Maintaining Sensor Networks Perpetually Via Wireless Recharging Mobile Vehicles

Weifa Liang[†], Wenzheng Xu[‡†], Xiaojiang Ren[†], Xiaohua Jia[¶], and Xiaola Lin[‡]
† Research School of Computer Science, Australian National University, Canberra, ACT 0200, Australia
‡ School of Information Science and Technology, Sun Yat-Sen University, Guangzhou, 510006, P. R. China
¶ Department of Computer Science, City University of Hong Kong, Hong Kong
Email: wliang@cs.anu.edu.au, wenzheng.xu3@gmail.com, richard.rxj@anu.edu.au,
csjia@cityu.edu.hk, linxl@mail.sysu.edu.cn

***Abstract*—The emerging wireless energy transfer technology based on magnetic resonant coupling is a promising technology for wireless sensor networks as it can provide a controllable and perpetual energy source to sensors. In this paper we study the use of minimum number of wireless charging mobile vehicles to charge sensors in a sensor network so that none of the sensors runs out of its energy, subject to the energy capacity imposed on mobile vehicles, for which we first advocate an flexible on-demand wireless charging paradigm that decouples sensor energy charging scheduling from data routing protocols design. We then formulate an optimization problem of scheduling mobile vehicles to charge lifetime-critical sensors with an objective to minimize the number of mobile vehicles deployed, subject to the energy capacity constraint on each mobile vehicle. As the problem is NP-hard, we devise an approximation algorithm with a provable performance guarantee for it. We finally evaluate the performance of the proposed algorithm through experimental simulations. Experimental results demonstrate that the proposed algorithm is promising, and the solution obtained is fractional of the optimal.**

Index Terms–*rechargeable sensor networks; wireless energy transfer; charging time scheduling; approximation algorithms.*

I. Introduction

Sensors in conventional wireless sensor networks are mainly powered by energy-limited batteries. Due to the limited storage capacity of each battery, the operational time of such networks usually is limited. To prolong the network lifetime, extensive studies in the past decade have been conducted, which include batch deployments of sensors, harvesting energy from surrounding environments, etc [2], [8], [21]. In spite of these intensive efforts, network lifetime remains the performance bottleneck which perhaps is one main obstacle in the wide scale deployment of wireless sensor networks.

To prolong sensor network operations, one obvious solution is to replace expired batteries with new ones, however, for large-scale wireless sensor networks [13], [19], [20], [22], it is painstaking and very costly. Worst of all, in some applications such as monitoring dangerous and/or polluted areas and surveillance, it is almost impossible to adopt this approach. Alternatively, another approach is to compensate the expired sensors by dispatching a new batch of sensors to the region of dead sensors. By doing so may prolong the network lifetime. However, this is achieved at the expense of long-term environmental contaminations as most batteries are made with poisonous chemical materials. Contrary to these mentioned solutions, in the past several years, a new technique for environmentally friendly WSN deployments has been exploited, that is, sensors can be powered by energy harvested from their surroundings [3], [4], [8], [11]. Although energy harvesting is an ideal solution, its success remains very limited in practice as time-varying energy harvesting sources pose a great challenge. For example, in a solar harvesting system, statistics has shown that the differences of energy generating rates in shadowy, cloudy and sunny days can be up to three orders of magnitude [9].

Complementary to the energy harvesting technique, the recent breakthrough of a wireless energy transfer technology based on strongly coupled magnetic resonances has attracted scientists' attention [6], [7], which adds a new dimension to prolong the lifetime of sensor networks. By exploiting a novel technique called magnetic resonance, Kurs *et al.* [6] showed that the wireless energy transfer (transferring electric energy from one storage device to another without any plugs or wires) is not only efficient but also immune to its surrounding environment. This technique can potentially provide sensors with steady and high recharging rates.

