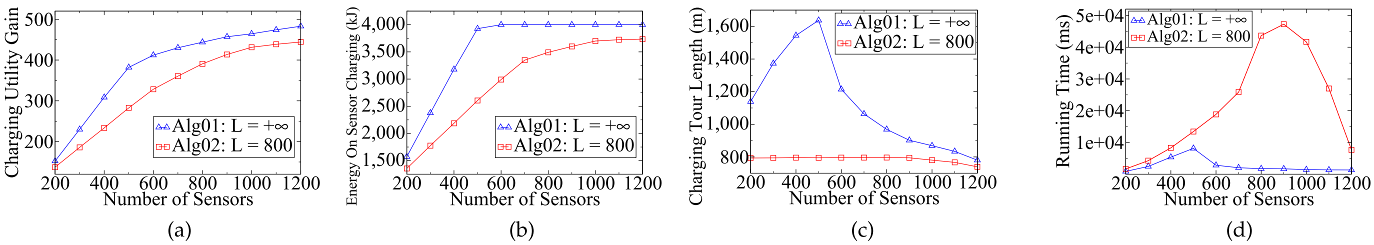


Fig. 3. Performance of `Algorithm` 1 and `Algorithm` 2 in a WRSN consisting of sensors from 200 to 1,200. (a) The accumulative charging utility gain. (b) The total energy on sensor charging. (c) The tour length of the mobile charger. (d) The running times.

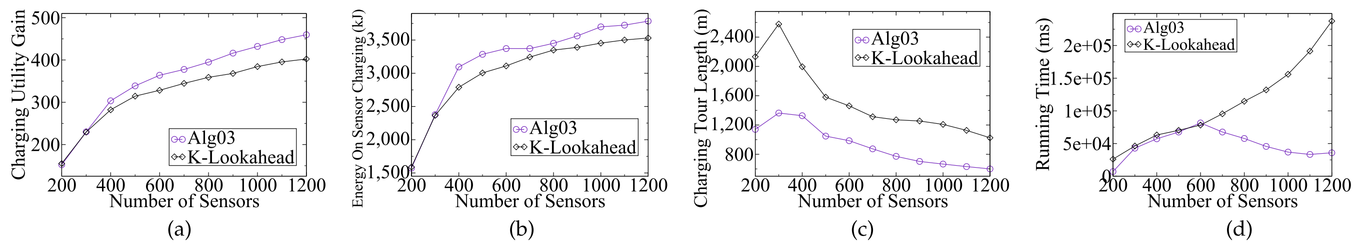


Fig. 4. Performance of `Algorithm` 3 and algorithm `K-Lookahead` in a WRSN consisting of sensors from 200 to 1,200. (a) The accumulative charging utility gain. (b) The total amount of energy consumption on sensor charging. (c) The tour length of the mobile charger. (d) The running times.

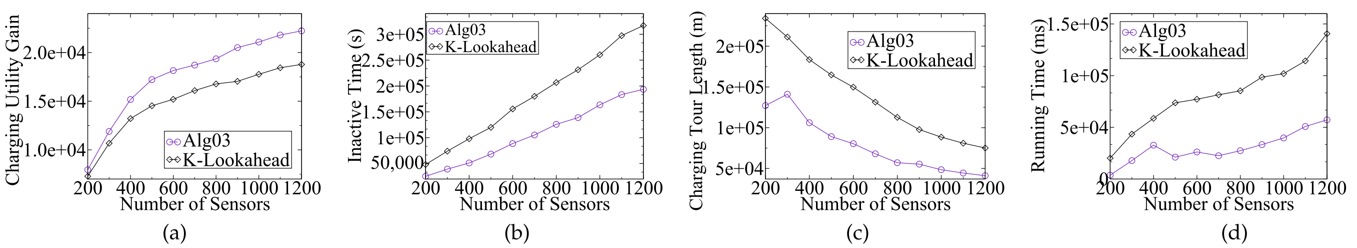


Fig. 5. Performance of `Algorithm` 3 and algorithm `K-Lookahead` in a WRSN under a real sensor energy consumption model [12]. (a) The accumulative charging utility gain. (b) The accumulative inactive time of sensors out of energy. (c) The tour length of the mobile charger. (d) The running times.

B. Algorithm Performance for the Charging Utility Maximization Problem

We first evaluate the performance of `Algorithm` 1 and `Algorithm` 2 for the length-constrained utility maximization problem, by varying the number of sensor nodes from 200 to 1,200 while fixing the tour length L at 800 meters.

