# Maximizing Sensor Lifetime via Multi-node Partial-Charging on Sensors

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Abstract—In this paper, we study the employment of a mobile charger to charge lifetime-critical sensors under the multi-node partial-charging model, in which the charger can simultaneously charge the sensors within its charging range and each sensor may be partially charged each time. We notice that existing studies only scheduled the charger to minimize the number of dead sensors, but did not consider the charging scheduling for the sensors that have already run out of their energy, and the dead sensors will be last charged by the mobile charger. Then, their dead durations may be very long. In this paper, we consider not only how to minimize the number of dead sensors but also reduce the dead durations of sensors. To this end, we first formulate a sensor lifetime maximization problem, which is to find a charging tour for a mobile charger to charge sensors, such that the sum of sensor lifetimes is maximized. We then propose a novel  $\frac{1}{3}$ -approximation algorithm for the problem. We finally evaluate the performance of the proposed algorithm through experiments. Experimental results show that both the average and maximum sensor dead durations by the proposed algorithm are up to 70% shorter than those by existing algorithms.

Index Terms—Wireless rechargeable sensor networks, multi-node charging, partial charging, approximation algorithm

### 1 Introduction

Wireless sensor networks (WSNs) are widely used in various applications, including air pollution monitoring in smart cities, bush fire monitoring, flooding monitoring, etc [2], [6], [9], [20]. Since the amount of energy stored in every sensor battery is limited, the sensor will run out of its energy due to sensing, data transmission, and data reception [13]. To replenish energy to sensors, a promising approach is to deploy *mobile chargers* to charge lifetime-critical sensors by utilizing a wireless charging technique [11], [12], [24], as mobile chargers are able to provide stable and high-efficient energy supplies to sensors.

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The scheduling of mobile chargers to charge sensors has attracted lots attentions in the past decade. Most existing studies adopted a *one-to-one charging model* [3], [5], [10], [26], [28], [29], [34], in which a mobile charger charges only a single sensor each time. However, since it takes a while (e.g., 30-80 min) to fully charge a commercial off-the-shelf sensor battery (e.g., Lithium battery) [28], some sensors may run out of their energy before the mobile charger charges them.

To improve charging efficiency and shorten sensor energy expiration durations, some studies adopted a multi-node fullcharging model [15], [19], [30], [31], [32], in which sensors within the charging range of a mobile charger can be simultaneously charged by the mobile charger, and the sensors will be fully charged. A recent experimental result showed that the maximum charging range of a mobile charger can be as far as 15 meters [22]. However, the energy charging efficiency decreases with the increase on the distance between a mobile charger and a sensor. That is, the further away from the mobile charger, the less energy the sensor will receive. Then, the charging time at a charging location under the multi-node full-charging model highly depends on the distance between the mobile charger and the farthest sensor in its charging range, as the sensor has the lowest charging rate. Therefore, it may take a long time to fully charge all sensors in the charging range of the mobile charger, and some other sensors may still run out of their energy before the mobile charger charges them.

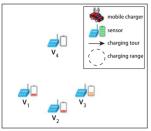
There are a few recent studies that adopted a novel *multi-node partial-charging model* [18], [35] to significantly shorten sensor dead durations, in which not only multiple sensors can be simultaneously charged by a mobile charger, but also each sensor may be partially charged each time.

We note that the studies in [18], [35] did not consider the charging scheduling of sensors that have run out of their

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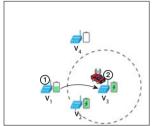
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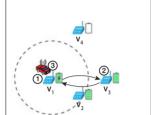




sors  $v_1, v_2$ , and  $v_3$  are 5, 10, and tially charges sensors  $v_1$  and  $v_2$ 40 minutes, respectively, while for 30 minutes at the location of  $v_4$  has run out energy for 20 sensor  $v_1$ min

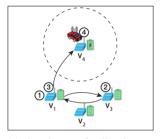
(a) the residual lifetimes of sen- (b) the mobile charger first par-





(c) the charger then fully (d) charges sensors  $v_2$  and  $v_3$  for recharges sensor  $v_1$  to its full one hour at the location of  $v_3$ 

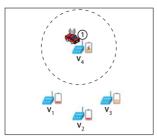
the further energy capacity for 30 minutes at the location of sensor  $v_1$ 

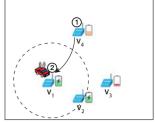


(e) the charger finally charges sensor  $v_4$  to its full energy capacity for one hour

Fig. 1. An illustration of the charging schedulings in existing studies under the multi-node partial-charging model, where the dead durations of sensors  $v_1, v_2, v_3$  and  $v_4$  are 0, 0, 0, and 140 minutes (=20+30+60 +30), respectively.

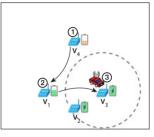
energy, and the dead sensors will be last charged by the mobile charger. Thus, the dead duration of a dead sensor may be very long. For example, Fig. 1a shows a sensor network consisting of four sensors  $v_1, v_2, v_3$ , and  $v_4$ , where the residual lifetimes of sensors  $v_1, v_2, v_3$  are 5, 10, and 40 min, respectively, while sensor  $v_4$  has already run out of its energy for 20 min. For the sake of convenience, assume that the traveling time of the mobile charger between any two sensors can be ignored, as the mobile charger traveling time is much shorter than the sensor charging time [33]. Figs. 1b, 1c, 1d, and 1e show the charging scheduling delivered by existing studies in [18], [35] under the multi-node partial-charging model. Specifically, Fig. 1b shows that the mobile charger first partially charges sensors  $v_1$  and  $v_2$  for 30 minutes at the location of  $v_1$ . Assume that the amounts of energy charged to sensors  $v_1$  and  $v_2$  in the 30 minutes are able to support their operations for 2 weeks and 1 week, respectively, as the energy charging efficiency for  $v_1$  is larger than that of  $v_2$ . The mobile charger then moves to the location of  $v_3$  and it takes

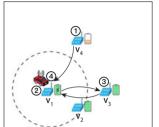




tially charges sensor  $v_4$  for 5 minutes at the location of  $v_4$ 

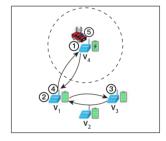
(a) the mobile charger first par- (b) the mobile charger then partially charges sensors  $v_1$  and  $v_2$ for 30 minutes at the location of sensor  $v_1$ 





(c) the charger fully charges (d) the sensors  $v_2$  and  $v_3$  for one hour recharges sensor  $v_1$  to its full at the location of  $v_3$ 

charger further energy capacity for 30 minutes at the location of sensor  $v_1$ 



(e) the charger finally recharges sensor  $v_4$  to its full energy capacity at the location of  $v_4$ 

Fig. 2. An illustration of the proposed charging scheduling under the multi-node partial-charging model, where the dead durations of sensors  $v_1, v_2, v_3$  and  $v_4$  are 0, 0, 0, and 20 min, respectively.

some energy has already been charged to sensor  $v_2$  when the mobile charger stayed at  $v_1$ . The mobile charger further returns to the location of  $v_1$  and it takes 30 minutes to fully charge sensor  $v_1$ , see Fig. 1d. It can be seen that none of the three sensors  $v_1, v_2$  and  $v_3$  runs out its energy. On the other hand, since sensor  $v_4$  has run out of energy initially, the mobile charger finally charges  $v_4$  (see Fig. 1e) and its dead duration is as long as 140 minutes (= 20 + 30 + 60 + 30).

We note that the energy expirations of some sensors for a long time may lead to severe consequences to the sensor network. For example, consider a sensor network for early forest fire detections [8], [23]. The energy expiration of a sensor for several hours may delay the detection of a forest fire. Such a detection delay may incur significant damages and casualties, as the forest fire may quickly spread by strong wind in a very short time [8], [23], [33].

In this paper, we consider not only how to minimize the number of dead sensors but also reduce the dead durations of the sensors. To this end, we aim to maximize the lifetime of all sensors, thereby minimizing the dead durations of the one hour to fully charge sensors  $v_2$  and  $v_3$ , see Fig. 1c, where sensors. Fig. 2 illustrates the proposed charging schedu Authorized licensed use limited to: CITY UNIV OF HONG KONG. Downloaded on October 24,2023 at 14:26:49 UTC from IEEE Xplore. Restrictions apply. sensors. Fig. 2 illustrates the proposed charging scheduling

in this paper. Specifically, Fig. 2a shows that the mobile charger first charges sensor  $v_4$  for only 5 min and the amount of energy charged to  $v_4$  in the 5 minutes can support its operation for some time, e.g., 3 hours, since the energy consumption rate of a sensor usually is low. The mobile charger then charges sensors  $v_1$  and  $v_2$  for 30 minutes at the location of  $v_1$ , see Fig. 2b. The charger moves to the location of  $v_3$  and it takes one hour to fully charge sensors  $v_2$  and  $v_3$ , see Fig. 2c. The charger further recharges sensor  $v_1$  to its full energy capacity for 30 minutes at the location of  $v_1$ , see Fig. 2d. Finally, the charger recharges sensor  $v_4$  to its full energy capacity at the location of  $v_4$ , see Fig. 2e. It can be seen that not only three sensors  $v_1, v_2$ , and  $v_3$  do not run out of their energy before the mobile charger charges them, but also the dead duration of sensor  $v_4$  is only 20 min, which is much shorter than its dead duration 140 minutes in the charging schedulings by the existing studies under the multi-node partial-charging model [18], [35] (see Fig. 1), where sensor  $v_4$  has already run out of its energy for 20 min before the mobile charger charges any sensor.

The novelties of this paper are two-fold. On one hand, unlike existing studies that ignored the charging scheduling of sensors that have already run out of energy, in this paper we study a sensor lifetime maximization problem, which is to find a charging tour for a mobile charger, such that the sum of sensor lifetimes is maximized, thereby shortening sensor dead durations. On the other hand, we propose the very first  $\frac{1}{3}$ -approximation algorithm for the problem.

The challenges of the problem considered in this paper are as follows. (i) How to determine the charging locations of a mobile charger and the charging time at each location to ensure that each energy-critical sensor will be charged to its full energy capacity? (ii) Since sensors can be partially charged each time, how to schedule the mobile charger to maximize the lifetime of sensors? In this paper, we will tackle the two mentioned challenges.