A. Related work

With the advance in the efficient wireless energy transfer technology based on strongly magnetic resonances, wireless energy replenishment has been studied and adopted for lifetime prolongation of WSNs in literature. However, applying this technology to sensor networks is still in its infancy stage. Several studies have been conducted in the past few years, and most of these studies considered sensor energy recharging and data flow routing jointly. For example, Shi *et al.* [12] are the very first to conduct a theoretical study on the efficient usage of the wireless charging technique in sensor networks by employing a wireless charging vehicle to periodically charge sensors such that the network can operate perpetually. They formulated an optimization problem that maximizes the ratio of the vacation time of the vehicle over each charging cycle, under the assumption that data rates of sensors are given. They also developed a near-optimal solution for their problem with performance guarantee. They later extended their solution to general cases where multiple sensors within the vicinity of a mobile charger can be charged at the same time [15], or the mobile vehicle charges sensors and collects sensing data simultaneously along its tours [14], [16]. Xu *et al.* [17] considered the problem of scheduling k mobile chargers to replenish a set of to-be-charged sensors, such that the maximum time spent among the k chargers is minimized, for which they proposed constant approximation algorithms. Xu *et al.* [18] considered the problem of scheduling multiple mobile charging vehicles to maintain the perpetual operation of sensors in a rechargeable sensor network for a period of T, so that the total traveling distance of the mobile vehicles is minimized, for which they proposed an approximation algorithm with a guaranteed performance ratio. Ren *et al.* [10] recently presented an algorithm for scheduling a mobile charger to charge on-demand sensors under the mobile charger traveling distance constraint. Zhao *et al.* [24] proposed a joint design of energy replenishment and data gathering by exploiting sink mobility. They first identified the sensors to be recharged, and then found an optimal data gathering scheme such that the network utility can be maximized, while maintaining the perpetual operations of the network. However, these joint consideration of energy replenishment and data flow routing in literature may have limited applications, due to their unrealistic assumptions such as (i) the energy consumption rate and/or data generation rate does not change over time; (ii) the flow conservation at each sensor node is maintained; and (iii) reliable wireless communications among the nodes are always assumed. In contrast, in reality sensing data rates of sensors are usually closely related to specific applications of the sensor network. The flow conservation prevents data aggregation at intermediate nodes while data aggregation at sensor nodes can not only reduce data traffics but also bring node energy savings [5]. Also, it is well-known that wireless communication is notoriously unreliable [23], retransmissions at some nodes at some unexpected time intervals may lead to substantial energy consumption of the nodes. In this paper we advocate that sensor energy charging and sensing data routing should be considered separately. In other words, we decouple the energy charging process from the design of data flow routing protocols. The benefits by doing so include: (i) the proposed on-demand wireless charging schema can be applicable to sensor networks for different purposes, such as periodic monitoring, event detections, surveillance coverage, and so on. (ii) There is no need that the charging vehicles patrol the monitoring region periodically, instead, only is there such a need from sensors, the system then determines how many mobile vehicles to be deployed, this will lead to significant operational cost savings. (iii) The routing protocol designers can concentrate on routing protocol functionalities for specific applications, rather than taking energy recharging constraints into account.

B. Contributions

The main contributions of this paper are as follows. Unlike most existing studies that either focused on periodic recharging or joint considering energy replenishment and data routing protocols design, we first advocate a flexible on-demand charging paradigm that decouples the energy recharging process from the design of data flow routing. We then formulate a novel optimization problem of minimizing the number of mobile charging vehicles needed, subject to the energy capacity constraint on each mobile vehicle and we show that the problem is NP-hard. We also devise the very first approximation algorithm with a provable performance guarantee for the problem. We finally conduct extensive simulation experiments to evaluate the performance of the proposed algorithm, and experimental results demonstrate that the proposed algorithm is promising, and the solution delivered is fractional of the optimum.

The remainder of the paper is organized as follows. Section II introduces preliminaries, including the system model, notions, and the problem definition. Section III details a 5-approximation algorithm for finding optimal p closed tours, which will be used as a main subroutine of the proposed algorithm. Section IV proposes an approximation algorithm for the optimal number of mobile vehicles deployment problem. Section V evaluate the performance of the proposed algorithm, and Section VI concludes the paper.

II. PRELIMINARIES

In this section we introduce the system model, wireless energy charging paradigm, and the problem definition.

