MULTIPLE END-TO-END DELAY CONSTRAINED MULTICASTING IN MULTI-HOP OPTICAL WDM NETWORKS

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ABSTRACT

This paper considers the problem of establishing multiple multicast sessions in a multi-hop optical wavelength division multiplexing (WDM) network simultaneously, such that the sum of the cost of realizing these sessions is minimized and at the same time, the end-to-end delay between each source-destination pair is bounded. The cost of a multicast session is expressed in terms of the cost of using a wavelength on a link and the cost of wavelength conversion at a node. The end-to-end delay is bounded by the sum of communication delays on links and the wavelength conversion delays at intermediate nodes. In this paper, we present a solution to the problem by formulating it into an Integer Linear Program (ILP) and solving the ILP on a representative sized mesh network.

1. INTRODUCTION

Wavelength Division Multiplexing (WDM) technique is a promising candidate for next-generation high speed Internet and is capable of supporting traffic intensive, high bandwidth applications such as multimedia communications, distributed gaming, medical imaging, video-conferencing, interactive learning, [1]. These applications demand high throughput and stringent end-to-end delay requirements. WDM uses multiple wavelengths that can be multiplexed in a single fiber link at very high bit rates (potentially Tbps), aggregating to high throughput. Abundant bandwidth coupled with the advances in optical networking technology gives WDM the potential to satisfy these stringent Quality of Service (QoS) performance parameters.

WDM networks can be characterized into two types – single-hop and multi-hop networks. In *single-hop* networks, a message is transmitted from a source node to a destination node in an all-optical transmission medium, and uses the *same* wavelength along the links of the path. In this scenario, the available bandwidth can be efficiently utilized as overheads due to wavelength conversion and electronic processing at intermediate nodes can be avoided. However, this leads to *high blocking probability* as the *same wavelength* has to be used on all the links of the transmission path. The probability of setting up a session is *increased* by using *multi-hop* WDM networks as they allow *wavelength conversion* to occur at intermediate nodes. In this paper, we will concentrate on multi-hop networks.

Multicasting is a prominent service that forms an integral part of the communication networks already deployed, and is crucial to the communication networks that will be deployed in the future. In *multiple multicasting*, if sufficient resources are available, then two or more multicast sessions have to be established *simultaneously*, such that the QoS requirements of the application are guaranteed – in particular, meeting the end-to-end delay bound between each source-destination pair of *every* multicast session.

Multicasting in WDM networks can be efficiently setup using the concept of *light-trees* [2]. In this approach, an inherent optical multicasting capability (splitting of light) is introduced in the wavelength routing switches (nodes) that can significantly improve the throughput and hence the performance of the network. These switches can have wavelength conversion functionalities built into them, hence the probability of successfully establishing one or more multicast sessions is increased [3].

2. RELATED WORK

For traditional electronic networks, multicast routing has been extensively investigated and several routing algorithms have been proposed. There is an *essential difference* between multicasting in traditional networks and multicasting in WDM networks. Multicasting in the former is focused on constructing a delay bounded multicast tree while minimizing the total cost. Here, only a *single* cost and delay function is taken into consideration, namely the *cost of using a particular link* and the *communication delay* over it [4] (and the references therein). However, in WDM networks, wavelengths constitute an important resource and hence the *cost of wavelength conversion* together with the *delay incurred due to wavelength conversion* at intermediate nodes are of paramount importance. Therefore, the optimization objective in this case should not only consider the cost of using a wavelength on a link, but also the cost of wavelength conversion at an intermediate node, thereby involving *more than one cost metric*. Wavelength conversion delay at a node is the delay incurred to convert an incoming wavelength to an outgoing, but *different* wavelength. Due to speed and other physical limitations of wavelength converters, wavelength conversion delay plays an important role in bounding the end-to-end delay between source-destination pairs. Hence, this parameter has to be considered in any efficient routing strategy. Thus, while bounding the end-to-end delay, we should consider not only the communication delays on links, but also the delays incurred due to wavelength conversions at intermediate nodes of the routing path. Also, as wavelengths are the critical network resources, assigning them to communication links efficiently are fundamental to WDM networks. Therefore, application of these multicast routing algorithms (multicast routing algorithms designed for traditional electronic networks) to WDM network environment is limited, and

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calls for the design of effective Routing and Wavelength Assignment (RWA) strategies.