It can be seen from Figure 3(a) that the charging utility gain by `Algorithm` 2 is around from 73.99% to 90.11% of the one by `Algorithm` 1, due to the fact that the tour length of the mobile charger in `Algorithm` 1 is unbounded while it is bounded in `Algorithm` 2. Figure 3(b) and 3(c) demonstrated that the amount of energy consumed on sensor charging by `Algorithm` 1 nearly reaches the energy capacity IE of the mobile charger, while in the solution delivered by `Algorithm` 2, the mobile charger does not run out of its energy for sensor charging due to the limited tour length constraint. Also, as depicted in Figure 3(c), the tour length of the mobile charger by `Algorithm` 1 is always longer than the tour length constraint L in `Algorithm` 2. It can be seen from Figure 3(d) that `Algorithm` 2 takes a much longer time than that of `Algorithm` 1 as refining its initial candidate solution takes time. However, with the increase on network size, the initial candidate solution by `Algorithm` 2 is very likely to be feasible and the running time of the algorithm decreases.

We then investigate the performance of `Algorithm` 3 against algorithm `K-Lookahead` for the charging utility maximization problem, by varying the number of sensors from 200 to 1,200 in a moderate size network and from 10,000 to 100,000 in a large network while fixing K at 2, under both synthetic and real sensor energy consumption models.

Under the synthetic sensor energy consumption model, it can be seen from Figure 4(a) that `Algorithm` 3 outperforms algorithm `K-Lookahead` in all cases, and the performance gap between them becomes larger with the growth of the network size. It can also be seen from Figure 4(b), and 4(c) that the tour length of the mobile charger by `Algorithm` 3 is less than that by algorithm `K-Lookahead`, this implies that `Algorithm` 3 will have more energy on sensor charging. Furthermore, the running time of algorithm `K-Lookahead` grows rapidly with the increase on network size.

Under the real sensor energy consumption model in [12], the performance of `Algorithm` 3 against algorithm `K-Lookahead` is evaluated as follows. Once the number of sensors running out of their energy reaches 10 percent of the network size, the mobile charger will be dispatched by the base station from its depot to charge the sensors. Figure 5 depicts the performance curves of these two algorithms. Notice that algorithm `K-Lookahead` only serves as a benchmark,

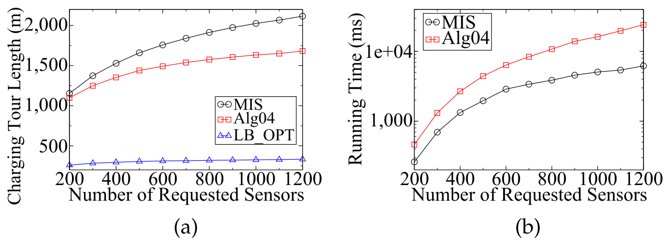


Fig. 6. Performance of different algorithms in a WRSN consisting of sensors from 200 to 1,200. (a) The charging tour length. (b) The running times.

thus, the actual energy charging process is performed by `Algorithm` 3 in each round, and 50 rounds of charging process are recorded. From Figure 5(a), it can be seen that `Algorithm` 3 outperforms algorithm `K-Lookahead` in all cases, and the performance gap between them becomes larger and larger with the increase on the network size. Charging utility delivered by `Algorithm` 3 is always more than that by algorithm `K-Lookahead`. Also, it can be seen from Figure 5(c) that the tour length of the mobile charger by `Algorithm` 3 is less than that by algorithm `K-Lookahead`, thus `Algorithm` 3 will have more energy on sensor charging. As a result, as shown in Figure 5(b), the accumulative inactive time of all sensors in the network by `Algorithm` 3 is much less than that by algorithm `K-Lookahead` in all cases. Figure 5(d) depicts the running times of the two algorithms.

C. Algorithm Performance for the Charging Tour Length Minimization Problem

We finally study the performance of `Algorithm` 4 against a heuristic `MIS`, by varying the number of sensors from 200 to 1,200 in a moderate size network. We also evaluate the solution from an optimal solution by adopting a conservative estimation on the optimal solution. That is, we make use of the lower bound L_{OPT} on the optimal solution calculated by Ineq. (7) as an estimation of the optimal solution, and we term this lower bound curve `LB_OPT`. Notice that the lower bound L_{OPT} may be far from the optimal solution OPT.

It can be seen from Figure 6(a) that `Algorithm` 4 delivers a solution within 4.22 to 5.06 times the lower bound of the corresponding optimal solutions. This empirical approximation ratio is much better than its analytical counterpart 40. In addition, the charging tour length by `Algorithm` 4 is much less than the one by algorithm `MIS` in all cases, and their performance gap becomes larger and larger with the increase on the network size. Figure 6(b) depicts the running times of the mentioned two algorithms.