A. System Model

We consider a wireless sensor network $G_s = (V_s, E_s)$, where V_s is a set of sensors and a base station. There is an edge in E_s between any two sensors or each sensor and the base station if they are within the transmission range of each other. All sensing data will be relayed to the base station through multi-hop relays. Such a network is illustrated in Fig. 1. Assume that there is sufficient energy supplies to the base station, thus, there is no energy constraint on the base station. Each sensor $v_i \in V_s$ is powered by a rechargeable battery with energy capacity B_i.

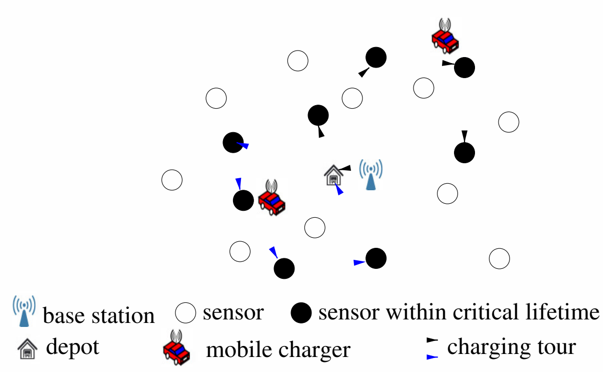


Fig. 1. A rechargeable sensor network

To maintain the network long-term operations, its sensors will be charged at certain time points by wireless charging mobile vehicles. We thus assume that there is a depot in the monitoring region, where there are a number of mobile vehicles available to meet sensor charging demands. Assume that each mobile vehicle with a full energy capacity IE has a charging rate μ and a constant traveling speed s. Let η be the energy consumption rate of each mobile vehicle on traveling per unit-length. All mobile vehicles will start from the depot when performing their charging duties and return to the depot after finishing their charging tours to recharge themselves and wait for their next round scheduling. Since the energy capacity IE of each mobile vehicle is limited, the total travel length and the number of to-be-charged sensors by each mobile vehicle will be constrained by its energy capacity IE.

The residual lifetime of each sensor $v_i \in V_s$ at time t is defined as $\gamma_i(t) = \frac{RE_i(t)}{\rho_i(t)}$, where $RE_i(t)$ and $\rho_i(t)$ are the residual energy and energy consumption rate of v_i at time t, respectively. The base station keeps a copy of the energy depletion rate $\rho_i(t)$ and the residual energy $RE_i(t)$ of each sensor $v_i \in V_s$. We assume that each sensor is able to monitor its residual energy $RE_i(t)$ and estimate its energy consumption rate $\rho_i(t)$ in the near future through predictions. Thus, each sensor can estimate its residual lifetime $\gamma_i(t)$. We also assume that the energy consumption rate of each sensor does not change within a recharging round or such minor changes can be neglected, as the duration of a recharging round usually is short (e.g. a few hours). But it is allowed to change at different rounds. Recall that for each sensor $v_i \in V$, there is a record of its energy consumption rate $\rho_i(t)$ at the base station, and this value is subject to be updated if the energy consumption profile of the sensor in the next time period experiences significantly changes. To accurately measure the energy consumption rate of each sensor, each sensor adopts a lightweight prediction technique to estimate its energy consumption rate in the near future, e.g., each sensor can make use of a linear regression, $\hat{\rho}_i(t) = \omega\rho_i(t-1)+(1-\omega)\hat{\rho}_i(t-1)$, where $\hat{\rho}_i$ is the estimation and ρ_i is the actual value at that momentLet $\theta > 0$ be a small given threshold, the energy consumption rate updating is as follows. For each sensor $v_i \in V_s$, if $|\hat{\rho}_i(t) - \hat{\rho}_i(t-1)| \le \theta$, no updating report from sensor v_i will be forwarded to the base station; otherwise, the updated energy consumption rate and its residual energy of v_i will be sent to the base station. The base station then performs the updating related to v_i accordingly.

B. Wireless energy recharging paradigm

We notice there is no such need that every sensor must be charged at each round. Also, sensor charging tours are not necessarily periodic, instead sensors should be charged in an on-demand fashion. The rationale behind this is that in some applications such as event detections, if there are no events happening in a monitoring area, sensors usually perform duty-cycling and run much longer than they are keeping at wake-up statuses. When an event does occur, the sensors around the event region will keep in wake-up statuses to capture the event and report their sensing results to the base station, while for the sensors being not in the event region, they continue maintaining their wakeup-and-sleep duty-cycling statuses, thus consuming much less energy. It can be seen from this case that not all sensors in the network need to be recharged in each energy recharging round, only the sensors in the regions that the event happened are needed to be recharged.