The work in [5] addresses the problem of multiple multicasting in WDM networks, its objective is to minimize the network congestion. [6] focuses on single-hop networks in an attempt to minimize the number of distinct wavelengths required for constructing a set of QoS multicast trees at a suboptimal cost, with an assumption that the cost and delay functions are correlated. [7] focuses on establishing a multicast tree taking into account the cost of wavelength utilization and conversion along with wavelength conversion delay to bound the end-to-end delay between a source node and a set of destination nodes. However, this ignores the communication delay on a link.

To the best of our knowledge, we have not seen a formulation that aims at realizing *multiple*, *end-to-end delay constrained* multicast sessions *simultaneously*, taking into account i) the cost of utilizing a wavelength on a link, ii) the cost of wavelength conversion at an intermediate node, iii) the communication delay on a link, and iv) the delay due to wavelength conversion at an intermediate node.

In this paper we address the above problem and present ILP formulations for efficient RWA at a *globally optimum cost*. An important feature of our work is that the cost and delay input metrics are independent of each other. We decouple the cost of utilizing a wavelength on a link from the cost of wavelength conversion at intermediate nodes. Also, the communication delay on a link is decoupled from the delay incurred due to wavelength conversion.

3. NODE ARCHITECTURE

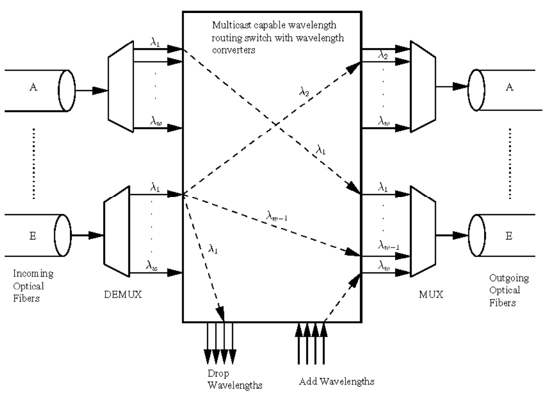

Fig. 1. Multicast capable wavelength routing switch with wavelength converters

An optical WDM network consists of a number of nodes with communication fiber links interconnecting the nodes. Each node in our study is a multicast capable wavelength routing switch with wavelength converters, capable of converting any incoming wavelength to any outgoing wavelength among the wavelengths available in the network. The structure of such a switch is shown in Fig. 1. For efficient multicasting, the incoming optical signal is split using optical splitters (i.e. replicated into two or more identical copies) for routing [2]. Optical signal λ_1 on incoming optical fiber E is split into three identical copies at the node. One copy is dropped off locally on wavelength λ_1 while the other two are routed to outgoing fibers A and E respectively. Wavelength conversion functionalities are useful to such switches as they help reduce the blocking probability of the sessions. As a result, the signal on outgoing fiber A occupies wavelength λ_2 while the signal on outgoing fiber E occupies wavelength λ_{w-1}. Observe that the signal λ_1 from incoming optical fiber A is switched on the same wavelength to the outgoing optical fiber E without any replication. These switches can also perform the local Add/Drop functionalities. As the output signal weakens because of optical splitting, they may have to be amplified before the signal is routed to the respective outgoing fiber links. In the absence of wavelength conversion, the multicast session is known to exhibit the wavelength continuity constraint.

4. MATHEMATICAL FORMULATION

In this section we solve the problem by modeling it as an integer linear program. The following are the notations that will be used in the literature.