The *contributions* of this paper are summarized as follows. (i) We formulate a novel sensor lifetime maximization problem, which is to find a charging tour for a mobile charger with energy capacity, such that the sum of sensor lifetimes is maximized. (ii) To the best of our knowledge, we propose the very first  $\frac{1}{3}$  approximation algorithm for the problem. (iii) We evaluate the performance of the proposed algorithm through experiments. Experimental results show that the proposed algorithm is very promising. Specifically, both the average and maximum sensor dead durations by the proposed algorithm are up to 70% shorter than those by existing algorithms.

The rest of this is organized as follows. Section 2 reviews related work. Section 3 introduces the network model and charging model, and defines the problem. Sections 4 proposes an approximation algorithm for the problem and Section 5 analyzes its approximation ratio. Section 6 evaluates the performance of the proposed algorithm. Finally, Section 7 concludes this paper.

### 2 Related Work

Wireless energy transfer technique [11] has been considered as a promising approach to prolong the lifetime of sensor networks. Most existing studies adopted the *one-to-one full*-Authorized licensed use limited to: CITY UNIV OF HONG KONG. Downloaded on October 24,2023 at 14:26:49 UTC from IEEE Xplore. Restrictions apply.

*charging model* [10], [26], [29], [34], in which a mobile charger charges only one sensor each time and the sensor is fully charged before the charger charges the next sensor. For example, Shi et al. [26] were the first to apply the wireless charging technique for charging sensors, and they scheduled a mobile charger to charge sensors periodically such that the network can operate perpetually. Xu et al. [34] studied the deployment of multiple mobile chargers to collaboratively charge sensors for a monitoring period, where different sensors have different energy consumption rates. They proposed a charging algorithm for dispatching the mobile chargers in the monitoring period, such that no sensors run out of energy and the total traveling distance of the mobile chargers is minimized. Wang et al. [29] devised an adaptive charging scheduling algorithm to maximize the amount of energy charged to sensors minus the mobile chargers traveling energy consumption, subject to the energy capacity constraint on each mobile charger, where each charged sensor is replenished before its energy expiration. Jiang et al. [10] jointly considered the charging tour planning and mobile charger depot positioning for largescale WSNs. They studied the problem of minimizing the number of deployed mobile chargers to maintain perpetual network operations.

Some studies adopted the one-to-one partial-charging model [14], [16], [33], in which a mobile charger charges only one sensor each time and the sensor may be partially charged, rather than fully charged. For example, Liang et al. [14] considered two different charging scenarios where sensors can be charged multiple times per tour or not. They devised approximation algorithms to maximize the sum of charging rewards collected from all charged sensors per tour under the energy capacity constraint on the mobile charger. Xu et al. [33] applied partial-charging for increasing sensor lifetimes and proposed an algorithm to schedule a mobile charger to charge sensors by determining not only the charging sequence of sensors but also the amount of energy charged to a sensor each time. They aimed to maximize the sum of sensor lifetimes and minimize the travel distance of the mobile charger. Lin et al. [16] further improved the algorithm in [33] by proposing a mixed partial and full charging scheme.

Some recent studies adopted the multi-node full-charging model to improve charging efficiencies [15], [17], [19], [27], [30], [31], [32], where a mobile charger can simultaneously charge all sensors within the charging range of the mobile charger. Xie et al. [31] were the first to apply this model for WSNs by deploying a mobile charger to periodically travel inside the network and charge sensors. They aimed to maximize the ratio of the vacation time of the mobile charger at its service station to the total period, while ensuring that none of the sensors run out of energy. Ma et al. [19] devised approximation algorithms to find a charging tour under the energy capacity constraint on the mobile charger, such that the accumulative utility of the sensors charged by the mobile charger is maximized. Wang et al. [30] considered the trade-off between the mobile charger movement cost and the sensor charging efficiency, and proposed a bundle charging scheme to cluster sensors in the network into a set of charging bundles. They then found the optimal bundle

charge sensors in bundles, such that the total energy consumption of the mobile charger is minimized. Lin et al. [17] studied a directional energy transfer model, and investigated a problem of determining the charging time at each charging location as well as the charging direction of the mobile charger at the location, such that the total charging time is minimized. Lin et al. [15] later considered that obstacles around a sensor will affect the charging efficiency for the sensor. They proposed a charging model with obstacles to solve the charging utility maximization problem. To further improve charging scalability, Xu et al. [32] recently studied the multi-node full-charging model with multiple mobile chargers to charge large-scale WSNs. They proposed an approximation algorithm to find a charging tour for each mobile charger such that the longest charging tour time among multiple tours is minimized, while ensuring that no sensors are simultaneously charged by multiple mobile chargers. Tomar et al. [27] proposed an on-demand charging scheduling scheme with multiple mobile chargers, which first assigns sensors to the mobile chargers by the charging load of sensors, then utilizes a fuzzy logic to determine the charging sequence of sensors.

Unlike the aforementioned studies, in this paper we utilize a multi-node partial-charging model to effectively shorten sensor dead durations and improve charging efficiency, where sensors within the charging range of a mobile charger can be partially and simultaneously charged. Then, not only the charging efficiency is improved by the multi-node charging model, but also more sensors are charged before their energy expirations by smartly determining the charging sequence and charging time at each charging location.

We also note that there are a few studies adopting the multi-node partial-charging model [18], [35]. Liu et al. [18] studied a problem of minimizing the number of dead sensors under a multi-node partial-charging model and proposed a greedy charging scheduling algorithm by the residual lifetimes of sensors. Yang et al. [35] investigated a problem of maximizing the total charging utility of sensors, where the charging utility of a sensor is counted if it is charged before its energy expiration. However, both the studies in [18], [35] did not consider the charging scheduling for the sensors that have already run out of energy or cannot be charged before their energy expirations. Then, the dead durations of the sensors may be very long. On the other hand, in this paper, we not only consider how to minimize the number of dead sensors but also reduce the dead durations of the sensors that cannot be charged before their energy expirations. To this end, we aim to maximize the lifetime of all sensors, thereby minimizing the dead durations of the sensors.

### **PRELIMINARIES**

In this section, we first introduce the network model, then present the charging model, and finally define the problem precisely.

#### 3.1 **Network Model**

We consider a large-scale Wireless Rechargeable Sensor Network  $G_s = (V_s, E_s)$  deployed in a two-dimensional space for surveillance, etc. The network consists of a set  $V_s =$  $\{v_1, v_2, \dots, v_{n_s}\}\$  of  $n_s$  stationary sensors. Denote by  $(x_i, y_i)$ the coordinate of a sensor  $v_i$  in  $V_s$ . Assume that the coordinate of each sensor is known. For any two sensors  $v_i$  and  $v_i$ , there is an edge  $(v_i, v_i)$  in  $E_s$  between the two sensors if they are within the communication range of each other. In addition, there is a base station in the network for collecting data from sensors. The sensing data from each sensor are sent to the base station directly or through multi-hop relays along a given routing path, e.g., along the routing path with the minimum energy consumption [25].

Each sensor  $v_i \in V_s$  consumes energy on data sensing, transmission, and reception. Denote by  $b_i(t)$  the data sensing rate of sensor  $v_i$  at time t, where the value of  $b_i(t)$  is determined by the application of the sensor network. Following the routing path in the network, the data transmission rate  $b_i^{Tx}(t)$  and the data reception rate  $b_i^{Rx}(t)$  of sensor  $v_i$  at time t can be calculated, where  $b_i^{Tx}(t) = b_i(t) + b_i^{Rx}(t)$ . Then, the energy consumption rate  $\rho_i(t)$  of sensor  $v_i$  at time t can be calculated, by adopting a real sensor energy consumption model from [13], by considering its energy consumptions on data sensing, transmission, and reception, respectively.

Assume that each sensor  $v_i \in V_s$  is powered by a rechargeable battery with a limited energy capacity  $B_i$ . Denote by  $RE_i(t)$  the residual energy of sensor  $v_i$  at time t. Then, the residual lifetime of sensor  $v_i$  at time t is  $l_i(t) = \frac{RE_i(t)}{\rho_i(t)}$ . Sensor  $v_i$  sends a charging request  $REQ_i = \langle t, v_i, l_i(t), \rho_i(t), B_i - v_i \rangle$  $RE_i(t)$  > to the base station for energy charging when its residual lifetime  $l_i(t)$  falls below a given threshold  $l_c$  (e.g., 2 hours) at some time t, where  $B_i - RE_i(t)$  is the amount of tobe-charged energy of sensor  $v_i$ . The set  $V_c$  of to-be-charged sensors includes the sensors with their residual lifetimes no greater than  $\alpha \cdot l_c$ , e.g.,  $\alpha = 5$ , i.e.,  $V_c = \{v_i \mid v_i \in V_s, l_i(t) \leq$  $\alpha \cdot l_c$  \}. Let  $n = |V_c|$ .

### 3.2 Charging Model

To maintain long-term operations of the sensor network  $G_s$ , assume that there is a mobile charger located at a depot r. The mobile charger stays and recharges itself at depot rwhen there are no charging requests. Once the base station receives a charging request, the mobile charger will be dispatched to wirelessly charge sensors in  $V_c$  by traveling along a certain charging tour, assuming that there is a server located at depot *r* for calculating the charging tour. Note that the charging algorithm proposed in this paper can be easily extended to the case with multiple mobile chargers, by adopting a similar strategy from the work in [32].

Ideally, the mobile charger can stay at any location for charging sensors in the network, which means that there are infinite number of candidate charging locations for the mobile charger. This however makes the scheduling of the mobile charger intractable. We here only consider a finite number of candidate charging locations where the mobile charger can stay.

We construct a set S of candidate charging locations as follows. Initially, the locations of to-be-charged sensors in  $V_c$  are added to S, i.e.,  $S = \{s_1, s_2, ..., s_n\}$ , where  $s_i$  is the location of sensor  $v_i$ . Then, the middle location  $s_k$  between any two sensors  $v_i$  and  $v_i$  in  $V_c$  is added to S if their Euclidean distance is no greater than twice the maximum energy environmental monitoring, disaster monitoring, industrial transmission range  $\gamma$  (e.g.,  $\gamma=15$  m [22]), i.e.,  $d(s_i,s_j)\leq 2\gamma$ . Authorized licensed use limited to: CITY UNIV OF HONG KONG. Downloaded on October 24,2023 at 14:26:49 UTC from IEEE Xplore. Restrictions apply. Otherwise  $(d(s_i, s_j) > 2\gamma)$ , the middle location  $s_k$  between sensors  $v_i$  and  $v_j$  will not be added to S, since both of them will not be able to receive any energy from the mobile charger at location  $s_k$ . Denote by m the number of candidate charging locations in S in the end, i.e., m = |S|.