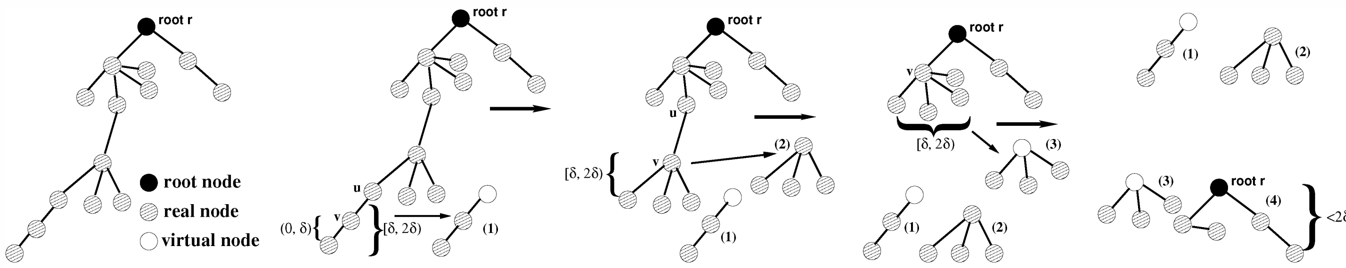
Let γ_{max} be the longest duration of a mobile vehicle tour for charging all sensors in the network. Clearly, $\gamma_{max} \le \frac{L_{TSP}}{s} + \frac{\min\{IE,\ \sum_{v_i \in V_s} B_i\}}{\mu}$ as all energy in one charging vehicle IE is used to replenish sensors or all the sensors in the network are required to be recharged in the worst scenario, where B_i is the battery capacity of sensor v_i, L_{TSP} is the length of a TSP tour including all sensors and the depot, and s is the travelling speed of each mobile vehicle. In other words, to ensure that none of the sensors fails due to its energy expiration, each sensor should be recharged when its residual lifetime is no greater than γ_{max}.

We say that a sensor v_i at time t is in its *critical lifetime interval* if $\gamma_{\max} \le \gamma_i(t) \le \alpha \cdot \gamma_{max}$ with a constant $\alpha \ge 1$. Following the definition of the critical lifetime interval, it can be seen that only the sensors within their critical lifetime intervals need to be recharged to avoid running out of their energy completely. Without loss of generality, in the rest of this paper, we assume that V is the set of sensors within their critical lifetime intervals that will be recharged in the next round. Clearly, $V \subseteq V_s$.

We propose a flexible on-demand wireless energy charging paradigm as follows. When the residual lifetime of a sensor falls in the pre-defined critical lifetime interval, the sensor will send an energy-recharging request to the base station for its energy replenishment, where the request contains its identity, the amount of the residual energy, and the energy consumption rate of the sensor. Once the base station receives the requests from sensors, it then performs a scheduling to dispatch a number of mobile vehicles to recharge the requested sensors in the network. Hence, the result of each scheduling consists of the number of wireless charging mobile vehicles needed, the closed tours for each vehicle, and the recharging duration at each sensor node along the tour for each of these vehicles. Finally, the mobile vehicles are dispatched from the depot to perform the charging task. We ignore sensor energy depletions per charging tour, as the amount of energy consumed by each sensor in this period is several orders of magnitude less than its energy capacity.