- **Inputs to the problem**

1. A network $G = (V, E)$ representing a physical topology, consists of $|V| = n$ nodes and $|E| = m$ links interconnecting the nodes. Each link in the physical topology is bidirectional and modeled as a pair of unidirectional links.
2. $W = \{\lambda_1, \lambda_2, \lambda_3, \ldots\ldots, \lambda_w\}$ is the set of available wavelengths in the network.
3. Every node $v \in V$ is equipped with a $d(v) \times d(v)$ multicast capable wavelength routing switch, capable of converting any wavelength λ' on an incoming link $(u, v) \in E$ to any wavelength λ'' on an outgoing link $(v, t) \in E$ among W wavelengths. $d(v)$ denotes the physical degree of node v. This represents the number of incoming (and hence the number of outgoing) physical fiber links at node v.
4. Wavelength utilization cost $\omega_{u,v}^{\lambda'}$, represents the cost of using a wavelength on a physical link between nodes u and v, where $(u, v) \in E$.
5. Every node $v \in V$ is associated with a cost C_v^λ, that represents the cost of converting an incoming wavelength to one or more outgoing wavelengths at node v. If an incoming wavelength does not undergo wavelength conversion at node v, $C_v^\lambda = 0$.
6. $\delta_{u,v}$ denotes the communication delay on a link between nodes u and v, where $(u, v) \in E$.
7. δ_v^λ is the time required to convert an incoming wavelength to an outgoing wavelength at node $v \in V$. If an incoming wavelength does not undergo wavelength conversion at node v, $\delta_v^\lambda = 0$.
8. A group of k multicast sessions that have to be established. Each multicast session is represented as $S_i = \{s_i, d_{i_1}, d_{i_2}, d_{i_3}, \ldots\ldots\ldots, d_{i_p}\}$. The first element of set S_i is the source node, s_i, and the remaining elements of the set are the destination nodes respectively. For k multicast sessions, we have $1 \le i \le k$.
9. Δ_{i_j} is the end-to-end delay bound between the source node s_i and a destination node d_{i_j} for session i, where s_i, $d_{i_j} \in S_i$.

- **Boolean variables used in the formulation**

1. $e_{u,v}^{i,\lambda'} = 1$, if link between nodes u and v is used by multicast session i on wavelength λ', where $(u,v) \in E$, $\lambda' \in W$; otherwise $e_{u,v}^{i,\lambda'} = 0$.
2. $v_v^i = 1$, if node v is included in multicast session i, i.e. if v is a node in the tree generated for multicast session i, where $v \in V$; otherwise $v_v^i = 0$.
3. $f_{(u,v),\, d_{ij}}^{i,\lambda'} = 1$, if one unit of commodity for destination node d_{ij} is used on the link between the nodes u and v on wavelength λ' for the multicast session i, where $(u,v) \in E$, $\lambda' \in W$; otherwise $f_{(u,v),\, d_{ij}}^{i,\lambda'} = 0$.
4. $v_{v,(u,v),\, \lambda'}^{i,\, d_{ij}} = 1$, if node v is on the path in the tree from source node s_i to destination node d_{ij} for the multicast session i and performs wavelength conversion from incoming wavelength λ' on link $(u,v) \in E$ to any outgoing wavelength λ'', where $v \in V$, and $\lambda', \lambda'' \in W$; otherwise $v_{v,(u,v),\, \lambda'}^{i,\, d_{ij}} = 0$.
5. $wc_v^i = 1$, if node v performs wavelength conversion for multicast session i, where $v \in V$; otherwise $wc_v^i = 0$.

In our formulation, the wavelength conversion cost is independent of the wavelength conversion delay and vice-versa. In other words, we decouple the cost of wavelength conversion from the wavelength conversion delay. Consider a scenario of a multicast tree where two or more connections from a source node to multiple destination nodes share a few nodes along the path. In this case, all the connections undergo wavelength conversion jointly at the shared node but suffer from wavelength conversion delays individually. Therefore, the wavelength conversion cost must be accounted for only once and in order to verify the end-to-end delay requirement, the wavelength conversion delay must be accounted for separately, i.e. on a per-connection basis. Hence the variables wc_v^i and $v_{v,(u,v),\, \lambda'}^{i,\, d_{ij}}$ will be associated with the wavelength conversion cost and the wavelength conversion delay respectively.

- **Objective Function**

The objective is to minimize the total cost (due to wavelength utilization and wavelength conversion) of establishing k multicast sessions.

$$Minimize \quad \sum_{i=1}^{i=k} \sum_{u,v} \sum_{\lambda'=1}^{\lambda'=w} e_{u,v}^{i,\lambda'} . \omega_{u,v}^{\lambda'} + \sum_{i=1}^{i=k} \sum_{v=1}^{v=n} wc_v^i . C_v^{\lambda} \tag{1}$$

subject to the following constraints

- **Tree creation constraints**

$$\forall i, \forall p \in S_i : \quad v_p^i = 1 \tag{2}$$

$$\forall i, \forall v = s_i : \quad \sum_{\lambda'} \sum_{u} e_{u,v}^{i,\lambda'} = 0 \tag{3}$$

$$\forall i, \forall v \neq s_i : \quad \sum_{u} \sum_{\lambda'} e_{u,v}^{i,\lambda'} = v_v^i \tag{4}$$

