Charging Your Smartphones on Public Commuters via Wireless Energy Transfer

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***Abstract*—Smartphones now become an indispensable part of our daily life. However, their continuing operations consume lots of battery energy. For example, a fully-charged smartphone usually cannot support its continuing operation for a whole day. A fundamental problem related to this energy issue is how to prolong the smartphone lifetime so that it can last as long as possible to meet its user needs. Wireless energy transfer has been demonstrated as a promising technique to address this challenge. In this paper, we study the smartphone charging problem, using wireless chargers deployed on public commuters, e.g., subway trains, to charge energy-critical smartphones when their users take subway trains to work or go home. Since the residual energy of different smartphones are significantly different, the charging satisfactions of different users are essentially different too. In this paper we formulate this charging problem as a novel optimization problem that allocates limited wireless chargers on subway trains to charge energy-critical smartphones such that the overall charging satisfaction of mobile users is maximized, for a given monitoring period (e.g., one day). Specifically, we first devise a $\frac{1}{3}$-approximation algorithm if the travel trajectory of each smartphone user in the monitoring period is given; otherwise, we devise an online algorithm dealing with dynamic energy-critical smartphone charging requests. We finally evaluate the performance of the proposed algorithms through experimental simulations with a real dataset of subway-taking in San Francisco. The experimental results show that the proposed algorithms are very promising, and 93.9% of energy-critical user smartphones can be satisfactorily charged in one-day monitoring period.**

I. Introduction

With the advance on micro-electronic technology and wireless communication, more and more people nowadays rely heavily on portable mobile devices, such as smartphones, tablets, and Apple watches, for entertainment and business purposes. Especially, smartphones now become an indispensable part of our daily life. The *eMarketer* reported that there were more than 4 billion smartphone users globally in 2014 and this number is expected to grow to 5 billion in 2017 [14]. However, smartphones are very energy-consuming, and a fully-charged smartphone usually cannot support its continuing operation for a whole day [11].

The limited energy capacities of smartphones bring their users many inconveniences. Some users cannot continue using their smartphones any more later of the day (e.g., afternoon) due to their energy depletions. Others get to turn off some valuable yet energy-consuming functionalities, such as GPS, 3G/4G, and WiFi, to prolong smartphone lifetimes, which makes users cannot use many features of smartphones provided such as Twitter, google maps, Email, YouTube, eBay, Pinterest, etc. Furthermore, users must charge their smartphones quite often in order to continue their operations. Alternatively, a smartphone can be charged by a portable charger if needed. It however is inconvenient for its user to bring the charging device with him/her all time. A fundamental problem related to smartphone charging is: is there any convenient way to charge energy-critical smartphones so that users can enjoy all applications provided by smartphones at all times without worrying whether there is enough energy left? In this paper we tackle this challenge, using wireless energy chargers installed on subway trains to charge energy-critical smartphones.

The recent breakthrough on wireless energy transfer technology based on strongly coupled magnetic resonances has drawn lots of attentions in the research community [6]. Kurs *et al.* demonstrated that it is possible to achieve an approximate 40% efficiency of wireless power transfer for powering a $60W$ light bulb within two meters without any wire lines [6]. This technology has many advantages in comparison with other charging technologies, including high wireless energy transfer efficiency over a mid-range, minor radiations, immunity to the neighboring environment, no requirement of line-of-sight or any alignment, and charging multiple mobile devices simultaneously [12]. Several commercial products of wireless energy transfer technology now are available in the market, e.g., smartphones, electric vehicles, and sensors [8], [5], [12]. It thus is envisioned smartphones supporting wireless energy transfer will be pervasive in the near future.

The wireless energy transfer technique will revolutionize the way people charge their smartphones. Subway is one of the most popular transportation means in modern metropolitan cities including New York, London, Tokyo, Hong Kong, and Beijing that people take a subway train to work or go home. For example, the average daily subway ridership in New York alone is about 5.5 millions in a weekday [10]. Let us consider an application scenario where there are multiple chargers deployed on every subway train and every charger can charge a smartphone via wireless energy transfer if the smartphone is within the charging range of the charger, e.g., 2 to 3 meters. When a smartphone user takes on a subway train, his/her smartphone can send a charging request automatically if its residual energy falls below a given threshold, e.g., only 20% energy left (we refer to such smartphones as energy-critical smartphones), assuming that the user pays the subway company some fee for such service, e.g., \$5 per month. Once the request is received, a charger nearby the user is very likely to be allocated to charge the smartphone wirelessly.

In this paper we consider a charging satisfaction maximization problem on subway trains as follows. People take on a subway train at one station and take off at another station, or they may interchange for another train. Therefore, each user has several opportunities to get his/her smartphones

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charged when he/she is on a subway train. Furthermore, some people will take more stations than others before they take off. Since there are only limited numbers of chargers installed on subway trains and the number of charging requests may be far larger than the sum of the service capacities of all chargers, it thus is desirable to "fairly" replenish energy to the requested smartphones such that as many users as possible are satisfied. Otherwise, some users with sufficiently residual energy will be fully charged while others with barely left energy will miss the charging opportunities. We thus model the charging satisfaction of each requested user as a non-decreasing submodular function of the amount of energy charged to the smartphone of the user, where a submodular function usually is used to characterize the diminishing return property. For example, the charging satisfaction of a user v_j is $f_j(B_j) = \log_2(RE_j + B_j + 1) - \log_2(RE_j + 1)$, where RE_j is the residual energy of user v_j before the user is charged and B_j is the amount of energy charged to the user. The charging satisfaction maximization problem is to allocate the chargers to replenish energy to smartphones of requested users for a given monitoring period (e.g., one day), such that the sum of charging satisfaction of all users is maximized.

Unlike existing solutions to prolonging smartphone lifetimes by forcing users to turn off some useful yet energy-consuming functionalities (e.g., 3G/4G), we make use of wireless energy chargers deployed on subway trains to charge energy-critical smartphones, and introduce a novel metric to characterize the charging satisfactions of smartphone users so that their smartphones are 'fairly' charged.

The main contributions of this paper are as follows. We are the first to consider the use of wireless energy chargers installed on subway trains to charge energy-critical smartphones of users when they are in subway trains. To fairly charge energy-critical smartphones, we formulate a novel optimization problem of allocating wireless chargers to charge the smartphones for a given monitoring period, such that the overall charging satisfaction of mobile users is maximized, and we term this problem as the charging satisfaction maximization problem. We then devise a $\frac{1}{3}$-approximation algorithm for the problem if the travel trajectory of each smartphone user is given; otherwise, we devise a heuristic algorithm. We finally evaluate the performance of the proposed algorithms using a real dataset of subway-taking in San Francisco. Experimental results show that the proposed algorithms are very promising, and as high as 89.5% and 93.9% of energy-critical users can be charged by the online and approximation algorithms, respectively.