In this paper, we adopt a *multi-node charging model*, which is based on the WISP-reader charging model [7], [24]. When the mobile charger stays at a candidate location  $s_j \in S$ , a set  $N(s_j)$  of sensors within the maximum energy transmission range  $\gamma$  can be simultaneously charged, where  $N(s_j) = \{v_i \mid v_i \in V_c, d(v_i, s_j) \leq \gamma\}$ ,  $d(v_i, s_j)$  is the Euclidean distance between sensor  $v_i$  and location  $s_j$ .

Notice that the energy transfer efficiency decreases with the increase on the distance  $d(v_i, s_j)$ . Following an existing study [7], the reception power  $P_r(v_i, s_j)$  of sensor  $v_i$  from the mobile charger at location  $s_j$  is

$$P_r(v_i, s_j) = \begin{cases} \frac{\alpha_0}{(d(v_i, s_j) + \beta_0)^2} P_t, & d(v_i, s_j) \le \gamma \\ 0, & d(v_i, s_j) > \gamma \end{cases}$$
(1)

where  $\alpha_0$  and  $\beta_0$  are two constants determined by the hardware of the mobile charger, and  $P_t$  is the output power of the mobile charger. We assume that when  $P_r(v_i,s_j)$  is very small when  $d(v_i,s_j) > \gamma$ , the received energy at sensor  $v_i$  can be ignored.

Unlike most existing studies that adopted the multi-node full-charging model, in which the mobile charger charges sensors within its maximum charging range to their full energy capacities at each charging location. In this paper, we consider a *multi-node partial-charging model*. That is, no sensors are required to be fully charged once the mobile charger starts to charge them, but can be charged multiple times until they are fully charged.

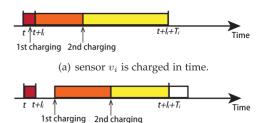
We define a charging tour C of the mobile charger for charging sensors in  $V_c$  as follows. Let  $C = \langle (r,0) \rightarrow (s'_1,\delta_1) \rightarrow (s'_2,\delta_2) \rightarrow \cdots \rightarrow (s'_{n'},\delta_{n'}) \rightarrow (r,0) \rangle$ , which means that the mobile charger starts from depot r, moves to a charging location  $s'_1$  in S and charges for a period of  $\delta_1$ , moves to a charging location  $s'_2$  and charges for a period of  $\delta_2,\ldots$ , then travels to a charging location  $s'_{n'}$  and charges for a period of  $\delta_n$ , finally returns to depot r, where r is a positive integer to be determined. Note that the mobile charger may charge at the same location multiple times in tour C. Tour C is feasible if each sensor  $v_i \in V_c$  is fully charged, i.e.,

$$\sum_{j=1}^{n'} P_r(v_i, s_j') \cdot \delta_j \ge B_i - RE_i, \ \forall \ v_i \in V_c, \tag{2}$$

where  $P_r(v_i, s_j') \cdot \delta_j$  is the amount of energy charged to sensor  $v_i$  by the mobile charger at location  $s_j'$ ,  $P_r(v_i, s_j')$  is the reception power of sensor  $v_i$  from the mobile charger located at  $s_j'$ ,  $\delta_j$  is the charging time of the mobile charger at location  $s_j'$ , and  $B_i - RE_i$  is the energy demand of sensor  $v_i$ .

### 3.3 Problem Definition

It is desirable that every sensor is charged before its energy expiration. We say that a sensor  $v_i$  is charged in time if  $v_i$  does not run out of energy before its charging energy demand  $B_i - RE_i$  is met. We assume that the energy consumption rate  $\rho_i$  of  $v_i$  can be considered as constant within



(b) sensor  $v_i$  has depleted its energy for a period of time before it is charged for the first time

Fig. 3. Sensor  $v_i$  is charged in time or runs out of its energy for a period of time.

the period of a charging tour, but may change over time. Without loss of generality, denote by  $l_i$  the residual lifetime of sensor  $v_i$  at time t, where  $l_i = \frac{RE_i}{\rho_i}$  and  $RE_i$  is the residual energy of  $v_i$ . Let  $T_i = \frac{B_i - RE_i}{\rho_i}$ . It can be seen that the residual lifetime of sensor  $v_i$  is prolonged from  $l_i$  to  $l_i + T_i$ , if  $v_i$  is charged in time. In other words, sensor  $v_i$  will not run out of energy in the time interval  $[t+l_i,\ t+l_i+T_i]$ , if  $v_i$  is charged in time. For example, Fig. 3a shows that sensor  $v_i$  is charged twice by the mobile charger, and  $v_i$  does not run out of energy from  $t+l_i$  to  $t+l_i+T_i$ .

When there are many lifetime-critical sensors to be charged in the network, sensor  $v_i$  may run out of its energy within the time interval  $[t+l_i,\ t+l_i+T_i]$ . Fig. 3b shows that sensor  $v_i$  has depleted its energy for some time before it is charged by the mobile charger for the first time. Denote by  $T_i^d$  the total dead duration of  $v_i$  within the time interval  $[t+l_i,\ t+l_i+T_i]$ . We define the normalized lifetime  $\eta_i$  of each sensor  $v_i \in V_c$  to represent its survival probability within the time period  $[t+l_i,t+l_i+T_i]$  by the charging tour C, i.e.,

$$\eta_i = 1 - \frac{T_i^d}{T_i},\tag{3}$$

where  $\eta_i \in [0, 1]$ ,  $T_i$  is the duration of the time interval  $[t + l_i, t + l_i + T_i]$ , and  $T_i = \frac{B_i - RE_i}{\rho_i}$ .

It can be seen that the normalized lifetime  $\eta_i$  of sensor  $v_i$  is 1, if  $v_i$  is charged in time, i.e.,  $T_i^d=0$  and  $\eta_i=1-\frac{0}{T_i}=1$ , see Fig. 3a. On the other hand, the normalized lifetime  $\eta_i$  of  $v_i$  is 0, if  $v_i$  is not charged at all within  $[t+l_i,\ t+l_i+T_i]$ , i.e.,  $T_i^d=T_i$  and  $\eta_i=1-\frac{T_i}{T_i}=0$ .

We consider the sum of normalized lifetimes  $\eta_{sum}$  of tobe-charged sensors in  $V_c$  by the charging tour C, i.e.,

$$\eta_{sum} = \sum_{i: \in V_o} \eta_i,\tag{4}$$

Since the normalized lifetime  $\eta_i$  indicates the survival probability of each sensor  $v_i \in V_c$ , the sum of normalized lifetimes  $\eta_{sum}$  means the expected number of live sensors, if the mobile charger charges sensors in  $V_c$  along tour C. We investigate an important sensor lifetime maximization problem, which is to find a charging tour C for the mobile charger, such that the sum  $\eta_{sum}$  of normalized lifetimes of sensors in  $V_c$  is maximized, which is precisely defined as follows.

Given a set  $V_c$  of to-be-charged sensors in WSN  $G_s$  at some time t, the energy consumption rate  $\rho_i$ , the residual lifetime  $l_i$ , and the charging energy demand  $B_i - RE_i$  of each sensor  $v_i \in V_c$ , the sensor lifetime maximization problem is

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to find a closed charging tour  $C = \langle (r,0) \rightarrow (s'_1,\delta_1) \rightarrow (s'_2,\delta_2) \rightarrow \cdots \rightarrow (s'_{n'},\delta_{n'}) \rightarrow (r,0) \rangle$  and schedule a mobile charger to charge sensors in  $V_c$  along tour C, such that the sum  $\eta_{sum}$  of normalized lifetimes of the sensors in  $V_c$  is maximized, subject to the constraint that the amount of energy charged to each sensor  $v_i \in V_c$  from the mobile charger at different charging locations is no less than its energy demand  $P_i - RE_i$ , i.e.,

maximize 
$$\eta_{sum}$$
, (5)

subject to

$$\sum_{j=1}^{n'} \delta_j \cdot P_r(v_i, s_j') \ge B_i - RE_i, \ \forall \ v_i \in V_c.$$
 (6)

### 3.4 NP-Hardness of the Problem

In the following, we show that the sensor lifetime maximization problem is NP-hard by a reduction from the deadline TSP problem [1].

**Theorem 1.** The sensor lifetime maximization problem is NP-

**Proof.** We show the NP-hardness of the problem by reducing the decision version of the deadline TSP problem to a special case of the problem considered. Given a metric graph G=(V,E), a starting node r in V, a deadline function  $D:V\mapsto \mathcal{Z}^{\geq 0}$ , and a length function  $\mathcal{L}:E\mapsto \mathcal{Z}^{\geq 0}$ , the decision version of the deadline TSP problem is to decide whether there is a closed tour C starting from r and returning to r, such that every node is visited before its deadline in tour C, where a node  $v_i$  is visited before its deadline  $D(v_i)$  if the length of the path in tour C from r to  $v_i$  is no greater than  $D(v_i)$  [1].

Given a metric graph  $G=(V\!,E)$ , a starting node r in V, a deadline function  $D: V \mapsto \mathbb{Z}^{\geq 0}$ , and a length function  $\mathcal{L}: E \mapsto \mathcal{Z}^{\geq 0}$ , we now construct an instance of the sensor lifetime maximization problem. The set  $V_c$  of tobe-charged sensors is  $V \setminus \{r\}$ , r is the depot, the residual lifetime  $l_i$  of each sensor  $v_i$  is the deadline  $D(v_i)$ , the maximum charging distance of the mobile charger is  $\gamma = 0$ , which indicates that the mobile charger must charge sensors one by one. The charging rate of the mobile charger is very large, then the charging time of each sensor is very small, i.e., ignored. Finally, the traveling time between two sensors  $v_i$  and  $v_j$  is the length  $\mathcal{L}(v_i, v_j)$  in graph G. It can be seen that in this special case of the sensor lifetime maximization problem, every sensor will be charged only once, since its charging time is ignored and multiple chargings on the sensor are unnecessary.