C. Problem definition

Given a sensor network $G_s = (V_s, E_s)$ consisting of sensors, one stationary base station, and a depot with multiple mobile vehicles, following the above wireless energy charging paradigm, assume that at a specific time point, the base station receives energy charging requests from the sensors within their critical lifetime intervals. The base station then starts a new round scheduling by dispatching a certain number of mobile vehicles to charge these sensors so that none of sensors will run out of its energy. Let V be the subset of sensors in G_s to-be-charged (within critical lifetime) in the next round ($V \subseteq V_s$), see Fig. 1. Assume that for each sensor $v_i \in V$, its energy consumption rate ρ_i does not change during each charging round (or such changes are marginal), and its residual energy



(a) A spanning tree T in G (b) Case one (c) Case two (d) Case three (e) obtained four subtrees

Fig. 2. An illustration of tree decompositions

RE_i is given (at the base station), *the optimal number of mobile vehicles deployment problem* is to find a scheduling of mobile vehicles to fully charge the sensors in V by providing a closed tour for each mobile vehicle such that the number of mobile vehicles deployed is minimized, subject to the energy capacity constraint of mobile vehicles IE. It is obvious that the problem is NP-hard by a polynomial reduction from the well known Traveling Salesman Problem (TSP).

In the following we define the p-traveling salesman problem (p-TSP tours), which will serve as a subroutine of the proposed algorithm for the optimal number of mobile vehicles deployment problem. Specifically, given a node and edge weighted metric graph $G = (V, E; h, w)$, a root node $r \in V$, and an integer $p \geq 1$, where $h : V \mapsto R^{\geq 0}$ and $w : E \mapsto R^{>0}$, *the p-optimal closed tour problem* in G is to find p node-disjoint closed tours covering all nodes in V except the root r that appears in each of the tours such that the maximum total cost among the p closed tours is minimized, where *the total cost* of a closed tour is the weighted sum of nodes and edges in it.

III. Algorithm for the p-optimal closed tour problem

In this section we devise a 5-approximation algorithm for the p-optimal closed tour problem in a node and edge weighted metric graph $G(V, E; h, w)$, which will be used as a subroutine for the problem of concern in this paper. As a special case of the p-optimal closed tour problem when $p = 1$ is the well-known TSP problem which is NP-hard, the p-optimal closed tour problem is NP-hard, too. In the following we start by introducing a popular technique transforming a tree into a closed tour in G. We then introduce a novel tree decomposition. We finally present an approximation algorithm for the problem.

A. A closed tour derived from a tree

We first introduce the technique of transforming a tree in G to a closed tour by the following lemma.

Lemma 1: Given a node and edge weighted metric graph $G = (V, E; h, w)$ with sets V and E of nodes and edges, $h : V \mapsto R^{\geq 0}$ and $w : E \mapsto R^{>0}$, and the edge weight follows the triangle inequality, let $T = (V, E_T; h, w)$ be a spanning tree of G rooted at r. Let C be the traveling salesman tour of G derived from T through performing the pre-order traversal on T and pruning, then *the total cost* of C, $WH(C)$, is no more twice the total cost of T, $WH(T)$, i.e., $WH(C) \leq 2WH(T) = 2(\sum_{v \in V} h(v) + \sum_{e \in E_T} w(e))$.

Proof: Let $H(X)$ be the weighted sum of nodes in X and $W(Y)$ be the weighted sum of edges in Y. As the weighted sum of the edges in C, $W(C)$, is no more than $2\sum_{e \in E_T} w(e)$, and the weighted sum of nodes in C, $H(C)$, is the same as the one in T. Thus, the total cost of C is $WH(C) = W(C) + H(C) \leq 2W(T) + H(T) \leq 2(W(T) + H(T)) = 2WH(T)$. ■

B. Tree decomposition

Given a metric graph $G = (V, E; h, w)$, let $T = (V, E_T; h, w)$ be a spanning tree in G rooted at node r, let $\delta \geq \max_{v \in V}\{h(v), 2w(v, r)\}$ be a given value. We decompose the tree into a set of subtrees such that the total cost of each subtree is no more than 2δ as follows.

Let (u, v) be a tree edge in T, where u is the parent of v and v is a child of u. Also, let T_v be a subtree of T rooted at node v. We perform a depth-first search on T starting from root r until the total cost of the leftover tree $WH(T_r) < 2\delta$. Fig. 2 demonstrates an example of the tree decomposition procedure.