VII. Conclusion

In this paper we studied the use of a mobile charger to charge multiple sensors simultaneously in a WRSN. We first formulated a novel charging utility maximization problem under the energy capacity constraint on the mobile charger. We also studied the charging tour length minimization problem if all requested sensors must be charged and the mobile charger has sufficient energy to do so. We showed that both problems are NP-hard, and instead developed efficient approximation and heuristic algorithms for them. We finally evaluated the proposed algorithms through experimental simulations. Simulation results demonstrate that the proposed algorithms are promising, and outperform other heuristics significantly.

Appendix

Proof for Theorem 1

Proof: We prove the NP-hardness of the charging utility maximization problem through a reduction from a well known NP-hard problem – the knapsack problem that is defined as follows. Given a bin capacity B, and a set A of items with each item $a_j \in A$ having a specified size $s(a_j)$ and a profit $p(a_j)$, the problem is to pack as many items as possible to the bin such that the total profit is maximized, subject to the bin capacity B.

We show that an instance of the knapsack problem can be reduced to an instance of the charging utility maximization problem. Specifically, the bin corresponds to the mobile charger with energy capacity B. A set $A = \{a_1, \ldots, a_n\}$ of items corresponds to a set of sensors to be charged by the mobile charger in the network. Each sensor a_j will be charged with an amount of energy $s(a_j)$ to its full capacity, while the charging utility gain is $p(a_j)$ for all j with $1 \leq j \leq n$.

We assume that no two sensors are within the energy charging range of the mobile charger at the same time. We further assume that the energy consumption of the mechanical movement of the mobile charger can be neglected. The charging utility maximization problem is to charge a subset of sensors by the mobile charger such that the accumulative charging utility gain is maximized, subject to the energy capacity B of the mobile charger. It can be seen that a solution to this special charging utility maximization problem returns a solution to the knapsack problem, and the reduction is polynomial. The theorem thus follows. □

Proof for Theorem 2

Proof: We show that the decision version of the length-constrained charging utility maximization problem is NP-hard, by a reduction from the following TSP problem.

Given a complete graph $G[V]$ with $|V| = n$, assume that each edge in $G[V]$ is assigned a weight either 1 or 2, the decision version of TSP in $G[V]$ is to determine whether there is a Hamiltonian cycle C in $G[V]$ such that the weighted sum of the edges in C is n.

We construct an instance $G'[V]$ of the length-constrained utility maximization problem from the instance of the TSP problem, where $G'[V]$ is a complete graph with $|V| = n$, too. Each edge is assigned a weight that is identical to its corresponding one in $G[V]$, which is the length of the mobile charger travelling along the edge. Each node in $G'[V]$ is assigned a weight of 1, corresponding to the amount of energy to charge the node, and the charging utility gain assigned to it. We assume that the mobile charger has an energy charging range $\gamma = 0$, i.e., the mobile charger only visits each sensor to charge it. Given a depot node v_0 in $G'[V]$ and an integer n, assume that the energy capacity IE of the mobile charger is n, and its tour length constraint L is n, the decision version of the length-constrained utility maximization problem is to determine whether there is a closed charging tour including the depot v_0 for the mobile charger such that the total charging utility gain collected from the nodes in C is n, subject to that the total energy consumption in C is no more than IE $(= n)$, and the total tour length of the mobile charger is no more than L $(= n)$. Clearly, if there is an optimal solution to the length-constrained utility maximization problem, there is an optimal solution to the TSP problem. □

Proof for Theorem 3

Proof: The NP-hardness of the charging tour length minimization problem, can be shown by a reduction from the well-known NP-hard TSP problem as follows.

A set V of vertices in a TSP instance corresponds to a set of sensors to be charged by the mobile charger plus the depot node v_0. We assume that the mobile charger has an energy charging range $\gamma = 0$. That is to say, the mobile charger only visits each requested sensor to charge it. We aim to find a closed tour with the minimum length containing all sensor nodes. It can be seen that a solution to this special charging tour length minimization problem is a solution to the TSP problem. The theorem thus follows. □

Proof for Lemma 1

Proof: We first show Claim (i). For each sensor in $OPT \setminus S_{k-1}$, the ratio of its charging utility gain with its neighbors' to its charging cost is bounded by $\frac{g'(v_k)}{\Delta'(v_k)}$, since the greedy strategy always selects a sensor v_k with the maximum ratio $\rho(v_k)$ from set $V_s \setminus S_{k-1}$. Since the total charging energy IE is a constraint on the charging utility gain of the set of sensors in $OPT \setminus S_{k-1}$ with their neighbors (in other words, the total energy received by sensors in $OPT \setminus S_{k-1}$ and their neighbors must be less than IE), we have

$$OPT - g(S_{k-1}) \le g(OPT \setminus S_{k-1}) \le IE \cdot \frac{g'(v_k)}{\Delta'(v_k)}. \quad (8)$$