Given the maximum sum OPT of normalized sensor lifetimes in an optimal solution to the sensor lifetime maximization problem in this special case, it can be seen that there is an r-rooted closed tour C such that every node in G is visited before its deadline in tour C if OPT = n; and there is no such a tour if OPT < n, where  $n = |V_c| = |V| - 1$ . Then, the decision version of the deadline TSP problem is reduced to the sensor lifetime maximization problem in this special case. Since the former problem is NP-hard, the latter one is NP-hard, too. The theorem then follows.  $\Box$ 

Notice that it is unlikely to design an algorithm that finds an optimal solution to the sensor lifetime maximization problem in polynomial time, assuming that  $P \neq NP$ . Therefore, in this paper we propose a  $\frac{1}{3}$ -approximation algorithm for the problem, which finds a solution no less than  $\frac{1}{3}$  of the optimal solution for any problem instance in polynomial time

## 4 ALGORITHM FOR THE SENSOR LIFETIME MAXIMIZATION PROBLEM

In this section, we propose a  $\frac{1}{3}$ -approximation algorithm for the sensor lifetime maximization problem, where multiple sensors can be simultaneously charged by the mobile charger and each sensor may be partially charged each time.

#### 4.1 Basic Idea

The basic idea behind the proposed algorithm is that, we reduce the sensor lifetime maximization problem to the submodular function maximization problem under the constraints of two matroids. Since there is a  $\frac{1}{3}$ -approximation algorithm for the latter problem [4], the algorithm then returns a  $\frac{1}{3}$ -approximate solution to the original problem.

### 4.2 Reduction to the Submodular Function Maximization Problem

### 4.2.1 Preliminaries of the Reduction

Recall that the energy demand of each sensor  $v_i \in V_c$  is  $B_i - RE_i$ , where  $B_i$  is the battery capacity of sensor  $v_i$  and  $RE_i$  is its residual energy. Then, the maximum charging time to fully charge sensor  $v_i$  by the mobile charger at a location  $s_j$  in S is  $\frac{B_i - RE_i}{P_r(v_i, s_j)}$ , where  $P_r(v_i, s_j)$  is the reception power of sensor  $v_i$  from the mobile charger at location  $s_j$ , see Eq. (1). Since the mobile charger can charge multiple sensors simultaneously within its maximum charging range, the maximum time for the mobile charger at location  $s_j$  to fully charge the sensors within its maximum charging range is  $\max_{v_i \in N(s_j)} \{\frac{B_i - RE_i}{P_r(v_i, s_j)}\}$ , where  $N(s_j)$  is the set of sensors within the maximum charging range of the mobile charger at location  $s_j$ .

In order to charge more sensors before their energy expirations, we propose a novel multi-node partial charging strategy as follows. Denote by  $\Delta$  the charging time unit, e.g.,  $\Delta=5$  minutes. This indicates that if the mobile charger charges at a location  $s_j$ , its charging time is one of the values in set  $\{\Delta, 2\Delta, ..., k_j\Delta\}$ , where  $k_j$  is the maximum number of charging time units for fully charging the sensors within the maximum charging range, i.e.,

$$k_j = \max_{v_i \in N(s_j)} \left\{ \left\lceil \frac{B_i - RE_i}{P_r(v_i, s_j) \cdot \Delta} \right\rceil \right\}. \tag{7}$$

Note that the value of charging time unit  $\Delta$  affects the charging efficiency of the network. If the value of  $\Delta$  is small, the mobile charger may stay for only a short time at each charging location. The traveling energy consumption of the charger may be very high due to frequent shuttles between different charging locations. On the other hand, if the value of  $\Delta$  is large, this indicates that the mobile charger stays for a long time at each charging location. Many sensors may run out of their energy before the charger charges them. We can october 24 2023 at 14:26:49 UTC from IEEE Xolorg. Restrictions apply

atter one is NP-hard, too. The theorem then follows.  $\qed$  run out of their energy before the charger charges them. We Authorized licensed use limited to: CITY UNIV OF HONG KONG. Downloaded on October 24,2023 at 14:26:49 UTC from IEEE Xplore. Restrictions apply.

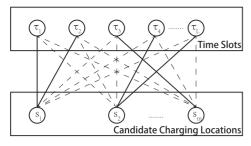


Fig. 4. An illustration of the constructed graph  $G_{charge}$ .

will study the optimal value of  $\Delta$  by experiments later, see Fig. 7 in Section 6.3.

Following an existing study [33], the total traveling time of the mobile charger per charging tour is much shorter than the total sensor charging time, but not negligible. For example, it may take less than half a minute for the charger to travel from its current charging location to the next charging location (e.g., the distance between two charging locations is 100 meters and the traveling speed of the charger is 5 m/s). On the other hand, it takes at least one charging time unit  $\Delta$  for the mobile charger to charge sensors at each charging location, e.g.,  $\Delta=5$  minutes. For the sake of convenience, the average traveling time of the charger between any two consecutive charging locations can be considered as a small constant  $t_{travel}$ , e.g.,  $t_{travel}=20$  seconds, while  $\Delta=5$  minutes. Denote by  $\tau$  the duration of a time slot, i.e.,  $\tau=\Delta+t_{travel}$ .

We divide time into equal-length time slots and the duration of each time slot is  $\tau$ . Then, the mobile charger stays at a single location  $s_j$  in S for charging sensors at each time slot, but may stay at different locations at different time slots. We index the time slots by  $\tau_1, \tau_2, \ldots, \tau_L$ , where L is the maximum number of time slots for fully charging sensors in  $V_c$ . For example, an upper bound on L can be estimated under the one-to-one charging model, i.e., the mobile charger charges sensors one-by-one, rather than under the multinode charging model. In this case, the value of L is no more than  $\sum_{v_i \in V_c} \lceil \frac{B_j - RE_i}{P_r(v_i, v_i) \cdot \Delta} \rceil$ , where  $B_i - RE_i$  is the amount of tobe-charged energy of sensor  $v_i$ ,  $P_r(v_i, v_i)$  is the reception power of sensor  $v_i$  if the mobile charger stays at the location of  $v_i$ , and  $\Delta$  is the charging time in a time slot. Let  $\mathcal T$  be the set of these L time slots, i.e.,  $\mathcal T = \{\tau_1, \tau_2, \ldots, \tau_L\}$ .

We construct a complete bipartite graph  $G_{charge} = (S \cup \mathcal{T}, E_c)$ , where S is the set of candidate charging locations (see the third paragraph in Section 3.2 for the definition of S) and  $\mathcal{T}$  is the set of the L time slots. There is an edge  $(s_j, \tau_l)$  in  $E_c$  for each location  $s_j$  in S and each time slot  $\tau_l$  in  $\mathcal{T}$ , which indicates that the mobile charger stays at location  $s_j$  in time slot  $\tau_l$ . Fig. 4 illustrates such a constructed graph.

Given any subset E' of  $E_c$ , a charging scheduling can be constructed from E'. For example, consider  $E' = \{(s_1, \tau_1), (s_1, \tau_2), (s_m, \tau_3), (s_2, \tau_4), (s_2, \tau_L)\}$  in Fig. 4. This means that the mobile charger charges at location  $s_1$  in time slots  $\tau_1$  and  $\tau_2$ , charges at location  $s_m$  in time slot  $\tau_3$ , and charges at location  $s_2$  in time slots  $\tau_4$  and  $\tau_L$ .

### 4.2.2 The Reduction to the Submodular Function Maximization Problem

Denote by f(E') the sum of normalized lifetimes of sensors in  $V_c$  if the mobile charger charges sensors by the scheduling

in E', see Eqs. (3) and (4). We later show that function f(.) is submodular, see Lemma 1 in Section 5.1. Specifically, given a set  $E_c$  and a function  $f: 2^{E_c} \mapsto \mathbb{R}^{\geq 0}$ , f is a nondecreasing submodular function if and only if it meets the following three properties [21]: (i)  $f(\emptyset) = 0$ ; (ii) Non-decreasement: for any two subsets  $E_1$  and  $E_2$  of  $E_c$  with  $E_1 \subseteq E_2$ , we have  $f(E_1) \leq f(E_2)$ ; (iii) Submodularity (i.e., diminishing return property): for any two subsets  $E_1$  and  $E_2$  of  $E_c$  with  $E_1 \subseteq E_2$  and any element e in  $E_c \setminus E_2$ , we have  $f(E_1 \cup \{e\}) - f(E_1) \geq f(E_2 \cup \{e\}) - f(E_2)$ .

We reduce the sensor lifetime maximization problem (defined by Eq. (5) and Ineq. (6)) to the submodular function maximization problem under the constraints of two matroids  $\mathcal{M}_S$  and  $\mathcal{M}_T$ , and the notion of a matroid is defined as follows. Given a set  $E_c$ , a matroid  $\mathcal{M}$  is a pair of  $(E_c, \mathcal{F})$  that has three following properties [21], where  $\mathcal{F}$  is a collection of subsets of  $E_c$ , i.e.,  $\mathcal{F} \subseteq 2^{E_c}$ . (i)  $\emptyset \in \mathcal{F}$ ; (ii) The hereditary property: for any two subsets  $E_1$  and  $E_2$  of  $E_c$  with  $E_1 \subseteq E_2$ , if  $E_2$  is contained in  $\mathcal{F}$ , then  $E_1$  is contained in  $\mathcal{F}$ , too. (iii) The independent set exchange property: for any two sets  $E_1$  and  $E_2$  in  $\mathcal{F}$  with  $|E_1| < |E_2|$ , then there is an element e in  $E_2$  such that  $E_1 \cup \{e\}$  is also contained in  $\mathcal{F}$ .

We first define a set system  $\mathcal{M}_S = (E_c, \mathcal{F}_S)$  on the edge set  $E_c$ , where  $\mathcal{F}_S$  is a collection of subsets of  $E_c$  such that, for any edge set E' in  $\mathcal{F}_S$  ( $E' \subseteq E_c$ ), the number of edges in E' sharing the same location node  $s_j$  in S is no more than  $k_j$ , where  $k_j$  is the maximum number of time slots that the mobile charger can stay at location  $s_j$ , and the value of  $k_j$  is defined in Eq. (7). It can be seen that  $\mathcal{M}_S$  indicates that the mobile charger stays at location  $s_j$  for no more than  $k_j$  time slots.