The rest of the paper is organized as follows. Section II reviews related work. Section III introduces preliminaries and defines the charging satisfaction maximization problem precisely. Sections IV and V propose approximation and online algorithms for the problem with and without the knowledge of travel trajectory of each user. Section VI evaluates the algorithm performance, and Section VII concludes the paper.

II. Related Work

The wireless power transfer technology based on strongly magnetic resonances has drawn a lot of attentions in many areas, such as smartphones [8], [12], electric vehicles [18], [5], wireless sensor networks [15], [16], [7], etc. For example, Zhu *et. al.* [18] studied the problem of scheduling n electric vehicles (EVs) to K deployed charging stations in a road network such that each of the EVs is fully charged and the average time spent on charging by each EV is minimized, where the average time for charging an EV includes the travel time to its assigned charging station, the queuing time, and the actual charging time. They also proposed two heuristics for the problem. Unlike their work, in this paper each to-be-charged smartphone of a user is not required to be fully charged on the subway. Instead, the sum of charging satisfactions of all mobile users should be maximized.

There are extensive studies in adopting wireless energy replenishment to prolong the lifetime of wireless sensor networks (WSNs) [15], [16], [7]. Xie *et. al.* [15] employed a wireless charging vehicle to periodically visit each sensor in a WSN, where the charging vehicle can charge each sensor wirelessly when the vehicle travels in the vicinity of the sensor. On the other hand, Xu *et. al.* [16] and Liang *et. al.* [7] employed multiple charging vehicles to replenish sensors energy in WSNs, in which Xu *et. al.* studied the problem of finding a series of charging scheduling for the charging vehicles to maintain the perpetual operations of the WSN for a given monitoring period such that the total travel distance of the vehicles for the period is minimized [16], and Liang *et. al.* considered the problem of dispatching the minimum number of charging vehicles to charge a set of to-be-charged sensors, assuming that the energy capacity of each charging vehicle is bounded [7]. Unlike these studies, in this paper the wireless chargers installed on subway trains are fixed and each smartphone may not be fully charged per charging. In fact, the amount of energy received by a smartphone relies on the duration of its user on trains and how much residual energy it has.

III. Preliminaries

A. System Model

We assume that the subway system in a metropolis consists of multiple subway trains and there are K chargers $C_1, C_2, \ldots, C_K$ deployed at K different places on subway trains. We consider the charging scheduling of the K chargers for a given monitoring period (e.g., one day), and we divide the period into T equal time slots with each time slot lasting δ units. We index the T time slots by $1, 2, \ldots, T$. Assume that each charger C_i has a charging capacity c_i, i.e., it can charge up to c_i smartphones at the same time, where $c_i \geq 1$ is a positive integer. We further assume that the output power of charger C_i is P_o $(Watts)$. Denote by d_{ijt} the Euclidean distance between charger C_i and smartphone v_j at time slot t, $1 \leq i \leq K$, $1 \leq j \leq n$ and $1 \leq t \leq T$. Following the seminal work of Kurs *et al.* [6], with the increase of distance d_{ijt}, the energy transfer efficiency μ_{ijt} of charger C_i charging smartphone v_j decreases. For example, Xie *et al.* [15] showed that $\mu_{ijt} = -0.0958 d_{ijt}^2 - 0.0377 d_{ijt} + 1$, where $0 \leq \mu_{ijt} \leq 1$. The reception power P_{ijt} of smartphone v_j from charger C_i at time slot t thus is

$$P_{ijt} = \mu_{ijt} P_o. \tag{1}$$

To ensure that the reception power P_{ijt} is large enough to charge smartphone v_j, we assume that charger C_i can replenish energy to smartphone v_j if d_{ijt} is no more than a maximum charging range D so that the energy transfer efficiency μ_{ijt} is no less than a threshold γ. For example, assume that $\mu_{ijt} \geq \gamma = 20\%$. The maximum charging range then is $D = 2.7$ m.

Assume that there are n_c users taking the subway for the period T, in which n $(\leq n_c)$ of them require to be charged at some time and each user can be charged by only one charger at each time slot. Let $V = \{v_1, v_2, \dots, v_n\}$ be the set of the n users. In the following, we use smartphone v_j and user v_j interchangeably. Assume user v_j sends a charging request at time slot t_j^S in a train and will take off the train at time slot t_j^F, clearly $t_j^S < t_j^F$. Then, user v_j can be charged within the time interval from t_j^S to t_j^F. We define the travel trajectory of user v_j between time slots t_j^S and t_j^F as the set of locations of the user on the train at every time slot in time interval $[t_j^S, t_j^F)$.

Denote by RE_j the residual energy of the smartphone of user v_j when the user sends a charging request with $1 \leq j \leq n$. Let binary variable x_{ijt} indicate whether charger C_i charges smartphone v_j at time slot t, i.e., $x_{ijt} = 1$ if smartphone v_j is charged by charger C_i at time slot t; $x_{ijt} = 0$, otherwise, where $1 \leq i \leq K$, $1 \leq j \leq n$, and $1 \leq t \leq T$. The amount of energy charged into the smartphone of user v_j when he/she takes off the subway is

$$B_j = \sum_{t=t_j^S}^{t_j^F-1} \sum_{i=1}^{K} P_{ijt} \cdot \delta \cdot x_{ijt}, \quad \forall v_j \in V, \tag{2}$$

where P_{ijt} is the reception power of user v_j from charger C_i at time slot t and δ is the duration of each time slot.

B. User charging satisfaction

In this subsection we model the charging satisfaction of every user v_j, by incorporating the amount of residual energy RE_j of user v_j when the user sends a charging request, the amount of energy B_j charged to user v_j, and his/her average energy consumption rate ρ_j. Before we proceed, we introduce non-decreasing submodular functions as follows.

Submodular functions usually are used to characterize the diminishing return property [3]. Let E be a finite set and z be a real-valued function: $z : 2^E \mapsto \mathcal{R}^{\geq 0}$, function z is a non-decreasing submodular function if and only if it has the following three properties [3]. (i) $z(\emptyset) = 0$; (ii) Non-decreasement: $z(S_1) \leq z(S_2)$ for any two sets $S_1, S_2 \subseteq E$ with $S_1 \subseteq S_2$; and (iii) Diminishing return property (submodularity): $z(S_1 \cup \{e\}) - z(S_1) \geq z(S_2 \cup \{e\}) - z(S_2)$ for any two sets S_1 and S_2 with $S_1 \subseteq S_2 \subset E$ and $e \in E \setminus S_2$.