Following the definition of charging utility gain of a sensor that $g'(v_k) = g(S_k) - g(S_{k-1})$, we have

$$g(S_k) - g(S_{k-1}) \ge \frac{\Delta'(v_k)}{IE}(OPT - g(S_{k-1})). \quad (9)$$

We then show Claim (ii) by mathematical induction on the number of iterations k with $1 \le k \le k'$. Initially, when $k = 1$, $g(S_1) = g'(v_1) = \sum_{u \in N_c^+(v_1)} \big(f(C_u) - f(RE_u)\big)$. We need to show that $g'(v_1) \ge OPT \cdot \frac{\Delta'(v_1)}{IE}$. This follows from the fact that the ratio $\frac{g'(v_1)}{\Delta'(v_1)}$ for sensor v_1 is the maximum one among all sensors in V_s, and the charging cost of the optimal solution is bounded by IE.

Suppose that $g(S_k) \ge OPT \cdot \Big(1 - \prod_{j=1}^{k} \big(1 - \frac{\Delta'(v_j)}{IE}\big)\Big)$ holds for iteration k with $1 \le k < k'$. We now show that it also holds for $k + 1$. By Claim (i), we have

$$\begin{aligned} g(S_k) &\ge g(S_{k-1}) + \frac{\Delta'(v_k)}{IE}\Big(OPT - g(S_{k-1})\Big) \\ &= (1 - \frac{\Delta'(v_k)}{IE}) \cdot g(S_{k-1}) + \frac{\Delta'(v_k)}{IE} \cdot OPT \\ &\ge (1 - \frac{\Delta'(v_k)}{IE}) \cdot \Big(OPT \cdot \Big(1 - \prod_{j=1}^{k-1} \big(1 - \frac{\Delta'(v_j)}{IE}\big)\Big)\Big) \\ &\quad + \frac{\Delta'(v_k)}{IE} \cdot OPT \quad \text{by inductive hypothesis,} \\ &= \Big(1 - \frac{\Delta'(v_k)}{IE} - \big(1 - \frac{\Delta'(v_k)}{IE}\big) \cdot \prod_{j=1}^{k-1} \big(1 - \frac{\Delta'(v_j)}{IE}\big)\Big) \\ &\quad \cdot OPT + \frac{\Delta'(v_k)}{IE} \cdot OPT \\ &= OPT \cdot \Big(1 - \prod_{j=1}^{k} \big(1 - \frac{\Delta'(v_j)}{IE}\big)\Big). \end{aligned} \quad (10)$$

The lemma thus follows. □

Proof for Theorem 4

Proof: By Lemma 1, we have $g(S_{k'}) \ge OPT \cdot (1 - \prod_{j=1}^{k'} \big(1 - \frac{\Delta'(v_j)}{IE}\big))$. Following `Algorithm` 1, we have $E(S_{k'}) = E(S_{k'-1}) + \Delta'(v_{k'}) > IE$, as the addition of sensor $v_{k'}$ to $S_{k'}$ will violate the charging energy capacity constraint IE of the mobile charger. Then,

$$\begin{aligned} g(S_{k'}) &\ge OPT\Big(1 - \prod_{j=1}^{k'} \big(1 - \frac{\Delta'(v_j)}{E(S_{k'})}\big)\Big) \\ &= OPT\Big(1 - \prod_{j=1}^{k'} \big(1 - \frac{\Delta'(v_j)}{\Delta'(v_1) + \Delta'(v_2) + \cdots + \Delta'(v_{k'})}\big)\Big) \\ &\qquad \text{as } E(S_{k'}) = \Delta'(v_1) + \Delta'(v_2) + \cdots + \Delta'(v_{k'}), \\ &\ge OPT \cdot \big(1 - (1 - \frac{1}{k'})^{k'}\big) \\ &\qquad \text{by Lemma 2 where } A = IE,\ b = 1, \text{ and } a_i = \Delta'(v_i), \\ &\ge OPT \cdot (1 - 1/e) \quad \text{when } k' \to \infty. \end{aligned} \quad (11)$$

On the other hand, the maximum charging utility gain can be achieved by charging sensor $v_{max} \in V_s$ and its neighbors in $N_c(v_{max})$, we thus have $g(S_{k'-1}) + g'(v_{max}) \ge g(S_{k'-1}) + g'(s_{k'}) = g(S_{k'}) \ge OPT \cdot (1 - \frac{1}{e})$.

Since `Algorithm 1` will select the larger one between $g(S_{k'-1})$ and $g'(v_{max})$, the solution delivered by `Algorithm` 1 is at least $\frac{1}{2}(1 - \frac{1}{e}) \cdot OPT$.