Similarly, we can define another set system  $\mathcal{M}_{\mathcal{T}} = (E_c, \mathcal{F}_{\mathcal{T}})$  on the edge set  $E_c$ , where  $\mathcal{F}_{\mathcal{T}}$  is a collection of subsets of  $E_c$  such that, for any edge set E' in  $\mathcal{F}_{\mathcal{T}}$  ( $E' \subseteq E_c$ ), there are no two edges in E' sharing the same time slot node in  $\mathcal{T}$ . It can be seen that  $\mathcal{M}_{\mathcal{T}}$  implies that the mobile charger stays at most one location in each time slot.

It can be seen that the set of feasible solutions to the sensor lifetime maximization problem can be defined as  $\mathcal{F}_S \cap \mathcal{F}_T$ .

We later show that both  $\mathcal{M}_S$  and  $\mathcal{M}_T$  are matroids, see Lemma 2 in Section 5.2. Then, the sensor lifetime maximization problem can be reduced to maximize the submodular function f(.) subject to the constraints of both matroids  $\mathcal{M}_S$  and  $\mathcal{M}_T$ .

### 4.3 Approximation Algorithm

Since the sensor lifetime maximization problem can be reduced to the submodular maximization problem under the constraints of p=2 matroids, following the study in [4], a greedy heuristic always delivers a performance-guaranteed solution with an approximation ratio of  $\frac{1}{p+1}=\frac{1}{3}$ . Specifically, the greedy heuristic finds the solution E' to the problem in an iterative way. Initially,  $E'=\emptyset$ . At each iteration, the greedy heuristic identifies an edge  $(s_j,\tau_l)$  in  $E_c\setminus E'$  (edge  $(s_j,\tau_l)$  means that the mobile charger charges at location  $s_j$  in time slot  $\tau_l$ ), such that the *increased* sum of normalized lifetime by  $(s_j,\tau_l)$ , i.e.,  $f(E'\cup\{(s_j,\tau_l)\})-f(E')$ , is maximized, subject to the constraint that  $E'\cup\{(s_j,\tau_l)\}$  is contained in both matroids  $\mathcal{M}_S$  and  $\mathcal{M}_T$ . The greedy heuristic continues until the value of  $f(E'\cup\{(s_j,\tau_l)\})$  is no greater than the value of f(E'), i.e.,  $f(E'\cup\{(s_j,\tau_l)\}) \leq f(E')$ . E' is the final solution to the problem.

in  $V_c$  if the mobile charger charges sensors by the scheduling the final solution to the problem. Authorized licensed use limited to: CITY UNIV OF HONG KONG. Downloaded on October 24,2023 at 14:26:49 UTC from IEEE Xplore. Restrictions apply.

### **Algorithm 1.** Approximation Algorithm AlgMaxLife for the Sensor Lifetime Maximization Problem

**Input:** a set  $V_c$  of to-be-charged sensors, a set S of candidate charging locations, and L time slots  $\tau_1, \tau_2, \dots, \tau_L$ .

**Output:** a charging tour C of the mobile charger so that the sum of normalized sensor lifetimes is maximized

- 1: Calculate the residual lifetime  $l_i$  of each sensor  $v_i$  in  $V_c$ . Let  $T_i = \frac{B_i RE_i}{\rho_i}$ , where  $B_i RE_i$  is the energy demand of sensor  $v_i$  and  $\rho_i$  is the energy consumption rate of  $v_i$ .
- 2: For each sensor  $v_i \in V_c$ , let  $T_{i,1}^d = 0$ ,  $T_{i,2}^d = T_i$ , where  $T_{i,1}^d$  and  $T_{i,2}^d$  are the dead durations of sensor  $v_i$  before and after time slot  $\tau_1$ , respectively. Let  $[t_i^s, t_i^f]$  be the dead interval of sensor  $v_i$  after  $\tau_1$ . Initially,  $t_i^s = t + l_i$  and  $t_i^f = t + l_i + T_i$ .

```
3: Let E' \leftarrow \emptyset;
 4: for l \leftarrow 1 to L do
       /* Find an edge (s_i, \tau_l) such that the increased sum of
       normalized lifetime is maximized */
       for j' \leftarrow 1 to m do
 6:
 7:
          if the mobile charger stays at location s_{i'} strictly less
          than k_{i'} times then
 8:
            Calculate the increased sum f(E' \cup \{(s_{j'}, \tau_l)\}) - f(E')
            of normalized lifetime by (s_{i'}, \tau_l);
 9:
            if f(E' \cup \{(s_{i'}, \tau_l)\}) - f(E') > f(E' \cup \{(s_i, \tau_l)\}) - f(E')
             f(E') then
10:
               Let j \leftarrow j'; /* find a better charging location */
             end if
11:
12:
          end if
13:
       end for
       if f(E' \cup \{(s_j, \tau_l)\}) \leq f(E') then
14:
15:
          Break the outer for loop.
16:
17:
       Let E' \leftarrow E' \cup \{(s_i, \tau_l)\};
       The mobile charger charges sensors within its maximum
       charging range at location s_i for \Delta time, and update dead
       intervals of the sensors;
19:
       For each sensor v_i in V_c, update its dead durations T_{i,1}^d
       and T_{i,2}^d before and after the next time slot \tau_{l+1},
```

respectively, and update the dead interval of

sensor  $v_i$  after time slot  $\tau_{l+1}$ ;

21: **return**the scheduling E'.

20: end for

It can be seen that, within each iteration, the greedy heuristic needs to find the edge  $(s_j, \tau_l)$  with the maximum increased sum of normalized lifetime among the edges in  $E_c \setminus E'$ . However, the number of edges  $E_c \setminus E'$  is  $O(|E_c \setminus E'|) = O(|E_c|) = O(mL)$ , where  $E_c = S \times T$ , m = |S| and L = |T|. We here propose a novel strategy to find the edge  $(s_j, \tau_l)$  only among O(m) edges, rather than among O(mL) edges, thereby reducing the time complexity by a factor of L. To this end, we will show an important property. That is, for any location  $s_j$  and any two time slots  $\tau_{l_1}$  and  $\tau_{l_2}$ , assume that  $l_1 < l_2$ . Then, the increased sum of normalized lifetime by edge  $(s_j, \tau_{l_1})$  is no less than the increased sum of normalized lifetime by edge  $(s_j, \tau_{l_2})$ , i.e.,

$$f(E' \cup \{(s_i, \tau_{l_1})\}) - f(E') \ge f(E' \cup \{(s_i, \tau_{l_2})\}) - f(E'). \tag{8}$$

Therefore, for each charging location  $s_j$ , we just need to find its earliest available time slot, and ignore latter time slots.

By utilizing the important property, we propose an efficient approximation algorithm for the problem. Recall that E' represents the solution to the problem. Initially,  $E' = \emptyset$ . The proposed algorithm performs at most L iterations, where L is the number of time slots in  $\mathcal{T}$ . At the lth iteration (i.e., time slot  $\tau_l$ ), the algorithm finds an edge  $(s_j, \tau_l)$  (i.e., the charging location  $s_j$  in time slot  $\tau_l$ ) such that the increased sum of normalized lifetime by edge  $(s_j, \tau_{l_1})$  is maximized, subject to the constraint that the mobile charger stays at location  $s_j$  no more than  $k_j$  times. The algorithm continues until the objective value of function f(.) does not increase. The detailed algorithm is presented in Algorithm 1.

### 5 ALGORITHM ANALYSIS

In this section, we analyze the performance of the proposed algorithm, including its approximation ratio and time complexity. We first show that function f(.) is submodular. We then prove that both  $\mathcal{M}_S$  and  $\mathcal{M}_T$  are matroids. Then, following the study in [4], the proposed algorithm delivers a  $\frac{1}{2}$ -approximate solution.

### 5.1 Submodularity of Function f

**Lemma 1.** Given any subset E' of  $E_c$ , let f(E') be the sum of normalized lifetimes of sensors in  $V_c$  by the scheduling of E'. Then, f(.) is a nondecreasing submodular function.

**Proof.** Recall that  $\eta_i(E')$  is the normalized lifetime of a sensor  $v_i$  in  $V_c$  by the scheduling of E'. Then,  $f(E') = \sum_{v_i \in V_c} \eta_i(E')$  by Eq. (4). We only show that  $\eta_i(.)$  is a non-decreasing submodular function for each sensor  $v_i$  in  $V_c$ . Therefore, f(.) also is a nondecreasing submodular function, since it is a nonnegative, linear combination of submodular functions [21].

In the following, we show that function  $\eta_i(.)$  meets the three properties of a nondecreasing submodular function. First, it can be seen that the normalized lifetime of sensor  $v_i$  is zero, i.e.,  $\eta_i(\emptyset) = 0$ , if  $v_i$  is not charged at all by the mobile charger, see Eq. (3).

We then prove that function  $\eta_i(.)$  meets (ii) the nondecreasing property. For any two subsets  $E_1$  and  $E_2$  of  $E_c$  with  $E_1 \subseteq E_2$ , following Eq. (3), it can be seen that the normalized lifetime of sensor  $v_i$  by the scheduling of  $E_1$  is no greater than that of  $E_2$ , i.e.,  $\eta_1(E_1) \le \eta_i(E_2)$ .

We finally show that function  $\eta_i(.)$  meets (iii) the diminishing return property. Consider any two subsets  $E_1$  and  $E_2$  of  $E_c$  with  $E_1 \subseteq E_2$  and any edge  $(s_j, \tau_l)$  in  $E_c \setminus E_2$ , where the edge  $(s_j, \tau_l)$  indicates that the mobile charger stays at location  $s_j$  in time slot  $\tau_l$ .

For any time t' in the time interval  $[t+l_i,\ t+l_i+T_i]$ , it can be seen that if sensor  $v_i$  is live at time t' in the scheduling of  $E_1$ , then  $v_i$  is also live at time t' in the scheduling of  $E_2$ , since  $E_1\subseteq E_2$ . Denote by  $T_i^d(E_1)$  and  $T_i^d(E_1,\tau_l)$  the dead durations of sensor  $v_i$  by the scheduling of  $E_1$  in the time intervals  $[t+l_i,\ t+l_i+T_i]$  and  $[t+\tau_l,\ t+l_i+T_i]$ , respectively. Similarly, denote by  $T_i^d(E_2)$  and  $T_i^d(E_2,\tau_l)$  the dead durations of sensor  $v_i$  by the scheduling of  $E_2$  in the time intervals  $[t+l_i,\ t+l_i+T_i]$  and  $[t+\tau_l,\ t+l_i+T_i]$ , respectively. Since  $E_1\subseteq E_2$ , we have  $T_i^d(E_1)\geq T_i^d(E_2)$  and  $T_i^d(E_1,\tau_l)\geq T_i^d(E_2,\tau_l)$ .