Note that the average energy consumption rates of different users may significantly vary, since there are various types of smartphones, e.g., smartphones manufactured by different companies, and the user behaviors of using their smartphones are different, e.g., different frequencies of using smartphones. Recall that B_j is the amount of energy charged to user v_j. Then, the smartphone operational time of user v_j can be prolonged from $\frac{RE_j}{\rho_j}$ to $\frac{RE_j+B_j}{\rho_j}$ after the amount of energy B_j has been charged into the smartphone.

Since smartphone users are sensitive to the residual operational time of their smartphones, we here use a non-decreasing submodular function $g(l_j)$ to characterize the satisfaction of a user v_j for the residual operational time l_j of his/her smartphone. For example, $g(l_j) = \log_2(l_j + 1)$. Given the residual energy RE_j of user v_j and his/her average energy consumption ρ_j, we model the satisfaction $f_j(B_j)$ of user v_j for charging an amount of energy B_j as

$$f_j(B_j) = g(\frac{RE_j + B_j}{\rho_j}) - g(\frac{RE_j}{\rho_j}), \tag{3}$$

where $\frac{RE_j}{\rho_j}$ and $\frac{RE_j+B_j}{\rho_j}$ are the residual lifetimes of user v_j before and after charging an amount of energy B_j, respectively. Note that our model $f_j(B_j)$ of characterizing the charging satisfaction has following important properties.

(i) A user v_j is more satisfied if more energy is replenished to his/her smartphone.

(ii) If two users v_1 and v_2 have the same energy consumption rate (i.e., $\rho_1 = \rho_2$) but different amounts of residual energy RE_1 and RE_2 (assuming that $RE_1 < RE_2$), then user v_1 is more satisfied than user v_2 if they both are charged with the same amount of energy.

(iii) If two users v_1 and v_2 have the same residual lifetime (i.e., $\frac{RE_1}{\rho_1} = \frac{RE_2}{\rho_2}$) but different energy consumption rates ρ_1 and ρ_2 (assuming that $\rho_1 < \rho_2$), then user v_1 is more satisfied than user v_2 if they both are charged with the same amount of energy B. The rationale behind is that the prolonged operational time $\frac{B}{\rho_1}$ of user v_1 is larger than that $\frac{B}{\rho_2}$ of user v_2, i.e., $\frac{B}{\rho_1} > \frac{B}{\rho_2}$.

C. Problem definitions

Since the number of to-be-charged smartphones usually is larger than the sum of the charging capacities of all chargers in trains, we consider scheduling the chargers to charge energy-critical smartphones for a given monitoring period consisting of T time slots, such that the overall charging satisfaction of smartphone users is maximized, where we say a smartphone is energy critical if its residual energy is below its defined energy threshold.

We distinguish our discussions into two different cases: offline charging scheduling and online charging scheduling. In the *offline charging scheduling*, the travel trajectory of each to-be-charged user v_j from time slot t_j^S that user v_j sends his/her charging request to the time slot t_j^F that the user takes off the subway is given, and such knowledge can be obtained through tracking the train-taking history of user v_j or from the ticket information of user v_j when he/she bought the ticket at a subway station. In the *online charging scheduling*, we assume that the user travel trajectory information is not available, due to personal security and privacy concerns.

Given K chargers $C_1, C_2, \dots, C_K$ deployed on subway trains with charging capacities $c_1, c_2, \dots, c_K$, respectively, n to-be-charged users $v_1, v_2, \dots, v_n$ with user v_j sending his/her charging request at time slot t_j^S, and the travel trajectories of these n users, *the offline charging satisfaction maximization problem* is to allocate the K chargers to charge the n to-be-charged smartphones for a given monitoring period T, so that the accumulative charging satisfaction of all smartphone users is maximized, i.e., our objective is to

$$maximize \quad \sum_{j=1}^{n} f_j(B_j), \tag{4}$$

subject to constraints (1), (2), (3), and following constraints.

$$\sum_{j=1}^{n} x_{ijt} \quad \leq \quad c_i, \qquad 1 \leq i \leq K, \;\; 1 \leq t \leq T, \tag{5}$$

$$\sum_{i=1}^{K} x_{ijt} \quad \leq \quad 1, \qquad 1 \leq j \leq n, \;\; 1 \leq t \leq T, \tag{6}$$

$$x_{ijt} \in \{0,1\}, \ 1 \le i \le K, 1 \le j \le n, 1 \le t \le T, \quad (7)$$

$$x_{ijt} = 0, \quad \textit{if } t < t_j^S, \ t \ge t_j^F, \textit{ or } d_{ijt} > D, \quad (8)$$

where constraint (5) ensures that each charger C_i can charge no more than c_i users at each time slot t, constraint (6) ensures that each user v_j can be charged by no more than one charger at each time slot t, constraint (7) ensures that each user v_j is either charged by a charger C_i at time slot t or not, and constraint (8) ensures that each user v_j will not be charged by any charger C_i when the user have not sent a charging request (i.e., $t < t_j^S$) or have taken off (i.e., $t \ge t_j^F$), or the distance d_{ijt} between the user and the charger is longer than the maximum charging range D (i.e., $d_{ijt} > D$).

The online charging satisfaction maximization problem can be similarly defined as follows. Given K chargers $C_1, C_2, \ldots, C_K$ deployed on subway trains, n to-be-charged users $v_1, v_2, \ldots, v_n$, assume that the residual energy of user v_j at t_j^S is RE_j and the energy consumption rate of user v_j is ρ_j, $1 \le j \le n$. The problem is to allocate the K chargers to charge the n to-be-charged smartphones for a given period T without the knowledge of the travel trajectory of each user in future, so that the sum of charging satisfaction of all smartphone users is maximized.

The travel trajectory information of each user can be used to significantly improve the user charging satisfaction, as users with plenty of charging opportunities can be distinguished from those with only a few charging opportunities. For example, assume that there are only two lifetime-critical users v_1 and v_2 within the charging range of a charger C_i and the residual lifetimes of v_1 and v_2 at some time slot t are 20 minutes and 10 minutes, respectively. We further assume that user v_1 will take off the train very soon (e.g., five minutes later) while user v_2 will take a longer trip on the train. For this scenario, a charging allocation algorithm $\mathcal{A}$ without the user trajectory information may allocate charger C_i to charge only user v_2 since the residual lifetime of user v_2 is less than user v_1. As a result, user v_1 will miss his/her only charging opportunity while user v_2 will be charged into a large amount of energy and the residual lifetime of user v_2 is prolonged from 10 minutes to, for example, 4 hours. Then, the sum of charging satisfactions of users v_1 and v_2 when they take off the trains by algorithm $\mathcal{A}$ is $0 + \log_2(4*3600+1) - \log_2(10*60+1) = 4.58$ by Eq. (3), assuming that $g(l_j) = \log_2(l_j + 1)$. Contrarily, another algorithm $\mathcal{B}$ with the trajectory information may allocate charger C_i to charge user v_1 before he/she takes off the train and then assign charger C_i to charger user v_2 after user v_1 has taken off the train. As a result, the residual lifetimes of users v_1 and v_2 when they take off the trains are prolonged to, for example, $20 + 30 = 50$ minutes and $4 - \frac{30}{60} = 3.5$ hours, respectively. Then, the sum of charging satisfaction of the two users by algorithm $\mathcal{B}$ is $\log_2(50*60+1) - \log_2(20*60+1) + \log_2(3.5*3600+1) - \log_2(10*60+1) = 5.71 > 4.58$.