The rest is to analyze the time complexity of `Algorithm` 1. The algorithm consists of $O(|V_s|)$ iterations. Within each iteration, identifying a sensor v_k with the maximum ratio $\rho(v_k)$ takes $O(|V_s|)$ time. The algorithm thus takes $O(|V_s|^2)$ time for $O(|V_s|)$ iterations. Choosing a single sensor v_{max} with the maximum charging utility gain takes $O(|V_s|)$ time, while finding a closed charging tour C takes $O(|V_s|^3)$ time. `Algorithm` 1 thus takes $O(|V_s|^3)$ time. □

Proof for Lemma 3

Proof: We show that each sensor in V' will be covered by $\cup_{u \in MIS} R(u, 2\gamma)$. Clearly, if a sensor is within disk $R(u, \gamma)$ for each $u \in MIS$, it will be covered (charged) as there is a closed tour with length L_I going through the center u of disk $R(u, \gamma)$ for each $u \in MIS$. Now, if a sensor $v \notin \cup_{u \in MIS} R(u, \gamma)$, then we show that it must be in $\bigcup_{u \in MIS} R(u, 2\gamma)$ by contradiction. Assume that v is not in $\bigcup_{u \in MIS} R(u, 2\gamma)$, then the Euclidean distance between v and a nearest node in its nearest disk $R(u', 2\gamma)$ with $u' \in MIS$ is larger than 2γ, this implies that node v should be included in the MIS by the construction of G'. This results in a contradiction. Thus, each sensor in V' will be covered by $\bigcup_{u \in MIS} R(u, 2\gamma)$. □

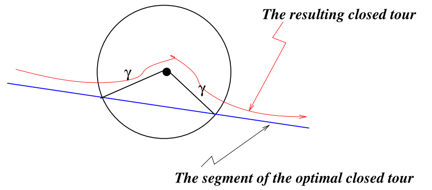


Fig. 7. An extended closed tour contains each center u of a disk for each $u \in MIS$.

Proof for Lemma 4

Proof: We show the claim by contradiction. Assume that there exist i and j with $i \neq j$ such that $N_c^+(u_i) \cap N_c^+(u_j) \neq \emptyset$. Let $v \in N_c^+(u_i) \cap N_c^+(u_j)$, then both sensors u_i and u_j can be charged by the mobile charger if it is located at v, then $d(u_i, u_j) \leq d(v, u_i) + d(v, u_j) = 2\gamma$, this contradicts the assumption that u_i and u_j are in an independent set MIS and their Euclidean distance is strictly greater than 2γ. □

Proof for Lemma 6

Proof: As L_{OPT} is an optimal tour visiting all disks $R(u_i, \gamma)$ for all $u_i \in MIS$, the area $A_{L_{OPT}}$ swept by a disk of radius 2γ, whose center moves along tour L_{OPT}, cover all of these disks. This area is bounded as follows.

$$p \cdot (\pi\gamma^2) \leq A_{L_{OPT}} \leq 4\gamma \cdot L_{OPT} + \pi(2\gamma)^2. \tag{12}$$

Then,

$$p \leq \frac{4\gamma L_{OPT} + 4\pi\gamma^2}{\pi\gamma^2} = \frac{4L_{OPT} + 4\pi\gamma}{\pi\gamma}. \tag{13}$$

The length of C' is no more than $L_{OPT} + 2\gamma \cdot |MIS| = L_{OPT} + 2p\gamma$, as at most the two endpoints of the segment in the optimal tour going through each disk can be extended to another closed tour that includes the centers of all disks centralized at all $u \in MIS$ (see Fig. 7). The length $|C'|$ of the optimal tour C' going through the centers of disks $R(u, \gamma)$ for all $u \in MIS$ thus is

$$\begin{aligned}
|C'| &\leq L_{OPT} + p \cdot 2\gamma \\
&\leq L_{OPT} + \frac{4\ L_{OPT} + 4\pi\gamma}{\pi\gamma} \cdot 2\gamma \quad \text{by Ineq. (13)}, \\
&\leq (1 + 8/\pi)L_{OPT} + 8\gamma \qquad (14)\\
&\leq (1 + 8/\pi)|C^*| + 8\gamma \quad \text{as } L_{OPT} \leq |C^*|. \qquad (15)
\end{aligned}$$

□

Proof for Lemma 7

Proof: We first show the length of tour $L(u)$ is no greater than $\sqrt{2} \cdot C_u$, where C_u is the circumference of disk $R(u, 2\gamma)$ while $C_u = 2\pi \cdot 2\gamma = 4\pi\gamma$ (see Fig. 8). As sensors are randomly distributed in the shadow area of concentric disks centered at u with radii γ and 2γ respectively. In the worst case all chosen sensors in $VR(u, 2\gamma)$ are either at nearby the circumference of the circle with radius γ or at nearby the circumference of the circle with radius 2γ, and we refer to them as *the internal-circumference* and *the outside-circumference* of the two circles centered at u. Let $L(u)$ be the closed subtour connecting these chosen sensors in V_u in clockwise order, which forms a zigzag-shape closed tour. The maximum length of $L(u)$ is calculated as follows.