$$\forall i, \forall u \neq d_{ij},\, j \geq 1 : \quad \sum_{v} \sum_{\lambda'} e_{u,v}^{i,\lambda'} \geq v_u^i \tag{5}$$

$$\forall i, u : \quad \sum_{v} \sum_{\lambda'} e_{u,v}^{i,\lambda'} \leq d(u) . v_u^i \tag{6}$$

$$\forall u, v : \quad \sum_{i} \sum_{\lambda'} e_{u,v}^{i,\lambda'} \leq w \tag{7}$$

$$\forall u, v, \lambda' : \quad \sum_{i} e_{u,v}^{i,\lambda'} \leq 1 \tag{8}$$

- **Integer commodity flow constraints**

$$\forall i, \forall u = s_i, j \geq 1 : \quad \sum_{v} \sum_{d_{ij}} \sum_{\lambda'} f_{(u,v),\, d_{ij}}^{i,\lambda'} = |S_i| - 1 \tag{9}$$

$$\forall i, \forall v = s_i, j \geq 1 : \quad \sum_{u} \sum_{d_{ij}} \sum_{\lambda'} f_{(u,v),\, d_{ij}}^{i,\lambda'} = 0 \tag{10}$$

$$\forall i, \forall v = d_{ij}, j \geq 1 : \quad \sum_{u} \sum_{\lambda'} f_{(u,v),\, d_{ij}}^{i,\lambda'} = 1 \tag{11}$$

$$\forall i, \forall v = d_{ij}, v \neq d_{iq}, j \geq 1, q \geq 1 : \quad \sum_{u} \sum_{\lambda'} f_{(u,v),\, d_{iq}}^{i,\lambda'} = \sum_{u} \sum_{\lambda'} f_{(v,u),\, d_{iq}}^{i,\lambda'} \tag{12}$$

$$\forall i, \forall d_{ij}, \forall v \notin S_i : \quad \sum_{u} \sum_{\lambda'} f_{(u,v),\, d_{ij}}^{i,\lambda'} = \sum_{u} \sum_{\lambda'} f_{(v,u),\, d_{ij}}^{i,\lambda'} \tag{13}$$

$$\forall i, u, v : \quad \sum_{d_{ij}} \sum_{\lambda'} f_{(u,v),\, d_{ij}}^{i,\lambda'} \leq [\,|S_i| - 1\,] . \sum_{\lambda'} e_{u,v}^{i,\lambda'} \tag{14}$$

$$\forall i, u, v, \lambda' : \quad \sum_{d_{ij}} f_{(u,v),\, d_{ij}}^{i,\lambda'} \leq [\,|S_i| - 1\,] . e_{u,v}^{i,\lambda'} \tag{15}$$

$$\forall i, u, v : \quad \sum_{\lambda'} e_{u,v}^{i,\lambda'} \leq \sum_{d_{ij}} \sum_{\lambda'} f_{(u,v),\, d_{ij}}^{i,\lambda'} \tag{16}$$

- **Wavelength conversion formulation**

$$\forall i, u, v, t, d_{ij}, j \geq 1 : \quad v_{v,(u,v),\, \lambda'}^{i,d_{ij}} - f_{(u,v),\, d_{ij}}^{i,\lambda'} - \sum_{t \neq u} \sum_{\lambda'' \neq \lambda'} f_{(v,\, t),\, d_{ij}}^{i,\lambda''} + 1 \geq 0 \tag{17}$$

- **Formulation of cost of wavelength conversion**

$$\forall i, v: \qquad wc_v^i \geq \frac{\sum_{d_{ij}} \sum_{\lambda'} \sum_{u} v_{v,(u,v),\lambda'}^{i,d_{ij}}}{|S_i| - 1} \tag{18}$$

- **End-to-end delay formulation**

$$\forall i, j \geq 1, \forall v \neq s_i, d_{ij}:$$
$$\sum_{v,u} \sum_{\lambda'} f_{(u,v),\,d_{ij}}^{i,\lambda'} \cdot \delta_{u,v} +$$
$$\sum_{v,u} \sum_{\lambda'} v_{v,(u,v),\lambda'}^{i,d_{ij}} \cdot \delta_v^{\lambda} \leq \Delta_{ij} \tag{19}$$