IV. Algorithm for the offline charging satisfaction maximization problem

In this section, we propose a novel $\frac{1}{3}$-approximation algorithm for the offline charging satisfaction maximization problem.

A. Algorithm

The basic idea behind the algorithm is as follows. It proceeds the charging allocation iteratively. Within each iteration, an pair $(C_{i^*}^{t^*}, v_{j^*}^{t^*})$ with the maximum increased satisfaction among all possible pairs is chosen, where charger C_{i^*} is allocated to charge user v_{j^*} at time slot t^*. In the following, we elaborate the approximation algorithm.

Given K chargers $C_1, C_2, \ldots, C_K$ with charging capacities $c_1, c_2, \ldots, c_K$, respectively, the n to-be-charged users $v_1, v_2, \ldots, v_n$ with user v_j sending a charging request at time slot t_j^S and taking off the subway at time slot t_j^F. Recall that the residual energy of user v_j is RE_j and its energy consumption rate is ρ_j. Also, its travel trajectory is given. The algorithm proceeds as follows.

Let $\mathcal{C} = \{C_1, C_2, \ldots, C_K\}$ and $V = \{v_1, v_2, \ldots, v_n\}$. We first construct a bipartite graph $G_t = (\mathcal{C}_t, V_t, E_t)$ for each time slot t, where $\mathcal{C}_t$ is the set of chargers at time slot t (i.e., $\mathcal{C}_t = \mathcal{C}$), V_t is the set of users that have charging opportunities on the subway at time slot t (i.e., $V_t = \{v_j \mid v_j \in V, t_j^S \le t < t_j^F\}$), where $t = 1, 2, \ldots, T$. For each charger $C_i^t \in \mathcal{C}_t$ and each user $v_j^t \in V_t$, there is an edge (C_i^t, v_j^t) in E_t if their Euclidean distance d_{ijt} at time slot t is no more than the maximum charging range D of charger C_i^t, i.e., $d_{ijt} \le D$. Then, the reception power P_{ijt} of charger C_i^t charging user v_j^t at time slot t is calculated by Eq. (1), and the amount of energy B_{ijt} charged to user v_j is $B_{ijt} = P_{ijt} \cdot \delta$, where δ is the duration of each time slot.

We then allocate the K chargers to charge the n users for the given period T iteratively. Let re_j be the amount of residual energy of user v_j after allocating some chargers to charge the user. Also, let c_i^t be the maximum residual number of users that charger C_i^t can charge at time slot t. Initially, $re_j = RE_j$, where RE_j is the residual energy of user v_j before charging and $1 \le j \le n$, and $c_i^t = c_i$, where $1 \le i \le K$ and $1 \le t \le T$. At each iteration, for each edge $(C_i^t, v_j^t) \in G_t$, recall that B_{ijt} is the amount of energy that can be charged to user v_j^t if charger C_i^t charges the user at time slot t, where $1 \le t \le T$. Then, the amount of increased satisfaction of user v_j is

$$\Delta(B_{ijt}) = g(\frac{re_j + B_{ijt}}{\rho_j}) - g(\frac{re_j}{\rho_j}). \quad (9)$$

We identify an edge $(C_{i^*}^{t^*}, v_{j^*}^{t^*})$ from the T graphs $G_1, G_2, \ldots, G_T$ such that the increased satisfaction of charging some user v_j^t by a charger C_t^t at time slot t is maximized, i.e., $(C_{i^*}^{t^*}, v_{j^*}^{t^*}) = \arg\max_{(C_i^t, v_j^t) \in G_1 \cup G_2 \cup \cdots \cup G_T} \{\Delta(B_{ijt})\}$. We then allocate charger C_{i^*} to charge user v_{j^*} at time slot t^*. We also increase the residual amount of energy re_{j^*} of user v_{j^*} by $B_{i^*j^*t^*}$ and reduce the maximum residual number of users $c_{i^*}^{t^*}$ that charger $C_{i^*}^{t^*}$ can charge at time slot t^* by one. Furthermore, we remove user node $v_{j^*}^{t^*}$ and its incident edges from graph G_{t^*} since user v_j can be charged by no more than one charger at time slot t^*, and remove charger node $C_{i^*}^{t^*}$ and its incident edges from graph G_{t^*} if $c_{i^*}^{t^*}$ has been decreased to zero as the number of users allocated to charger $C_{i^*}^{t^*}$ at time slot t^* now reaches its charging capacity c_i. The approximation algorithm continues until no edges are left in any of the T graphs $G_1, G_2, \ldots, G_T$. We detail the algorithm in `Algorithm 1`.

B. Algorithm analysis

We now analyze the approximation ratio of the proposed `Algorithm 1`. We start by introducing the definition of *matroids* as follows [3]. *A matroid* $\mathcal{M}$ is a pair of $(E, \mathcal{F})$ that

Algorithm 1 `ApproAlg`

Input: K deployed chargers $C_1, C_2, \ldots, C_K$ with charging capacities $c_1, c_2, \ldots, c_K$, n to-be-charged mobile users, the residual energy, energy consumption rate, and travel trajectory of each user, and a given period T.