Assume that node v is the node being visited at this moment, we distinguish the following three cases. (1) Case one: if $WH(T_v) < \delta$, $WH(T_v) + w(u, v) \geq \delta$ but less than 2δ, then a new tree $T_v \cup \{(v, u')\}$ is created with a *virtual node* u' with $h(u') = 0$. Split the subtree $T_v \cup \{(v, u')\}$ from the original tree, see Fig. 2 (b). (2) Case two: if $\delta \leq WH(T_v) < 2\delta$, split the subtree T_v from the original tree and remove edge (u, v), see Fig. 2 (c). (3) Case three: let $v_1^c, v_2^c, \ldots, v_k^c$ be the k children of v. Let l be the maximum children index so that $\delta \leq \sum_{j=1}^{l}(WH(T_{v_j^c}) + w(v_j^c, v)) < 2\delta$ with $1 \leq l \leq k$, then, a new subtree, $\cup_{j=1}^{l}(T_{v_j^c} \cup \{(v_j^c, v')\})$ rooted at the virtual node v', is created, which consists of these subtrees with $h(v') = 0$. Split this subtree from the original tree, see Fig. 2 (d). As a result, a set of subtrees is obtained, see Fig. 2 (e). The number of subtrees is bounded by the following lemma.

Lemma 2: Given a spanning tree $T = (V, E_T; h, w)$ of a graph $G = (V, E; h, w)$ with total cost $WH(T)$ and a value $\delta \geq \max_{v \in V}\{2w(r, v), h(v)\}$, T can be decomposed into p subtrees $T_1, T_2, \ldots, T_p$ with $WH(T_i) < 2\delta$ by the proposed tree-decomposition technique, $1 \leq i \leq p$. Then, $p \leq \lfloor \frac{WH(T)}{\delta} \rfloor$.

Proof: Following the tree decomposition on T, subtrees with the total cost in $[\delta, 2\delta)$ are split away from T until the total cost of the leftover tree is less than 2δ. Suppose that the split trees are $T_1, T_2, \ldots, T_p$ with $p \geq 2$. From the subtree construction, we know that $\delta \leq WH(T_i) < 2\delta$ for i with $1 \leq i \leq p-1$. The only subtree with the total cost less than δ is T_p. Note that prior to splitting T_{p-1}, the total cost of the remaining tree is at least 2δ. Therefore, the average total cost of T_{p-1} and T_p is no less than δ. In other words, the average total cost of all T_i is at least δ. Thus, $p \cdot \delta \leq WH(T)$, i.e., $p \leq \frac{WH(T)}{\delta}$. Since p is an integer, $p \leq \lfloor \frac{WH(T)}{\delta} \rfloor$. ■

C. Algorithm for finding p-optimal closed tours

Given a metric graph $G = (V, E; h, w)$ with root r and a positive integer p, we now devise an approximation algorithm for the p-optimal closed tour problem in G as follows. Let T be an MST of G rooted at r. The basic idea of the proposed algorithm is that we first perform a tree decomposition on tree T with $\delta = \max_{v \in V}\{WH(T)/p,\ 2w(v,r) + h(v)\}$ and we later show that δ is a lower bound on the optimal cost of the p-optimal closed tour problem. As a result, p' subtrees are derived from such decomposition, and p' closed tours are then derived from the p' subtrees. We then show that $p' \leq p$ and the maximum total cost of any closed tour among the p' closed tours is no more than 5δ.

Specifically, T is decomposed into no more than p' edge-disjoint subtrees, excepting the root node r which appears in one of these subtrees. Denote $T_1, T_2, \ldots, T_{p'}$ be the p' trees obtained by decomposing tree T. It can be observed that each T_i contains at least one real node and at most one virtual node, where a node v is a *real node* if $h(v) \neq 0$; otherwise, it is a virtual node. As a result, a forest $\mathcal{F}$ consisting of all the trees is found through the tree decomposition, and the number of trees in $\mathcal{F}$ is $p' \leq \lfloor WH(T)/\delta \rfloor$ and the total cost of each subtree is no more than 2δ by Lemma 2.