Equation (2) ensures that the source node and all the destination nodes of a multicast session are included in the tree. For example, for session 2, we have $v_{s_2}^2 = 1, v_{d_{21}}^2 = 1, v_{d_{22}}^2 = 1, v_{d_{23}}^2 = 1$; for session 3, we have $v_{s_3}^3 = 1, v_{d_{31}}^3 = 1, v_{d_{32}}^3 = 1, v_{d_{33}}^3 = 1$, and so on. Equation (3) ensures that the source node does not have any incoming edges as it is the root of the tree. Equation (4) ensures that any node, except the source node that belongs to the multicast tree, has only one incoming edge. It also ensures that there cannot be any loops in the tree. Equation (5) ensures that any node that belongs to the tree that is not a destination node, has at least one outgoing edge. The constraint in equation (6) ensures that the number of outgoing edges at a node $u \in V$ is constrained by the degree of the node, $d(u)$. The constraint in equation (7) restricts the total number of available wavelengths on a link to w. In other words, this constraint ensures that only w distinct sessions can be routed on a physical fiber link. No two sessions sharing a physical fiber link can have the same wavelength routed on that link. It implies that, if any two multicast sessions share a link $(u, v) \in E$, then the wavelength occupied by each of the two sessions on the link $(u, v) \in E$ should be different. This constraint is represented in equation (8).

For any multicast session S_i, every destination node should receive one unit of commodity. Therefore, the total outflow at the source node, s_i, must be equal to the number of destination nodes, $|S_i| - 1$. $|S_i|$ is the cardinality of session S_i and $|S_i| - 1$ is the number of destination nodes respectively. This is represented by equation (9). Equation (10) represents the condition that the total inflow to the source node for any multicast session is zero. Every destination node in a multicast session must receive one unit of commodity. If a destination node is a relay node, then the total outgoing flow is one less than the total incoming flow since one unit of commodity gets dropped locally at the destination node. Therefore, at any destination node $d_{ij} \in S_i$, the total incoming flow to d_{ij} must be equal to one. For all the remaining destination nodes $d_{iq} \in S_i$ where $d_{iq} \neq d_{ij}$, the total incoming flow at d_{ij} must be equal to the total outgoing flow. Observe that if d_{ij} is a destination node and a leaf node, the total incoming flow is equal to one and the total outgoing flow is zero. At all the remaining nodes $v \in V$ that belong to the multicast tree and are neither the source nor destination nodes, the total incoming flow at v must be equal to the total outgoing flow. These are represented by the equations (11), (12) and (13) respectively. For a multicast session S_i, the flow on link $(u, v) \in E$ is limited by the number of destination nodes of S_i. The flow to these destination nodes on the link $(u, v) \in E$ should be on the same wavelength. Also, if a link is occupied by a session, then there must be some flow on it, otherwise the flow on the link must be zero. These constraints are represented by the equations (14), (15) and (16) respectively.

In equation (17), $v_{v,(u,v),\lambda'}^{i,d_{ij}}$ is assigned the value one if the flow to a destination node d_{ij} is on wavelength λ' on an incoming link $(u, v) \in E$ at node v, and undergoes wavelength conversion at v, so that the resultant outgoing flow on any link $(v, t) \in E$ $(t \neq u)$ occupies a wavelength that is not the same as λ'.

Equation (18) ensures that the cost of wavelength conversion at a node v, for any multicast session S_i, is accounted for only once in the objective function. At node v, for a multicast session S_i, the maximum number of wavelength conversions possible is equal to the number of destination nodes for the session S_i, i.e. $|S_i| - 1$. As they face wavelength conversion jointly, the cost of wavelength conversion must be accounted for only once.

Finally, equation (19) represents the formulation for the end-to-end delay bound. A connection from a source node to a destination node can suffer from two delay parameters – communication delay on a link and the delay incurred due to wavelength conversion at intermediate nodes. Thus, the total end-to-end delay between a source node s_i and a destination node d_{ij} for a multicast session i is the sum of these two delays along its communication path.