Output: A charging allocation $\mathcal{A}$ that assigns the chargers to charge the users for the period T

Construct a bipartite graph $G_t = (\mathcal{C}_t, V_t, E_t)$ for each time slot t, where $\mathcal{C}_t$ is the set of the K chargers, V_t is the set of the users at time slot t, there is an edge (C_i^t, v_j^t) in E_t if the distance between charger C_i^t and user v_j^t at time slot t is no more than D, and $1 \leq t \leq T$;
For each edge (C_i^t, v_j^t) in the T graphs, compute the amount of energy B_{ijt} that can be charged to user v_j^t if allocating charger C_i^t to charge the user at time slot t by Eq. (1);
$\mathcal{A} \leftarrow \emptyset$; /* the charging allocation */
$re_j \leftarrow RE_j$, $1 \leq j \leq n$; /* the residual energy of user v_j */
$c_i^t \leftarrow c_i$, $1 \leq i \leq K$, $1 \leq t \leq T$; /* c_i^t is the maximum residual number of users that charger C_i^t can charge at time slot t */
while there is an edge in graphs $G_1, G_2, \ldots, G_T$ **do**
    For each edge $(C_i^t, v_j^t) \in G_1 \cup G_2 \cup \cdots \cup G_T$, compute the increased satisfaction $\Delta(B_{ijt})$ of user v_j Eq. (9);
    Find an edge $(C_{i^*}^{t^*}, v_{j^*}^{t^*})$ such that $(C_{i^*}^{t^*}, v_{j^*}^{t^*}) = \arg\max_{(C_i^t, v_j^t) \in G_1 \cup G_2 \cup \cdots \cup G_T} \{\Delta(B_{ijt})\}$;
    $\mathcal{A} \leftarrow \mathcal{A} \cup \{(C_{i^*}, v_{j^*}, t^*)\}$;
    $re_{j^*} \leftarrow re_{j^*} + B_{i^*j^*t^*}$, $\quad c_{i^*}^{t^*} \leftarrow c_{i^*}^{t^*} - 1$;
    Remove node $v_{j^*}^{t^*}$ and its incident edges from graph G_{t^*};
    Remove charger node $C_{i^*}^{t^*}$ and its incident edges from graph G_{t^*} if $c_{i^*}^{t^*}$ decreases to zero;
end while
return $\mathcal{A}$.

meets three properties, where E is a finite set and $\mathcal{F}$ is a family of subsets of E, i.e., $\mathcal{F} \subseteq 2^E$. (i) $\emptyset \in \mathcal{F}$; (ii) the hereditary property: $S_1 \subseteq S_2$ and $S_2 \in \mathcal{F}$ imply that $S_1 \in \mathcal{F}$ for any two subsets $S_1, S_2 \subseteq E$; and (iii) the independent set exchange property: for any two sets $S_1, S_2 \in \mathcal{F}$, if $|S_1| < |S_2|$, then, there is an element $e \in S_2 \setminus S_1$ such that $S_1 \cup \{e\} \in \mathcal{F}$. We state the following important lemma, which is the cornerstone of the approximation ratio analysis of `Algorithm` 1.

Lemma 1: [3] Let E be a finite set and $\mathcal{F}$ a non-empty collection of subsets of E which have the property that $S_1 \subseteq S_2 \subseteq E$ and $S_2 \in \mathcal{F}$ imply that $S_1 \in \mathcal{F}$. Given a non-decreasing submodular function $z : 2^E \mapsto \mathcal{R}^{\geq 0}$, a greedy heuristic always delivers a $\frac{1}{k+1}$-approximate solution to the problem $\max_{S \subseteq E}\{z(S) : S \in \mathcal{F}\}$, assuming that $(E, \mathcal{F})$ is described by the intersection of k matroids, where k is a positive integer.

Theorem 1: There is a $\frac{1}{3}$-approximation algorithm for the offline charging satisfaction maximization problem, which runs in time $O(nT^2 \log(nT) + KT)$, where n is the number of to-be-charged users, K is the number of deployed chargers, and T is the number of time slots in a given monitoring period.

Proof: We show the approximation ratio of `Algorithm` 1 by showing that (i) the objective function $\sum_{j=1}^{n} f_j(B_j)$ of the problem is a non-decreasing submodular function; and (ii) the constraints of (5) and (6) can be represented by $k = 2$ matroids. Following Lemma 1, it can be seen that `Algorithm` 1 delivers a $\frac{1}{k+1} = \frac{1}{3}$-approximate solution. We first can see that (i) the objective function $\sum_{j=1}^{n} f_j(B_j)$ is a non-decreasing submodular function, since $g(\cdot)$ is a given non-decreasing submodular function. We then prove (ii) that the constraints of (5) and (6) can be represented by two matroids, respectively. Recall that there is a bipartite graph $G_t = (\mathcal{C}_t, V_t, E_t)$ for each time slot t, where $1 \leq t \leq T$. Let $X = \mathcal{C}_1 \cup \mathcal{C}_2 \cup \cdots \cup \mathcal{C}_T$, $Y = V_1 \cup V_2 \cup \cdots \cup V_T$, and $E = E_1 \cup E_2 \cup \cdots \cup E_T$. Note that E is the set of the feasible charging allocations defined by constraints (7) and (8). We define a set system $\mathcal{M}_X = (E, \mathcal{F}_X)$ on the edge set E, where $\mathcal{F}_X$ is a family of subsets of E such that, for each edge set $S \in \mathcal{F}_X$ ($S \subseteq E$), the number of edges in S sharing the same endpoint C_i^t is no more than c_i for each charger node $C_i^t \in X$, and c_i is the maximum number of users that charger C_i can charge at time slot t. Similarly, we define another set system $\mathcal{M}_Y = (E, \mathcal{F}_Y)$, where $\mathcal{F}_Y$ is a family of subsets of E such that, for each edge set $S \in \mathcal{F}_Y$ ($S \subseteq E$), no two edges in S have the same endpoint in Y. Following the definitions of set systems $\mathcal{M}_X = (E, \mathcal{F}_X)$ and $\mathcal{M}_Y = (E, \mathcal{F}_Y)$, it can be seen that constraints (5) and (6) are represented by $\mathcal{M}_X$ and $\mathcal{M}_Y$, respectively. The proofs that $\mathcal{M}_X$ and $\mathcal{M}_Y$ are matroids are omitted, due to space limitation. It can be seen that the offline charging satisfaction maximization problem can be cast as a non-decreasing submodular function maximization problem, subject to the constraints of $k = 2$ matroids: $\mathcal{M}_X$ and $\mathcal{M}_y$, `Algorithm` 1 thus delivers a $\frac{1}{k+1} = \frac{1}{3}$-approximate solution by Lemma 1. Due to space limitation, the time complexity analysis of `Algorithm` 1 is omitted. ■

V. Algorithm for the Online Charging Satisfaction Maximization Problem

In the previous section, we have proposed an approximation algorithm for the charging satisfaction maximization problem, assuming that the travel trajectory of each user is given. However, such knowledge sometimes may not be available, due to personal security and privacy concerns. In this section, we study the charging satisfaction maximization problem without the knowledge of user trajectories, by devising an online algorithm.

A. The online algorithm

The basic idea behind the algorithm is that it finds a charging allocation with only the residual energy information of to-be-charged users provided so that the sum of the charging satisfaction of the users at each time slot is maximized. To this end, we reduce the problem to the maximum weight matching problem and an exact solution to the latter in turn returns a feasible solution to the former.