Consider the edge  $(s_i, \tau_l)$  (i.e., the mobile charger stays at location  $s_i$  in time slot  $\tau_l$ ), the maximum amount energy charged to sensor  $v_i$  is  $P_r(v_i, s_i)\Delta$ , where  $P_r(v_i, s_i)$  is the reception power of sensor  $v_i$  from the mobile charger at location  $s_i$  and  $\Delta$  the charging time within time slot  $\tau_l$ . It can be seen that, if the mobile charger charges at location  $s_i$ in time slot  $\tau_l$ , the dead duration of sensor  $v_i$  before time slot  $\tau_l$  will not change, but its dead duration after time slot  $\tau_l$  will decrease. Specifically, the dead duration of sensor  $v_i$ after time slot  $\tau_l$  in scheduling  $E_1$  decreases from  $T_i^d(E_1, \tau_l)$ to  $\max\{T_i^d(E_1, \tau_l) - \frac{P_r(v_i, s_j)\Delta}{\rho_i}, 0\}$ , where  $T_i^d(E_1, \tau_l)$  is the dead duration of sensor  $v_i$  after time slot  $\tau_l$  in scheduling  $E_1$ ,  $P_r(v_i, s_i)\Delta$  is the maximum amount of energy charged to sensor  $v_i$  in time slot  $\tau_l$  and  $\rho_i$  is the energy consumption rate of  $v_i$ . Then, the dead duration of sensor  $v_i$  in time interval  $[t + l_i, t + l_i + T_i]$  in scheduling  $E_1 \cup \{(s_j, \tau_l)\}$  is

$$T_i^d(E_1 \cup \{(s_j, \tau_l)\})$$

$$= T_i^d(E_1) - T_i^d(E_1, \tau_l)$$

$$+ \max \left\{ T_i^d(E_1, \tau_l) - \frac{P_r(v_i, s_j)\Delta}{\rho_i}, 0 \right\}$$
(9)

Similarly, the dead duration of sensor  $v_i$  in time interval  $[t + l_i, t + l_i + T_i]$  in scheduling  $E_2 \cup \{(s_i, \tau_l)\}$  is

$$T_i^d(E_2 \cup \{(s_j, \tau_l)\})$$

$$= T_i^d(E_2) - T_i^d(E_2, \tau_l)$$

$$+ \max \left\{ T_i^d(E_2, \tau_l) - \frac{P_r(v_i, s_j)\Delta}{\rho_i}, 0 \right\}$$
(10)

We now show that  $\eta_i(E_1 \cup \{(s_j, \tau_l)\}) - \eta_i(E_1) \ge \eta_i(E_2 \cup \{(s_j, \tau_l)\}) - \eta_i(E_2)$ . Following the definition of  $\eta_i(.)$ , we have

$$\eta_{i}(E_{1} \cup \{(s_{j}, \tau_{l})\}) - \eta_{i}(E_{1}) 
= 1 - \frac{T_{i}^{d}(E_{1} \cup \{(s_{j}, \tau_{l})\})}{T_{i}} - (1 - \frac{T_{i}^{d}(E_{1})}{T_{i}}), \text{ by Eq. (3)} 
= \frac{T_{i}^{d}(E_{1}) - T_{i}^{d}(E_{1} \cup \{(s_{j}, \tau_{l})\}}{T_{i}} 
= \frac{T_{i}^{d}(E_{1}, \tau_{l}) - \max\{T_{i}^{d}(E_{1}, \tau_{l}) - \frac{P_{r}(v_{i}, s_{j})\Delta}{\rho_{i}}, 0\}}{T_{i}}, \text{ by Eq. (9)}$$

Similarly, we have

$$\eta_{i}(E_{2} \cup \{(s_{j}, \tau_{l})\}) - \eta_{i}(E_{2}) \\
= \frac{T_{i}^{d}(E_{2}, \tau_{l}) - \max\{T_{i}^{d}(E_{2}, \tau_{l}) - \frac{P_{r}(v_{i}, s_{j})\Delta}{\rho_{i}}, 0\}}{T_{i}} \quad (11)$$

Recall that  $T_i^d(E_1, \tau_l) \geq T_i^d(E_2, \tau_l)$ , we distinguish into three cases by the value of  $\frac{P_r(v_i, s_j)\Delta}{\rho_i}$ : (1)  $T_i^d(E_1, \tau_l) \geq T_i^d(E_2, \tau_l) \geq \frac{P_r(v_i, s_j)\Delta}{\rho_i}$ ; (2)  $T_i^d(E_1, \tau_l) \geq \frac{P_r(v_i, s_j)\Delta}{\rho_i} \geq T_i^d(E_2, \tau_l)$ ; and (3)  $\frac{P_r(v_i, s_j)\Delta}{\rho_i} \geq T_i^d(E_1, \tau_l) \geq T_i^d(E_2, \tau_l)$ .

Case (1) where  $T_i^d(E_1, \tau_l) \ge T_i^d(E_2, \tau_l) \ge \frac{P_r(v_i, s_j)\Delta}{\rho_i}$ , we have that

$$\eta_{i}(E_{1} \cup \{(s_{j}, \tau_{l})\}) - \eta_{i}(E_{1}) \\
= \frac{T_{i}^{d}(E_{1}, \tau_{l}) - \max\{T_{i}^{d}(E_{1}, \tau_{l}) - \frac{P_{r}(v_{i}, s_{j})\Delta}{\rho_{i}}, 0\}}{T_{i}} \\
= \frac{T_{i}^{d}(E_{1}, \tau_{l}) - (T_{i}^{d}(E_{1}, \tau_{l}) - \frac{P_{r}(v_{i}, s_{j})\Delta}{\rho_{i}})}{T_{i}} \\
= \frac{T_{i}^{d}(E_{1}, \tau_{l}) - (T_{i}^{d}(E_{1}, \tau_{l}) - \frac{P_{r}(v_{i}, s_{j})\Delta}{\rho_{i}})}{T_{i}} \\
= \frac{\frac{P_{r}(v_{i}, s_{j})\Delta}{\rho_{i}}}{T_{i}} \\
= \frac{T_{i}^{d}(E_{2}, \tau_{l}) - (T_{i}^{d}(E_{2}, \tau_{l}) - \frac{P_{r}(v_{i}, s_{j})\Delta}{\rho_{i}})}{T_{i}} \\
= \frac{T_{i}^{d}(E_{2}, \tau_{l}) - \max\{T_{i}^{d}(E_{2}, \tau_{l}) - \frac{P_{r}(v_{i}, s_{j})\Delta}{\rho_{i}}, 0\}}{T_{i}} \\
= \eta_{i}(E_{2} \cup \{(s_{j}, \tau_{l})\}) - \eta_{i}(E_{2}). \tag{12}$$

Case (2) where  $T_i^d(E_1, \tau_l) \ge \frac{P_r(v_i, s_j)\Delta}{\rho_i} \ge T_i^d(E_2, \tau_l)$ , we have

$$\eta_{i}(E_{1} \cup \{(s_{j}, \tau_{l})\}) - \eta_{i}(E_{1}) \\
= \frac{T_{i}^{d}(E_{1}, \tau_{l}) - \max\{T_{i}^{d}(E_{1}, \tau_{l}) - \frac{P_{r}(v_{i}, s_{j})\Delta}{\rho_{i}}, 0\}}{T_{i}} \\
= \frac{T_{i}^{d}(E_{1}, \tau_{l}) - (T_{i}^{d}(E_{1}, \tau_{l}) - \frac{P_{r}(v_{i}, s_{j})\Delta}{\rho_{i}})}{T_{i}} \\
= \frac{T_{i}^{d}(E_{1}, \tau_{l}) - (T_{i}^{d}(E_{1}, \tau_{l}) - \frac{P_{r}(v_{i}, s_{j})\Delta}{\rho_{i}})}{T_{i}} \\
= \frac{P_{r}(v_{i}, s_{j})\Delta}{\rho_{i}} \\
\geq \frac{T_{i}^{d}(E_{2}, \tau_{l}) - 0}{T_{i}}, \text{ as } \frac{P_{r}(v_{i}, s_{j})\Delta}{\rho_{i}} \geq T_{i}^{d}(E_{2}, \tau_{l}) \\
= \frac{T_{i}^{d}(E_{2}, \tau_{l}) - \max\{T_{i}^{d}(E_{2}, \tau_{l}) - \frac{P_{r}(v_{i}, s_{j})\Delta}{\rho_{i}}, 0\}}{T_{i}} \\
= \eta_{i}(E_{2} \cup \{(s_{j}, \tau_{l})\}) - \eta_{i}(E_{2}). \tag{13}$$

Case (3) where  $\frac{P_r(v_i,s_j)\Delta}{\rho_i} \ge T_i^d(E_1,\tau_l) \ge T_i^d(E_2,\tau_l)$ , we have

$$\eta_{i}(E_{1} \cup \{(s_{j}, \tau_{l})\}) - \eta_{i}(E_{1}) \\
= \frac{T_{i}^{d}(E_{1}, \tau_{l}) - \max\{T_{i}^{d}(E_{1}, \tau_{l}) - \frac{P_{r}(v_{i}, s_{j})\Delta}{\rho_{i}}, 0\}}{T_{i}} \\
= \frac{T_{i}^{d}(E_{1}, \tau_{l}) - 0}{T_{i}}, \text{ as } \frac{P_{r}(v_{i}, s_{j})\Delta}{\rho_{i}} \ge T_{i}^{d}(E_{1}, \tau_{l}) \\
\ge \frac{T_{i}^{d}(E_{2}, \tau_{l}) - 0}{T_{i}}, \text{ as } T_{i}^{d}(E_{1}, \tau_{l}) \ge T_{i}^{d}(E_{2}, \tau_{l}) \\
= \frac{T_{i}^{d}(E_{2}, \tau_{l}) - \max\{T_{i}^{d}(E_{2}, \tau_{l}) - \frac{P_{r}(v_{i}, s_{j})\Delta}{\rho_{i}}, 0\}}{T_{i}} \\
= \frac{P_{r}(v_{i}, s_{j})\Delta}{\rho_{i}} \ge T_{i}^{d}(E_{2}) \\
= \eta_{i}(E_{2} \cup \{(s_{j}, \tau_{l})\}) - \eta_{i}(E_{2}). \tag{14}$$

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By combining Eqs. (12), (13), and (14), we conclude that  $\eta_i(E_1 \cup \{(s_j, \tau_l)\}) - \eta_i(E_1) \ge \eta_i(E_2 \cup \{(s_j, \tau_l)\}) - \eta_i(E_2)$ . Then, function  $\eta_i(.)$  is submodular, and function f(.) is submodular, too. The lemma then follows.