For each $T_i \in \mathcal{F}$, if it does not contain the root r, then, a tree $T'_i = T_i \cup \{(v_i, r)\}$ rooted at r is obtained by including node r and a tree edge (v_i, r) into T_i, where node v_i is a node in T_i and $w(v_i, r) = \min_{v \in T_i}\{w(v, r)\}$. The total cost of T'_i, $WH(T'_i)$, is $WH(T'_i) = WH(T_i) + w(v_i, r) \leq 2\delta + w(v_i, r) \leq 2.5\delta$ as $w(v_i, r) \leq \delta/2$. Otherwise (T_i contains node r), $T'_i = T_i$ and $WH(T'_i) = WH(T_i) \leq 2\delta$. We thus obtain a forest $\mathcal{F}' = \{T'_1, T'_2, \ldots, T'_{p'}\}$. From the trees in $\mathcal{F}'$, p' edge-disjoint closed tours with root r can be derived. Let $\mathcal{C}' = \{C'_1, C'_2, \ldots, C'_{p'}\}$ be the set of p' closed tours obtained by transforming each tree in $\mathcal{F}'$ into a closed tour. For each C'_i, we have that $WH(C'_i) \leq 2 \cdot WH(T'_i) \leq 5\delta$ by Lemma 1. As there are some C'_is containing virtual nodes that are not part of a feasible solution to the problem, a feasible solution can be obtained through a minor modification to $\mathcal{C}'$. That is, for each C'_i, if it contains a virtual node (as each C'_i contains at most one virtual node), a closed tour C_i with a less total cost than C'_i is obtained by removing the virtual node and two edges incident to the node from C'_i through short cutting, then $WH(C_i) \leq WH(C'_i)$ as the edge weight follows the triangle inequality property. Otherwise, $C_i = C'_i$. Clearly, each of the p' closed tours $C_1, C_2, \ldots, C_{p'}$ roots at r. The detailed algorithm is described in `Algorithm` 1.

Algorithm 1 *finding closed tours rooted at r with each having the bounded total cost*

Input: A metric graph $G = (V, E; h, w)$, a root $r \in V$, and a given value $\delta \geq \max_{v \in V}\{h(v), 2w(v, r)\}$.

Output: a set of closed tours covering all nodes in V so that the total cost of each tour is no more than 5δ.

let T be the MST of G and $WH(T)$ the total cost of T;

Let $\mathcal{F} = \{T_1, T_2, \ldots, T_{p'}\}$ be the forest obtained by tree decomposition on T;

Let $\mathcal{F}' = \{T'_1, T'_2, \ldots, T'_{p'}\}$ be a forest, where $T'_1 = T_i \cup \{(r, v_i)\}$ is derived by adding root r and an edge with the minimum edge weight between a node v_i in T_i and r if r is not in T_i; otherwise $T'_i = T_i$;

Let $\mathcal{C}' = \{C'_1, C'_2, \ldots, C'_{p'}\}$ be a set of p' closed tours, where C'_i is derived from T'_i;

Let $\mathcal{C} = \{C_1, C_2, \ldots, C_{p'}\}$ be a set of closed tours, where C_i is derived by removing the virtual node from $C'_i \in \mathcal{C}'$ if it does contain a virtual node. Otherwise, $C_i = C'_i$.

return $\mathcal{C}$.

D. Algorithm analysis

We show that `Algorithm` 1 delivers a 5-approximate solution by the following theorem.

Theorem 1: Given a metric graph $G = (V, E; h, w)$ and an integer $p \geq 1$, there is a 5-approximation algorithm for finding p-optimal closed tours. The time complexity of the proposed algorithm is $O(|V|^2)$.

Proof: In the following we first show that `Algorithm` 1 delivers a feasible solution to the p-optimal closed tour problem. We then show that the total cost of each closed tour in the solution is no more than 5δ. We thirdly show that δ ($= \max_{v \in V}\{WH(T)/p,\ 2w(v, r) + h(v)\}$) is a lower bound on the optimal cost of the problem. We finally analyze the time complexity of `Algorithm` 1.