The online algorithm is invoked at the beginning of every time slot for the entire period T and a charging allocation $\mathcal{A}_t$ will be delivered by the algorithm at each time slot t with $1 \leq t \leq T$. As a result, the union of charging allocations $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_T$ forms a feasible solution to the problem. Specifically, assume that the charging allocations $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_{t-1}$ for the previous $t-1$ time slots have been obtained, we now find a charging allocation $\mathcal{A}_t$ at time slot $t$to maximize the sum of charging satisfaction of the users. Recall that V_t is the set of users that have charging opportunities at time slot t, i.e., $V_t = \{v_j \mid v_j \in V, t_j^S \leq t < t_j^F\}$. Denote by re_j^t the amount of residual energy of user v_j at the beginning of time slot t.

We first construct a bipartite graph $G_t = (\mathcal{C}, V_t, E_t; w_t)$, where $\mathcal{C}$ is the set of the K chargers, there is an edge (C_i, v_j) in edge set E_t if the Euclidean distance between charger C_i

and user v_j is no more than the maximum charging range D. For each edge $(C_i, v_j) \in E_t$, its weight $w_t(C_i, v_j)$ is the net satisfaction by allocating charger C_i to charge user v_j at time slot t, i.e., $w_t(C_i, v_j) = g(\frac{re_j^t + B_{ijt}}{\rho_j}) - g(\frac{re_j^t}{\rho_j})$, where $\frac{re_j^t}{\rho_j}$ and $\frac{re_j^t + B_{ijt}}{\rho_j}$ are the residual lifetimes of user v_j before and after charging v_j at time slot t, respectively, re_j^t is the residual energy of the user at the beginning of time slot t, B_{ijt} is the amount of energy that can be charged to the user by charger C_i at time slot t, and ρ_j is its energy consumption rate.

We then construct another bipartite graph $G'_t = (\mathcal{C}', V_t, E'_t; w'_t)$ derived from $G_t = (\mathcal{C}, V_t, E_t; w_t)$ as follows. For each charger C_i in $\mathcal{C}$, there are c_i 'virtual charger' nodes $C_{i,1}, C_{i,2}, \ldots, C_{i,c_i}$ in $\mathcal{C}'$, which have the same location as charger C_i in graph G_t. Thus, the charging capacity of each 'virtual charger' is exactly one. Also, for each edge (C_i, v_j) in graph G_t, there are c_i edges $(C_{i,1}, v_j), (C_{i,2}, v_j), \ldots, (C_{i,c_i}, v_j)$ in edge set E'_t, and each of these c_i edges has the same weight as the original edge (C_i, v_j), i.e., $w'_t(C_{i,k}, v_j) = w_t(C_i, v_j)$, $1 \le k \le c_i$.

Consider the maximum weight matching problem in $G'_t = (\mathcal{C}', V_t, E'_t; w'_t)$ that is to find a matching M such that the weighted sum of edges in M is maximized. Given a maximum weight matching M in G'_t, a charging allocation $\mathcal{A}_t$ for the online charging satisfaction maximization problem then can be derived, by adding (C_i, v_j, t) to $\mathcal{A}_t$ for each matched edge $(C_{i,k}, v_j)$ in matching M, i.e., user v_j will be charged by charger C_i at time slot t. The detailed online algorithm is presented in `Algorithm` 2.

Algorithm 2 `OnlineAlg`

Input: K deployed chargers $C_1, C_2, \ldots, C_K$ with charging capacities $c_1, c_2, \ldots, c_K$, n_t to-be-charged users $v_1, v_2, \ldots, v_{n_t}$ with their amounts of residual energy re_j^t and energy consumption rates ρ_j at time slot t

Output: A charging allocation $\mathcal{A}_t$ that assigns the chargers to charge the users at time slot t

Construct a bipartite graph $G_t = (\mathcal{C}, V_t, E_t; w_t)$, where $\mathcal{C} = \{C_1, C_2, \ldots, C_K\}$, $V_t = \{v_1, v_2, \ldots, v_{n_t}\}$, there is an edge (C_i, v_j) in E_t if the distance between charger C_i and user v_j at time slot t is no more than D, and $w_t(C_i, v_j) = g(\frac{re_j^t + B_{ijt}}{\rho_j}) - g(\frac{re_j^t}{\rho_j})$ for each edge $(C_i, v_j) \in E_t$;

Construct another graph $G'_t = (\mathcal{C}', V_t, E'_t; w'_t)$ from graph $G_t = (\mathcal{C}, V_t, E_t; w_t)$, where there are c_i virtual charger nodes $C_{i,1}, C_{i,2}, \ldots, C_{i,c_i}$ in $\mathcal{C}'$ for charger C_i in $\mathcal{C}$, there are c_i edges $(C_{i,1}, v_j), (C_{i,2}, v_j), \ldots, (C_{i,c_i}, v_j)$ in E'_t for each edge $(C_i, v_j) \in E_t$, $w'_t(C_{i,k}, v_j) = w_t(C_i, v_j)$, and $1 \le k \le c_i$;

Find a maximum weight matching M in graph G'_t;

$\mathcal{A}_t \leftarrow \emptyset$; /* the charging allocation for time slot t*/

For each matched edge $(C_{i,k}, v_j) \in M$, add (C_i, v_j, t) to $\mathcal{A}_t$;

return $\mathcal{A}_t$.

Theorem 2: There is an algorithm for the online charging satisfaction maximization problem, it takes $O((n+K)^{2.5})$ time for charing scheduling at each time slot t with $1 \le t \le T$, where n is the number of to-be-charged mobile users and K is the number of chargers.

VI. Performance evaluation

A. Experimental settings

We consider the subway network in San Francisco in the States, which consists of 6 subway lines and 45 stations [1]. The running timetable of the subway trains is obtained from [1], which includes the arrival times of the stations of each train. For simplicity, we assume that the length, width, and height of each train are 100 meters, 3.2 meters, and 3.2 meters, respectively, and there are 300 seats along the two sides of each train [2]. We divide one day into equal length time slots with each time slot lasting $\delta = 1$ minute. We assume that the maximum charging range of each wireless charger is 2.7 meters [6]. We also assume that the charging capacity c_i of each charger C_i is 1 and the output power P_o of charger C_i is 10 $Watts$. We deploy 41 wireless chargers along the two sides of each train, where Fig. 1 illustrates such a deployment in a two-dimensional space and the height of each deployed charger is 0.4 meters (at the position below seats on the train).

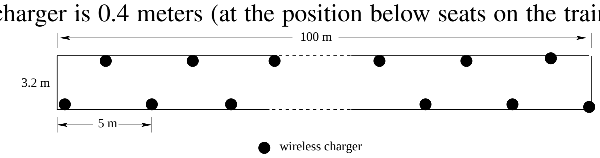


Fig. 1. The deployment of wireless chargers on each train.