### **5.2** Proof for the Matroids $\mathcal{M}_S$ and $\mathcal{M}_T$

We show that both  $\mathcal{M}_S$  and  $\mathcal{M}_T$  are matroids.

**Lemma 2.** Both  $\mathcal{M}_S$  and  $\mathcal{M}_T$  are matroids.

**Proof.** We here only show that  $\mathcal{M}_S$  is a matroid, and the proof for  $\mathcal{M}_T$  is similar, omitted. We show that  $\mathcal{M}_S = (E_c, \mathcal{F}_S)$  meets the three properties of matroids, see the definition of a matroid in the last paragraph of Section 3. First, it can be seen that  $\emptyset$  is contained in  $\mathcal{F}_S$ .

We then show that  $\mathcal{M}_S$  meets (ii) the hereditary property. For any two subsets  $E_1$  and  $E_2$  of  $E_c$  with  $E_1 \subseteq E_2$ , assume that  $E_2$  is contained in  $\mathcal{F}_S$ . Following the definition of  $\mathcal{F}_S$ , the number of edges in  $E_2$  sharing the same location node  $s_j$  in S is no more than  $k_j$ . Since  $E_1 \subseteq E_2$ , the number of edges in  $E_1$  sharing node  $s_j$  also is no more than  $k_j$ . Then,  $E_1$  is contained in  $\mathcal{F}_S$ , too.

We finally show that  $\mathcal{M}_S$  meets (iii) the independent set exchange property. Consider any two sets  $E_1$  and  $E_2$  in  $\mathcal{F}_S$ , assume that  $|E_1|<|E_2|$ . For any location  $s_j$  in S, denote by  $N_{E_1}(s_j)$  the set of edges in  $E_1$  sharing location node  $s_j$ . Since  $G_{charge}$  is a bipartite graph, it can be seen that  $N_{E_1}(s_{j_1})\cap N_{E_1}(s_{j_2})=\emptyset$  if  $j_1\neq j_2$ , see Fig. 4. Then,  $N_{E_1}(s_1),N_{E_1}(s_2),\ldots,N_{E_1}(s_m)$  form a partition of set  $E_1$ , and  $|E_1|=\sum_{j=1}^m|N_{E_1}(s_j)|$ . Similarly, for any location  $s_j$  in S, denote by  $N_{E_2}(s_j)$  the set of edges in  $E_2$  sharing location node  $s_j$ . Then,  $N_{E_2}(s_1),N_{E_2}(s_2),\ldots,N_{E_2}(s_m)$  form a partition of set  $E_2$  and  $|E_2|=\sum_{j=1}^m|N_{E_2}(s_j)|$ .

We claim there is a location node  $s_j$  in S such that the number of edges in  $E_1$  sharing node  $s_j$  is strictly less than the number of edges in  $E_2$  sharing node  $s_j$ , i.e.,  $|N_{E_1}(s_j)| < |N_{E_2}(s_j)|$ . We prove the claim by contradiction. Suppose that there are no such location nodes. Then, for each location node  $s_j$  in S, we have  $|N_{E_1}(s_j)| \geq |N_{E_2}(s_j)|$ . Therefore,  $|E_1| = \sum_{j=1}^m |N_{E_1}(s_j)| \geq \sum_{j=1}^m |N_{E_2}(s_j)| = |E_2|$ , which however contradicts the assumption  $|E_1| < |E_2|$ .

Since the number of edges in  $E_1$  sharing node  $s_j$  is strictly less than the number of edges in  $E_2$  sharing node  $s_j$  ( $|N_{E_1}(s_j)| < |N_{E_2}(s_j)|$ ), consider an edge  $(s_j, \tau_l)$  in  $E_2 \setminus E_1$  sharing node  $s_j$  (i.e.,  $(s_j, \tau_l) \in N_{E_2}(s_j) \setminus N_{E_1}(s_j)$ ). We claim that  $E_1 \cup \{(s_j, \tau_l)\}$  is contained in  $\mathcal{F}_S$  as follows.

For any location node  $s_{j'}$  in S, if  $s_{j'} \neq s_j$ , then the number of edges in  $E_1 \cup \{(s_j, \tau_l)\}$  sharing  $s_{j'}$  is equal to the number of edges in  $E_1$  sharing  $s_{j'}$ , i.e.,  $|N_{E_1 \cup \{(s_j, \tau_l)\}}(s_{j'})| = |N_{E_1}(s_{j'})| \leq k_j$ . On the other hand, if  $s_{j'} = s_j$ , notice that both  $E_1$  and  $E_2$  are contained in  $\mathcal{F}_S$ . Following the definition of  $\mathcal{F}_S$  and the fact that  $E_2 \in \mathcal{F}_S$ , we know that the number of edges in  $E_2$  sharing node  $s_j$  is no more than  $k_j$ . Then,  $|N_{E_1}(s_j)| < |N_{E_2}(s_j)| \leq k_j$ . Therefore,  $|N_{E_1 \cup \{(s_j, \tau_l)\}}(s_j)| = |N_{E_1}(s_j)| + 1 \leq |N_{E_2}(s_j)| \leq k_j$ . Following the definition of  $\mathcal{F}_S$ , we conclude that  $E_1 \cup \{(s_j, \tau_l)\}$  is contained in  $\mathcal{F}_S$ .  $\mathcal{M}_S = (E_c, \mathcal{F}_S)$  thus is a matroid. The lemma then follows.

### 5.3 Analysis of the Approximation Ratio and Time Complexity

**Theorem 2.** There is a  $\frac{1}{3}$ -approximation algorithm for the sensor lifetime maximization problem, i.e., Algorithm 1, with a time complexity of  $O(Lm\delta_{max})$ , where  $L=|\mathcal{T}|$ , m=|S|, and  $\delta_{max}$  is the maximum number of sensors within the maximum charging range of a charging location in S, i.e.,  $\delta_{max} = \max_{s_i \in S} \{|N(s_i)|\}$ .

**Proof.** Following Lemmas 1 and 2, the sensor lifetime maximization problem can be reduced to the submodular maximization problem under the constraints of two matroids. □

On the other hand, similar to the proof of Lemma 1, we can show that, for any location  $s_i$  and any two time slots  $\tau_{l_1}$  and  $\tau_{l_2}$  with  $l_1 < l_2$ , the increased sum of normalized lifetime by edge  $(s_i, \tau_{l_i})$  is no less than the increased sum of normalized lifetime by edge  $(s_j, \tau_{l_2})$ , i.e.,  $f(E' \cup \{(s_j, \tau_{l_1})\}) - f(E') \ge$  $f(E' \cup \{(s_i, \tau_{l_2})\}) - f(E')$ . Recall that at the beginning of the Ith iteration of Algorithm 1, the charging locations of the first l-1 time slots have been found. Let  $(s_i, \tau_{l'})$  be an edge in  $E_c \setminus E'$  with the maximum increased sum of normalized lifetime and  $E' \cup \{(s_j, \tau_{l'})\}$  is contained in both matroids  $\mathcal{M}_S$  and  $\mathcal{M}_T$ , i.e.,  $(s_j, \tau_{l'}) = \arg \max_{(s_{j''}, \tau_{l''}) \in E_c \setminus E'} \{ f(E' \cup e') \}$  $\{(s_{j''}, \tau_{l''})\}\}$  -  $f(E')\}$ . Then,  $l \leq l' \leq L$  and the mobile charger stays at location  $s_i$  less than  $k_i$  times. Since  $l \leq l'$ , we have  $f(E' \cup \{(s_i, \tau_l)\}) - f(E') \ge f(E' \cup \{(s_i, \tau_{l'})\}) - f(E')$ . Then, we know that  $(s_i, \tau_l)$  also is an edge in  $E_c \setminus E'$  with the maximum increased sum of normalized lifetime and  $E' \cup \{(s_i, \tau_l)\}\$  is contained in both matroids  $\mathcal{M}_S$  and  $\mathcal{M}_T$ . Therefore, following the study in [4], the proposed algorithm delivers a  $\frac{1}{3}$ -approximate solution.

The time complexity of Algorithm 1 is analyzed as follows. Following Algorithm 1, it finds the charging locations of L time slots  $\tau_1, \tau_2, \ldots, \tau_L$  one by one. For each time slot  $\tau_l$ , it calculates the increased sum of normalized lifetimes if charging at location  $s_j$  at time slot  $\tau_l$ . Since there are no more than m candidate charging locations in S and no more than  $\delta_{max}$  sensors within the maximum charging range of a charging location in S, the time complexity of Algorithm 1 is  $O(L \cdot |S| \cdot \delta_{max}) = O(L m \delta_{max})$ .

### 6 Performance Evaluation

In this section, we evaluate the performance of the proposed algorithm. We also study the impact of important parameters on the algorithm performance, including the network size, data sensing rates of sensors, and charging output power.

### 6.1 Experimental Environment

We consider a wireless rechargeable sensor network that consists of from 50 to 800 sensors, which are randomly deployed in a 200 m  $\times$  200 m square area. Assume that the base station and the depot r are co-located at the center of the square area. The energy capacity  $B_i$  of each sensor  $v_i$  is set to 10.8 kJ [26]. The data sensing rate  $b_i$  of each sensor  $v_i$  is randomly chosen from an interval  $[b_{min}, b_{max}]$ , where  $b_{min} = 1$  kbps and  $b_{max} = 50$  kbps. We adopt a real sensor energy consumption model from [13], and each sensor sends its data to the base station directly or through

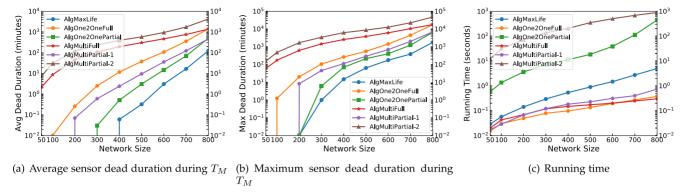


Fig. 5. Performance of different algorithms by varying the network size n from 50 to 800.

multi-hop relays along the routing path with the minimum energy consumption [25]. Each sensor  $v_i$  sends a charging request to the base station when its residual lifetime  $l_i$  falls below a given threshold  $l_c = 2$  hours.