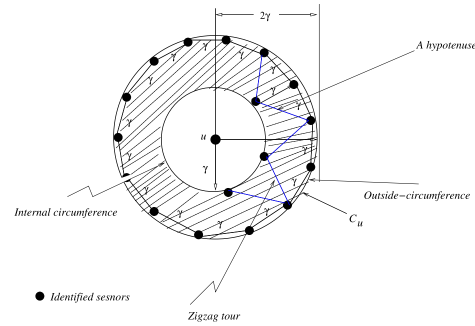


Fig. 8. An illustration of disk $R(u, 2\gamma)$ and all disks with radius γ at chosen sensors (black dots) can cover the sensors in disk $R(u, 2\gamma)$, where C_u is the outside-circumference, while the zigzag tour $L(u)$ consists of all chosen sensors in $VR(u, 2\gamma)$, which are the black nodes in the shadow (band) area between the internal-circumference and the outside-circumference of center u.

Since we aim to choose a set V_u of sensors covering all sensors in the shadow area of two concentric disks centered at u. In the worst scenario, there are at most $\frac{C_u}{\gamma} = \frac{2\pi \cdot 2\gamma}{\gamma} = 4\pi$ sensors located nearby the outside-circumference, corresponding the same number of hypotenuses between the nodes nearby the internal-circumference and the nodes nearby the outside-circumference. It can be seen that the length of each hypotenuse is $\sqrt{\gamma^2 + \gamma^2} = \sqrt{2}\gamma$. The length $L(u)$ of a subtour connecting all sensors in V_u thus is no more than $4\pi \cdot (\sqrt{2}\gamma) = 4\sqrt{2}\pi\gamma$. Then, $\sum_{u \in MIS} L(u) \leq 4\sqrt{2}\pi p\gamma$. □

Proof for Lemma 8

Proof: The length $|C|$ of the closed tour C including the depot v_0 of the mobile charger for charging all sensors in V' then is no more than

$$\begin{aligned}
|C| &\leq (|C'| + \sum_{u \in MIS} L(u))\beta \\
&\leq ((1 + 8/\pi)|C^*| + 8\gamma + 4\sqrt{2}\pi p\gamma)\beta \\
&\leq ((1 + 8/\pi)|C^*| + 8\gamma \\
&\qquad + 16\sqrt{2}(L_{OPT} + \pi\gamma))\beta \text{ by Ineq. (13)} \\
&\leq [(1 + 16\sqrt{2} + 8/\pi)|C^*| + (8 + 16\sqrt{2}\pi)\gamma]\beta \\
&\qquad \text{as } L_{OPT} \leq |C^*|. \qquad (16)
\end{aligned}$$

□

Proof for Theorem 7

Proof: We first show the approximation ratio of `Algorithm` 4 as follows. Recall that $|C^*|$ is the length of an optimal tour C^* to the charging tour length minimization problem. The length $|C|$ of the closed tour C delivered by `Algorithm` 4 is analyzed as follows.

$$\begin{aligned}
\frac{|C|}{|C^*|} &\leq \frac{[(1 + 16\sqrt{2} + \frac{8}{\pi})|C^*| + (8 + 16\sqrt{2}\pi)\gamma] \cdot \beta}{|C^*|} \\
&\qquad \text{by Ineq. (16)} \\
&\leq (1 + 16\sqrt{2} + \frac{8}{\pi})\beta + \frac{(8 + 16\sqrt{2}\pi)\gamma\beta}{|C^*|} \\
&\approx 39.26 + 118.63\gamma/|C^*| \quad \text{where } \beta = 1.5, \\
&\leq 40 \quad \text{as } \gamma \ll |C^*|,\ \gamma/|C^*| \approx 0. \qquad (17)
\end{aligned}$$

As $\gamma \ll |C^*|$, the approximation ratio of Algorithm 4 actually is around 40. Notice that the above approximation ratio estimation is very conservative, and the empirical approximation ratio of Algorithm 4 is no greater than 10.