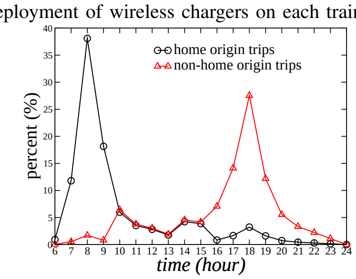
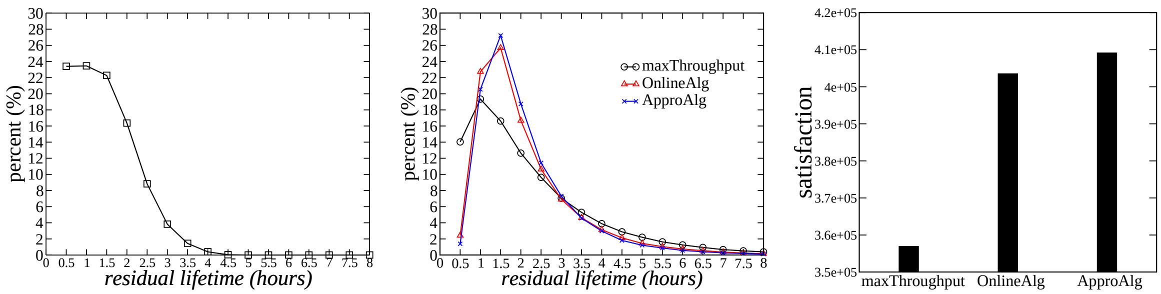


Fig. 2. The normalized number of users on trains over time in a weekday for home origin trips and non-home origin trips, respectively.

We adopt a real subway-taking dataset of San Francisco in a weekday of November 2014, which specifies the number of users between each pair of stations [9]. As a result, the total number of users n_c in a day is 424,763. Also, we generate the boarding time on trains for each user by referring to the 2008 station profile study of San Francisco subway [9]. Fig. 2 shows the normalized number of users on trains over time in a working day for home origin trips and non-home origin trips, respectively, where a home origin trip means that the trip of a user starts from home and a non-home origin trip indicates that the trip starts from locations other than home, such as work, school, etc. In addition, we assume that a user will randomly take a vacant seat if there are vacant seats available inn the train. Otherwise, the user will randomly stand on the train.

The battery capacity Cap_j of the smartphone of each user v_j is randomly chosen from $20\ kJ$ ($\approx 3.7\ V \cdot 1,500\ mAh$) to $40\ kJ$ ($\approx 3.7\ V \cdot 3,000\ mAh$). Also, the energy consumption rate ρ_j of each user v_j is randomly drawn from an interval $[0.5\ W,\ 1\ W]$. As a result, the lifetime of a fully charged smartphone can last from 5.5 ($\approx \frac{20\ kJ}{1\ W \cdot 3600\ s}$) hours to 22 ($\approx \frac{40\ kJ}{0.5\ W \cdot 3600\ s}$) hours. Furthermore, a fraction number α of users request to be charged (i.e., the number of to-be-charged is $\alpha \cdot n_c$), $0 \le \alpha \le 1$. The residual energy RE_j of each to-be-charged user v_j when issuing charging request is randomly chosen from an interval $[0,\ \beta \cdot Cap_j]$, where $0 \le \beta \le 1$ and Cap_j is the battery capacity of smartphone v_j. Function $g(l_j) = \log_2(l_j + 1)$ is used to characterize the satisfaction of each user v_j for the residual lifetime l_j of his/her smartphone, which is strictly concave and known to



(a) residual lifetime distribution of to-be-charged users when boarding the trains.

(b) residual lifetime distribution of to-be-charged users when taking off the trains.

(c) the satisfactions delivered by different algorithms

Fig. 3. Performance of algorithms `maxThroughput`, `OnlineAlg`, and `ApproAlg` when $\alpha = 0.5$ and $\beta = 0.1$.

achieve proportional fairness [17]. Given the residual energy RE_j of user v_j and his/her average energy consumption ρ_j, the satisfaction $f_j(B_j)$ of user v_j for charging an amount of energy B_j then is $f_j(B_j) = g(\frac{RE_j+B_j}{\rho_j}) - g(\frac{RE_j}{\rho_j}) = log_2(\frac{RE_j+B_j}{\rho_j}+1) - log_2(\frac{RE_j}{\rho_j}+1)$.

In addition to the two proposed algorithms `ApproAlg` and `OnlineAlg`, we also implement another charging allocation algorithm `maxThroughput`, which finds charging allocations such that the *sum of amounts of energy* charged to users is maximized. Similar to algorithm `OnlineAlg`, algorithm `maxThroughput` finds a charging allocation at each time slot t (i.e., in an online way). Unlike algorithm `OnlineAlg` that the weight of each edge (C_i, v_j) is the net satisfaction by allocating charger C_i to charge user v_j at time slot t, the weight of edge (C_i, v_j) in algorithm `maxThroughput` is the *amount of energy* charged to user v_j.

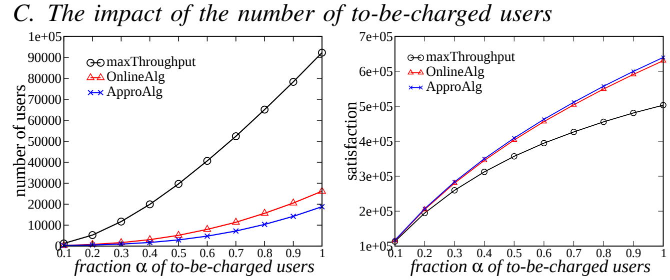
B. Performance evaluation of different algorithms

In the following we evaluate the performance of two proposed algorithms `ApproAlg` and `OnlineAlg` against the benchmark algorithm `maxThroughput`, assuming that half users on trains require to be charged, i.e., $\alpha = 0.5$. Fig. 3 (a) plots the residual lifetime distribution of to-be-charged users in the trains, from which it can be seen that most of users have short residual lifetimes. For example, about 23% of to-be-charged users have residual lifetime less than a half hour.