A mobile charger initially is located at the depot r. It travels at a speed of  $\lambda=5$  m/s. The charging output power  $P_t$  of the mobile charger is set to 5 Watts, and the maximum energy transmission range  $\gamma$  is 15 m [22]. The monitoring period  $T_M$  of the sensor network is one year. Every sensor is fully charged at the beginning of the monitoring period.

To evaluate the performance of the proposed algorithm AlgMaxLife, we compare against following five benchmark algorithms. (i) Algorithm AlgOne20neFull adopts the one-to-one full-charging model. It sorts to-be-charged sensors in increasing order by their residual lifetimes and charges the sensors one by one. (ii) Algorithm AlgOne20nePartial [33] adopts the one-to-one partial-charging model. It first constructs multiple virtual sensors for each to-becharged sensor, where each virtual sensor should be matched to a time slot. It then uses a matching-based method to find the charging scheduling so that the sum of sensor lifetimes is maximized. (iii) Algorithm AlgMultiFull [30] adopts the multi-node full-charging model. It continues to select charging bundles, such that each selected charging bundle can cover the maximum number of uncovered sensors, until all to-be-charged sensors are covered. (iv) Algorithm Alg-MultiPartial-1 [18] adopts the multi-node partial-charging *model*. It first finds a tentative charging tour by charging sensors in increasing order by their residual lifetimes, under the multi-node full-charging model. However, if a sensor cannot be charged before its energy expiration, the algorithm then shortens the charging durations of some sensors, thereby minimizing the number of dead sensors. (v) Algorithm Alg-MultiPartial-2 [35] also adopts the multi-node partialcharging model. It iteratively calculates the charging utility of each candidate charging location, and chooses the next charging location of the charger as the location with the maximum charging utility.

Each value in the figures is the average of the results of 50 different network topologies with the same network size. For each of the 50 network topologies, we randomly place all sensors in the monitoring area. Notice that all algorithms are implemented by the programming language C++. The algorithms are run on a server with an Intel(R) Xeon(R) CPU E5-2680 v4 @ 2.4GHz and a 64 GB RAM memory.

### 6.2 Algorithm Performance Evaluation

We first study the performance of different algorithms by increasing the network size  $n_s$  from 50 to 800. Fig. 5a shows that the average dead duration per sensor during the monitoring period  $T_M$  by each of the six algorithms becomes larger, since there are more to-be-charged sensors in each charging tour, and more sensors may run out of their energy before they are charged. Notice that since we use the logarithmic scale for the *y*-axis, the average dead duration of sensors by some algorithms sometimes are zero and thus cannot be plotted in the figure. For example, see the average dead duration of sensors by algorithm AlgMaxLife when the network size is no greater than 300. Fig. 5a also shows that the average dead duration per sensor delivered by the proposed algorithm AlgMaxLife is up to 70% shorter than those by the other five algorithms. For example, the average sensor dead durations by algorithms AlgMaxLife, AlgOne2OneFull, AlgOne2OnePartial, AlgMultiFull, AlgMultiPartial-1, and AlgMultiPartial-2 are 125, 1,418, 495, 1364, 454, and 4,215 minutes, respectively, when there are  $n_s = 800$ sensors. Fig. 5b demonstrates that the maximum sensor dead duration by algorithm AlgMaxLife is no more than 1,800 minutes when there are  $n_s = 800$  sensors, while the maximum sensor dead durations by the other five algorithms are at least 6,400 minutes. Fig. 5c plots the running times of different algorithms, from which it can be seen that the running time of the proposed algorithm AlgMaxLife is no more than 5 seconds, which is acceptable in a real sensor network.

We then investigate the performance of different algorithms with different parameters. Fig. 6a demonstrates that the average sensor dead duration by each of the six algorithms decreases with the growth of the charging output power  $P_t$  from 5 Watts to 10 Watts, as it takes less time to charge sensors, and more sensors are likely to be charged before running out of energy. Fig. 6a also shows that the average sensor dead duration per sensor by the proposed algorithm AlgMaxLife is from 50% to 70% shorter than those by the other five algorithms.

Fig. 6b shows that the average sensor dead duration by each of the algorithms increases with the growth of the maximum data sensing rate  $b_{max}$  from 10 kbps to 50 kbps while fixing  $b_{min}$  at 1 kbps. The average sensor dead duration by algorithm AlgMaxLife is much shorter than those by the other five algorithms. The rationale behind the phenomenon is that the energy consumption rates of sensors increase when the sensor data rates are larger. Then, there are more

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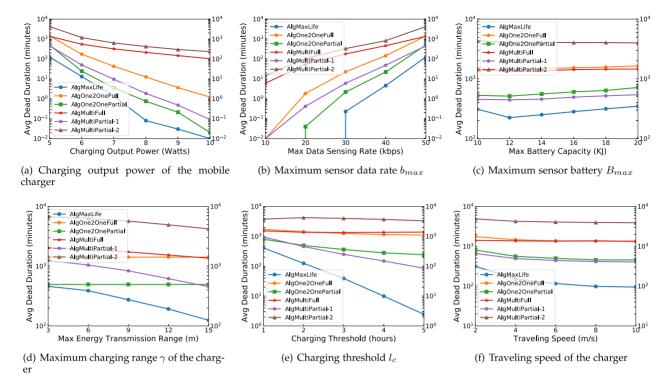


Fig. 6. Average sensor dead durations of different algorithms in the period  $T_M$  by evaluating various parameters.

to-be-charged sensors in each charging tour, and more sensors may run out of their energy before they are charged.

Fig. 6c plots that the average sensor dead duration by algorithm AlgMaxLife is the shortest one when the maximum sensor battery capacity  $B_{max}$  increases from 10 kJ to 20 kJ while fixing  $B_{min}$  at 10 kJ, where the battery capacity of each sensor is randomly chosen from an interval  $[B_{min}, B_{max}]$ . Specifically, the average sensor dead duration by algorithm AlgMaxLife is at least from 30% to 50% smaller than those by the other five algorithms.

Fig. 6d plots the performance of different algorithms when the maximum energy transmission range  $\gamma$  of the mobile charger increases from 3 meters to 15 meters from which it can be seen that the average sensor dead durations by the four algorithms AlgMaxLife, AlgMultiFull, AlgMultiPartial-1, and AlgMultiPartial-2 become smallers, when the maximum energy transmission range  $\gamma$  is larger. The rationale behind the phenomenon is that more sensors can be simultaneously charged by the mobile charger when  $\gamma$  is larger, thereby less sensors run out of energy. In contrast, the average sensor dead durations by both algorithms AlgOne20neFull and AlgOne2OnePartial do not change with the growth of the value of  $\gamma$ , as they adopt the one-to-one charging model. Fig. 6d also shows that the average sensor dead duration by algorithm AlgMaxLife is around from 4% to 70% shorter than those by the other five algorithms.

Fig. 6e studies the algorithm performance when the charging threshold  $l_c$  grows from one hour to five hours, where a sensor sends a charging request to the base station when its residual lifetime falls below  $l_c$  and the sensors with the residual lifetimes no greater than  $\alpha l_c$  will be charged by the mobile charger with  $\alpha = 5$ . Fig. 6e also shows that the average sensor dead duration by algorithm AlgMaxLife is from 50% to 90% shorter than those by the other five algorithms when  $l_c$  increases from one hour to five hours.

Fig. 6f evaluates the algorithm performance by increasing the traveling speed of the mobile charger from 2 m/s to 10 m/s, from which it can be seen that the average sensor dead duration by each of the six algorithms decreases when the mobile charger moves faster, since the waiting time of each sensor before its charging is shorter.

### 6.3 The Impact of the Charging Time Unit $\Delta$

We finally study the impact of the charging time unit  $\Delta$  on the algorithm performance by decreasing  $\Delta$  from  $\frac{T}{2}$  to  $\frac{T}{20}$ , where T is the time to fully charge an energy-empty sensor when the mobile charger is co-located at the sensor. Fig. 7 shows that the average sensor dead duration by algorithm AlgMaxLife-Given- $\Delta$  becomes smaller with the decrease of  $\Delta$  from  $\frac{T}{2}$  to  $\frac{T}{10}$ , while it slightly increases with the decrease of  $\Delta$  from  $\frac{T}{10}$  to  $\frac{T}{20}.$  The rationale behind is that more sensors are replenished before they run out of energy when the charging time unit  $\Delta$  decreases from  $\frac{T}{2}$  to  $\frac{T}{10}$ . However, when the value of  $\Delta$  is too small, e.g., smaller than  $\frac{T}{10}$ , the traveling time of the charger increases due to frequent shuttles between different charging locations.

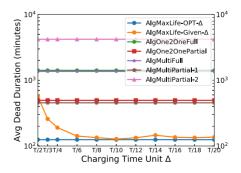


Fig. 7. Performance of different algorithms by decreasing the charging

### 7 CONCLUSION

Unlike existing studies focusing only on scheduling a mobile charger to minimize the number of dead sensors, they did not consider the charging scheduling for sensors that have already run out of energy, in this paper we considered not only how to minimize the number of dead sensors but also reduce the dead durations of sensors. To this end, we first formulated a novel sensor lifetime maximization problem, which is to find a charging tour for a mobile charger to charge sensors, such that the sum of sensor lifetimes is maximized. We then devised a  $\frac{1}{3}$ -approximation algorithm for the problem. We finally evaluated the performance of the proposed algorithm through experiments. Experimental results showed that both the average and maximum sensor dead durations by the proposed algorithms in the comparison.

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The first two authors Jingxiang Liu and Jian Peng contributed equally to this work, i.e., co-primary authors.

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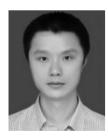


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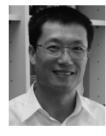
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