5. RESULTS AND DISCUSSION

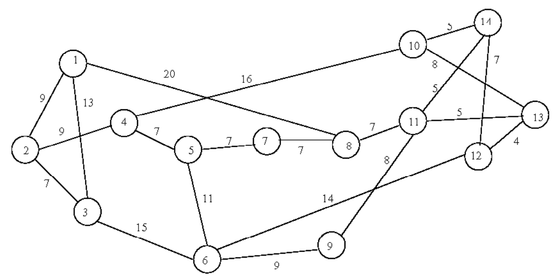

Fig. 2. 14-node NSFNET backbone network

To evaluate the effectiveness of our formulation, we conducted experiments by simulation on the NSFNET backbone shown in Fig. 2 using CPLEX [9]. The numbers in the figure along the links correspond to the communication delay over it. The cost of a link was set to a uniform random number [0,1] times the delay over the link, rounded to the nearest integer. Assuming a fixed delay for wavelength conversion at a node, the cost of wavelength conversion at a node was a uniform random number [0,1] times the delay, rounded to the nearest integer. The end-to-end delay between source-destination pairs were randomly generated integral values between [2,3] times the maximum link delay. Our experiments were aimed at analyzing the cost of multicast sessions as the number of wavelengths in the network, size of each session (size of a session is defined as the number of destination nodes for that session) and the number of sessions are varied. A multicast session instance comprises of a source node, destination nodes and an end-to-end delay bound between each source destination pair. As the number of multicast sessions required to be established were varied, 50 session instances were randomly generated and our formulation was executed on each of them. The experiment was performed by setting the size of each multicast session to 3, 6 and 8 nodes or roughly 20%, 40% and 60% of the total network size. For brevity, we will illustrate our results by establishing 3 and 4 multicast sessions with the size of each session set to 8 nodes.

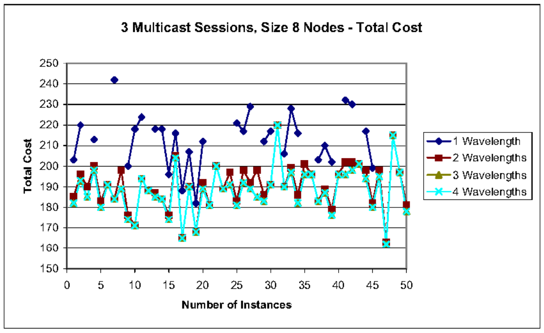

Fig. 3. Cost of 3 multicast sessions, each of size 8 nodes and number of wavelengths in the network varied from 1 to 4.

The graph in Fig. 3 illustrates the total cost of establishing 3 multicast sessions. With only one wavelength available for routing, the total cost ranged between 182 and 242 units. However, 40% of the multicast sessions could not be established because of insufficient resources. Observe the discontinuity in the plot shown in Fig. 3. This is natural to expect because some links have to be shared across 3 multicast sessions. Since only one wavelength was available, the session could not be established because no two sessions can share the same wavelength on the same link. With two wavelengths, all the sessions could be established successfully and the total cost now ranged between 163 and 220 units, a 10% reduction in the total cost bounds when compared to the previous case. Our formulation could successfully exploit the extra available wavelength and thereby globally reduce the total cost of establishing the multicast sessions. There was no marked change in the total cost when the number of available wavelengths exceeded two.

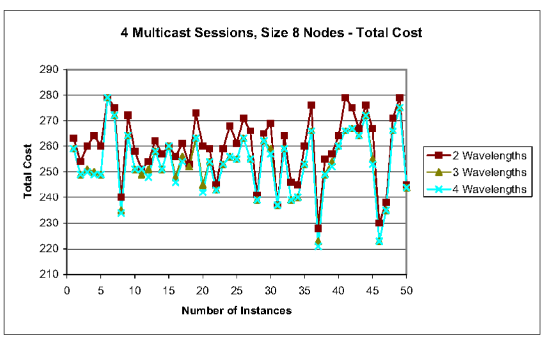

Fig. 4. Cost of 4 multicast sessions, each of size 8 nodes and number of wavelengths in the network varied from 1 to 4.

Next, we illustrate our results on 4 multicast sessions. No session could be established with only one wavelength available for routing. As the number of wavelengths in the network increased to two, all the sessions could be established successfully. The total cost curves for 4 multicast sessions are shown in Fig. 4. Similar results were obtained from other experiments.

6. CONCLUSION

In this paper we investigated the problem of establishing multiple multicast sessions in a multi-hop optical WDM network simultaneously, such that the end-to-end delay between each source-destination pair of these multicast sessions is met rigorously. We showed how our formulations can be applied to real world networks and illustrated our results using the representative NSFNET backbone mesh network. From a static design perspective, it might be desirable to run instances with large size for longer time so as to yield optimal solutions. Alternatively, we would be interested in obtaining faster but good sub-optimal solutions. Well known ILP relaxation techniques can be applied to our formulation to yield close to optimal solutions.

7. ACKNOWLEDGMENT

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