Fig. 3 (b) demonstrates the residual lifetime distribution of users when they take off the trains, from which it can be seen that the percentage of users with residual lifetime less than a half hour drops from 23% (see Fig. 3 (a)) to 14%, 2.4% and 1.4% in the charging allocations delivered by algorithms `maxThroughput`, `OnlineAlg`, and `ApproAlg`, respectively. As a result, algorithm `maxThroughput` can identify only a proportion of 39% ($= \frac{23-14}{23}$) lifetime-critical users, while algorithms `OnlineAlg`, and `ApproAlg` can identify most of energy-critical users, which are as high as 89.5% ($= \frac{23-2.4}{23}$) and 93.9% ($= \frac{23-1.4}{23}$), respectively. The rationale behind is that Algorithm `maxThroughput` finds charging allocations only by the amounts of energy charged to users, it thus fails to identify energy-critical users. Unlike algorithm `maxThroughput`, algorithms `OnlineAlg` and `ApproAlg` are able to identify energy-critical users since the net satisfaction gain by charging these users are significantly larger than that by charging those with long residual lifetimes. On the other hand, the number of users with energy critical residual lifetimes by algorithm `ApproAlg` is less than that by algorithm `OnlineAlg`, since algorithm `OnlineAlg` finds charging allocations without the knowledge of travel trajectory of each user as the one by algorithm `ApproAlg`. Thus, algorithm `ApproAlg` can distinguish the users with many charging opportunities from the users with only a few charging opportunities. Fig. 3 (b) also shows that the number of users with residual lifetimes between 1 hour and 3 hours delivered by algorithms `OnlineAlg` and `ApproAlg` are much larger than that by algorithm `maxThroughput`. In contrast, the numbers of users with residual lifetimes longer than 3 hours by algorithms `OnlineAlg` and `ApproAlg` are slightly less than that by algorithm `maxThroughput`, since the net satisfaction gain by charging the users with residual lifetime longer than 3 hours is marginal in algorithms `OnlineAlg` and `ApproAlg`.

Fig. 3 (c) plots the charging satisfaction performance delivered by the three mentioned algorithms, from which it can be seen that the total satisfaction delivered by algorithms `OnlineAlg` and `ApproAlg` are around 13.1% and 14.6% higher than that by algorithm `maxThroughput`. and the satisfaction delivered by algorithm `ApproAlg` is the highest one among them.

C. The impact of the number of to-be-charged users



(a) the number of users with residual lifetime less than a half hour when taking off the trains.

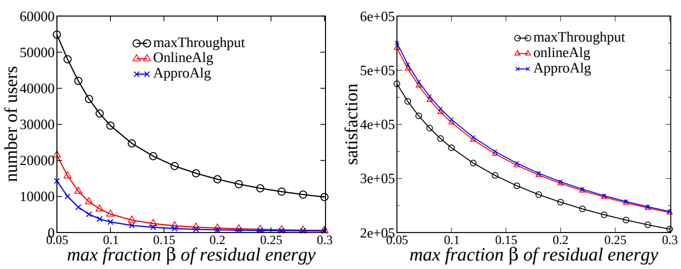
(b) satisfactions delivered by three algorithms

Fig. 4. Performance of algorithms `maxThroughput`, `OnlineAlg`, and `ApproAlg` by varying α from 0.1 to 1 when β is fixed at 0.1.

We then study the impact of the number of to-be-charged users on algorithm performance, by varying α from 0.1 to 1. Fig. 4 (a) plots the number of users with residual lifetime less than a half hour when they take off the trains by algorithms `maxThroughput`, `OnlineAlg`, and `ApproAlg`, respectively, from which it can be seen that the number of users in the charging allocations delivered by algorithms `OnlineAlg`, and `ApproAlg` are much less than that by algorithm `maxThroughput`. Furthermore, the number by algorithm `ApproAlg` always is the smallest one, which is only 71.7% of the number of users by algorithm `OnlineAlg` and 20.3% of the number of users by algorithm `maxThroughput` when all users request to be charged (i.e., $\alpha = 1$). The reason why algorithm `ApproAlg` outperforms algorithm `OnlineAlg` is that the former has the knowledge of user trajectories and more energy-critical users can be charged.

Fig. 4 (b) shows that the total satisfaction by algorithms `OnlineAlg` and `ApproAlg` is much higher than that by algorithm `maxThroughput`, and the gap between the first two algorithms and the third one grows bigger with the increase of α. Also, the satisfaction by algorithm `ApproAlg` always is the highest one. Note that although the satisfaction delivered by algorithm `ApproAlg` is only slightly higher than that by algorithm `OnlineAlg` (about from 0.87% to 1.48%), the charging allocations found by algorithm `ApproAlg` are much better than that by algorithm `OnlineAlg`, since there are much less number of users with energy critical residual lifetime by algorithm `ApproAlg` when users take off the trains, which has already been shown in Fig. 4 (a).

D. The impact of residual energy before charging



(a) the number of users with residual lifetime less than a half hour when taking off the trains.

(b) satisfactions delivered by three algorithms

Fig. 5. Performance of algorithms `maxThroughput`, `OnlineAlg`, and `ApproAlg` by varying β from 0.05 to 0.3 when $\alpha = 0.5$.

We finally investigate the impact of residual energy of to-be-charged users when they take on trains, by varying the maximum fraction β of residual energy of users from 0.05 to 0.3, where the residual energy RE_j of user v_j before charging is randomly chosen from an interval $[0, \ \beta \cdot Cap_j]$ and Cap_j the battery capacity of the user's smartphone. Fig. 5 (a) plots the number of users with residual lifetimes less than a half hour when they take off the trains, from which it can be seen that the number of users by each of the three mentioned algorithms will decrease with the increase of β, since there are more amounts of energy in the smartphones of these users before charging with a larger β. Also, the number of users by algorithms `OnlineAlg` and `ApproAlg` decreases to less than 1,000 while the number of users by algorithm `maxThroughput` is still more than 10,000 when $\beta > 0.2$. Again, the number of users by algorithm `ApproAlg` is the smallest one, which is around from 66.6% to 73.5% of that by algorithm `OnlineAlg` and even is about from 4.5% to 26.1% of that by algorithm `maxThroughput`.

Fig. 5 (b) implies that the satisfaction by each of the three algorithms decreases with the increase of the value of β. The rationale behind is that a user is less satisfied for charging an amount of energy if the user has more energy in his/her smartphone before charging. Fig. 5 (b) also shows the satisfactions by algorithm `ApproAlg` and `OnlineAlg` are around 13.6% and 15% higher than that by algorithm `maxThroughput`, respectively.

VII. CONCLUSION

In this paper, we considered the use of wireless energy chargers installed on subway trains to charge energy-critical smartphones of users through wireless energy transfer when users take subway trains to work or go home, for which we first formulated a problem of allocating wireless chargers to charge energy-critical smartphones for a given monitoring period, such that the overall charging satisfaction of mobile users is maximized. We then proposed a novel $\frac{1}{3}$-approximation algorithm for the problem, assuming that the travel trajectory of each to-be-charged smartphone user is given; otherwise, we devised an online algorithm. We finally evaluated the performance of the proposed algorithms, using a real dataset. Experimental results showed that the proposed algorithms are very promising, and as high as 89.5% and 93.9% of energy-critical users can be charged by the online and approximation algorithms on time.

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