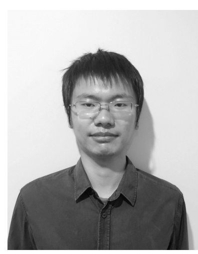
The construction of auxiliary graph G' and finding a maximal independent set MIS in G' takes $O(|V'|^2)$ time. Identifying a set V_u of sensors in $VR(u, 2\gamma)$ for each $u \in MIS$ takes $O(|V'|)$ time, as each sensor will be considered at most once. The construction of the closed tour C based on the identified sensor set $MIS \cup \cup_{u \in MIS} V_u$ takes $O(|V'|^3)$ time, by invoking the Christofides algorithm. Algorithm 4 thus takes $O(|V'|^3)$ time. □

ACKNOWLEDGEMENT

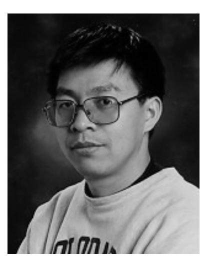
The authors would like to thank the three anonymous referees and the associate editor for their expertise comments and constructive suggestions, which have helped them improve the quality and presentation of the paper greatly.

REFERENCES

[1] N. Christofides, "Worst-case analysis of a new heuristic for the travelling salesman problem," Graduate School Ind. Admin., CMU, Pittsburgh, PA, USA, Tech. Rep. 388, 1976.
[2] A. Costanzo *et al.*, "Electromagnetic energy harvesting and wireless power transmission: A unified approach," *Proc. IEEE*, vol. 102, no. 11, pp. 1692–1711, Nov. 2014.
[3] (2017). *Cotasystem*. [Online]. Available:http://www.ossiainc.com
[4] H. Dai *et al.*, "Safe charging for wireless power transfer," *IEEE/ACM Trans. Netw.*, vol. 25, no. 6, pp. 3531–3544, Dec. 2017.
[5] A. Dumitrescu and J. S. B. Mitchell, "Approximation algorithms for TSP with neighborhoods in the plane," *J. Algorithms*, vol. 48, no. 1, pp. 135–159, Aug. 2003.
[6] G. Jiang, S.-K. Lam, Y. Sun, L. Tu, and J. Wu, "Joint charging tour planning and depot positioning for wireless sensor networks using mobile chargers," *IEEE/ACM Trans. Netw.*, vol. 25, no. 4, pp. 2250–2266, Aug. 2017.
[7] A. Kansal, J. Hsu, S. Zahedi, and M. B. Srivastava, "Power management in energy harvesting sensor networks," *ACM Trans. Embedded Comput. Syst.*, vol. 6, no. 4, Sep. 2007, Art. no. 32.
[8] L. Khelladi, D. Djenouri, M. Rossi, and N. Badachea, "Efficient on-demand multi-node charging techniques for wireless sensor networks," *Comput. Commun.*, vol. 101, pp. 44–56, Mar. 2017.
[9] S. Khuller, A. Moss, and J. S. Naor, "The budgeted maximum coverage problem," *Inf. Process. Lett.*, vol. 70, no. 1, pp. 39–45, Apr. 1999.
[10] A. Kurs *et al.*, "Wireless power transfer via strongly coupled magnetic resonances," *Science*, vol. 317, no. 5834, pp. 83–86, 2007.
[11] A. Kurs, R. Moffatt, and M. Soljačić, "Simultaneous mid-range power transfer to multiple devices," *Appl. Phys. Lett.*, vol. 96, no. 4, pp. 044102-1–044102-3, 2010.
[12] J. Li and P. Mohapatra, "Analytical modeling and mitigation techniques for the energy hole problem in sensor networks," *Pervasive Mobile Comput. J.*, vol. 3, no. 3, pp. 233–254, Jun. 2007.
[13] Z. Li, Y. Peng, W. Zhang, and D. Qiao, "J-RoC: A joint routing and charging scheme to prolong sensor network lifetime," in *Proc. 19th IEEE Int. Conf. Netw. Protocols*, Oct. 2011, pp. 373–382.
[14] W. Liang, W. Xu, X. Ren, X. Jia, and X. Lin, "Maintaining sensor networks perpetually via wireless recharging mobile vehicles," in *Proc. 39th Annu. IEEE Conf. Local Comput. Netw.*, Sep. 2014, pp. 270–278.
[15] W. Liang, W. Xu, X. Ren, X. Jia, and X. Lin, "Maintaining large-scale rechargeable sensor networks perpetually via multiple mobile charging vehicles," *ACM Trans. Sensor Netw.*, vol. 12, no. 2, 2016, Art. no. 14.
[16] W. Liang *et al.*, "Approximation algorithms for charging reward maximization in rechargeable sensor networks via a mobile charger," *IEEE/ACM Trans. Netw.*, vol. 25, no. 5, pp. 3161–3174, Oct. 2017.
[17] X. Lu, P. Wang, D. Niyato, D. I. Kim, and Z. Han, "Wireless charging technologies: Fundamentals, standards, and network applications," *IEEE Commun. Surveys Tuts.*, vol. 18, no. 2, pp. 1413–1452, 2nd Quart., 2016.
[18] (2017). *PRIMOVE e-Mobility Solution*. [Online]. Available: http://primove.bombardier.com
[19] X. Ren, W. Liang, and W. Xu, "Use of a mobile sink for maximizing data collection in energy harvesting sensor networks," in *Proc. IEEE 42nd Int. Conf. Parallel Process.*, Oct. 2013, pp. 439–448.
[20] X. Ren, W. Liang, and W. Xu, "Data collection maximization in renewable sensor networks via time-slot scheduling," *IEEE Trans. Comput.*, vol. 64, no. 7, pp. 1870–1883, Jul. 2015.
[21] S. Saxena, C. Le Floch, J. MacDonald, and S. Moura, "Quantifying EV battery end-of-life through analysis of travel needs with vehicle powertrain models," *J. Power Sources*, vol. 282, no. 8, pp. 265–276, May 2015.
[22] Y. Shi, L. Xie, Y. T. Hou, and H. D. Sherali, "On renewable sensor networks with wireless energy transfer," in *Proc. IEEE INFOCOM*, Apr. 2011, pp. 1350–1358.
[23] C. Wang, J. Li, F. Ye, and Y. Yang, "A mobile data gathering framework for wireless rechargeable sensor networks with vehicle movement costs and capacity constraints," *IEEE Trans. Comput.*, vol. 65, no. 8, pp. 2411–2427, Aug. 2016.
[24] (2017). *Wireless Power Consortium*. [Online]. Available: http://www.wirelesspowerconsortium.com/
[25] M. Wu, D. Ye, J. Kang, H. Zhang, and R. Yu, "Optimal and cooperative energy replenishment in mobile rechargeable networks," in *Proc. IEEE 83rd Veh. Technol. Conf. (Spring)*, May 2016, pp. 1–5.
[26] L. Xie, Y. Shi, Y. T. Hou, W. Lou, and H. D. Sherali, "On traveling path and related problems for a mobile station in a rechargeable sensor network," in *Proc. 14th ACM Int. Symp. Mobile Ad Hoc Netw. Comput.*, 2013, pp. 109–118.
[27] L. Xie *et al.*, "Multi-node wireless energy charging in sensor networks," *IEEE/ACM Trans. Netw.*, vol. 23, no. 2, pp. 437–450, Apr. 2015.
[28] L. Xie *et al.*, "A mobile platform for wireless charging and data collection in sensor networks," *IEEE J. Sel. Areas Commun.*, vol. 33, no. 8, pp. 1521–1533, Aug. 2015.
[29] W. Xu, W. Liang, X. Jia, and Z. Xu, "Maximizing sensor lifetime in a rechargeable sensor network via partial energy charging on sensors," in *Proc. IEEE 13th Annu. Conf. Sens. Commun. Netw.*, Jun. 2016, pp. 1–9.
[30] Z. Xu, W. Liang, and Y. Xu, "Network lifetime maximization in delay-tolerant sensor networks with a mobile sink," in *Proc. IEEE 8th Int. Conf. Distrib. Comput. Sensor Syst. (DCOSS)*, May 2012, pp. 9–16.
[31] X. Ye and W. Liang, "Charging utility maximization in wireless rechargeable sensor networks," *Wireless Netw.*, vol. 23, no. 7, pp. 2069–2081, 2017.
[32] J. Yick, B. Mukherjee, and D. Ghosal, "Wireless sensor network survey," *Comput. Netw.*, vol. 52, no. 12, pp. 2292–2330, Aug. 2008.



Yu Ma received the B.Sc. degree (Hons.) in computer science from The Australian National University in 2015, where he is currently pursuing the Ph.D. degree with the Research School of Computer Science. His research interests include software-defined networking, Internet of Things, and social networking.



Weifa Liang (M'99–SM'01) received the B.Sc. degree from Wuhan University, China, in 1984, the M.E. degree from the University of Science and Technology of China in 1989, and the Ph.D. degree from The Australian National University in 1998, all in computer science. He is currently a Full Professor with the Research School of Computer Science, The Australian National University. His research interests include design and analysis of energy efficient routing protocols for wireless ad hoc and sensor networks, cloud computing, software-defined networking, design and analysis of parallel and distributed algorithms, approximation algorithms, combinatorial optimization, and graph theory.

Wenzheng Xu (M'15) received the B.Sc., M.E., and Ph.D. degrees in computer science from Sun Yat-sen University, Guangzhou, China, in 2008, 2010, and 2015, respectively. He was a Visitor with The Australian National University and The Chinese University of Hong Kong. He is currently a Special Associate Professor with Sichuan University. His research interests include wireless ad hoc and sensor networks, mobile computing, approximation algorithms, combinatorial optimization, online social networks, and graph theory.