We first show that `Algorithm` 1 delivers a feasible solution. Let T be the MST of G. Since $\delta = \max_{v \in V}\{WH(T)/p, 2w(v, r) + h(v)\}$, clearly $\delta \geq \max_{v \in V}\{2w(v, r), h(v)\}$. A solution $\mathcal{C}$ which consists of p' closed tours rooted at r can be obtained by applying `Algorithm` 1 on T and $p' \leq \lfloor WH(T)/\delta \rfloor \leq WH(T)/\delta = \frac{WH(T)}{\max_{v \in V}\{WH(T)/p,\ 2w(v,r)+h(v)\}} \leq \frac{WH(T)}{WH(T)/p} = p$ by Lemma 2. Thus, $\mathcal{C}$ is a feasible solution.

We then show that the total cost of each closed tour in $\mathcal{C}$ is no more than 5δ. As each $C_i \in \mathcal{C}$ is derived from a $C'_i \in \mathcal{C}'$, we have $WH(C_i) \leq WH(C'_i) \leq 2\ WH(T'_i) \leq 2 \cdot 2.5\delta = 5\delta$ by Lemma 1.

We thirdly prove that δ is a lower bound on the optimal cost of the problem. Given a node and edge weighted metric graph $G = (V, E; h, w)$ with root r, an integer $p \geq 1$, partition the nodes in V into p disjoint subsets $X_1, X_2, \ldots, X_p$, and let C_j be the closed tour containing all nodes in X_j and the root r. The optimal partitioning is a partitioning such that the maximum value $\max_{1 \leq j \leq p}\{WH(C_j)\}$ is minimized. Let OPT be the total cost of the maximum closed tour in the optimal solution.

We show that $\delta \le OPT$ as follows.

Let $C_1^*, C_2^*, \ldots, C_p^*$ be the p closed tours in the optimal solution with the shared root r. Then, $WH(C_i^*) \le OPT$. Let e_i be the maximum weighted edge in C_i^*. Then, a tree $T' = \cup_{i=1}^p C_i^* \setminus \cup_{i=1}^p \{e_i\}$ rooted at r can be obtained by removing e_i from each tour C_i^*. We then have $WH(T') \le \sum_{i=1}^p WH(C_i^*) \le p{\cdot}OPT$. It can be seen that T' is a spanning tree in G. Since T is an MST of G, then $WH(T) \le WH(T')$. We thus have $WH(T)/p \le WH(T')/p \le OPT$.

On the other hand, each node $v \in V$ must be contained by one closed tour C_i^* in the optimal solution. Since tour C_i^* contains node v and the depot r, then the total cost of C_i^*, $WH(C_i^*)$, is at least $2w(v,r)+h(v)$, Thus, $2w(v,r)+h(v) \le WH(C_i^*) \le OPT, \forall v \in V$. Therefore, we have

$$\delta = \max_{v \in V}\{WH(T)/p,\ 2w(v,r)+h(v)\} \le OPT. \qquad (1)$$

We finally analyze the time complexity of `Algorithm 1`. Following the algorithm, the MST T of G can be found in $O(|V|^2)$ time. The tree decomposition and the formation of $\mathcal{F}'$ can be done in $O(|V|)$ time. For each $C_i' \in \mathcal{C}'$, the corresponding C_i can be found in $O(|E_i'|)$ time, where E_i' is the set of edges in C_i'. We also know that $E_i' \cap E_j' = \emptyset$ if $i \ne j$. As C_i' is derived from tree $T_i' \in \mathcal{F}'$, $|E_i'| \le 2|E(T_i')|$. Since $\sum_{i=1}^{p'} |E(T_i')| \le |V|-1$, $\sum_{i=1}^{p'} |E_i'| \le \sum_{i=1}^{p'} 2|E(T_i')| \le 2|V|$. Thus, `Algorithm 1` takes $O(|V|^2)$ time. ■

IV. Approximation Algorithm for the Optimal Number of Mobile Vehicles Deployment Problem

In this section we provide an approximation algorithm for the optimal number of vehicles deployment problem. As each mobile vehicle consumes energy in traveling and charging sensors per tour, the total amount of energy consumed by a mobile vehicle per tour is bounded by its energy capacity IE.

A. Algorithm

The basic idea of the proposed approximation algorithm is to reduce the optimal number of mobile vehicles deployment problem into a p closed tour problem with the total cost of each closed tour being bounded. A solution to the latter will return a solution to the former as follows.

Recall that we assume that the base station knows both the residual energy RE_i and the energy consumption rate ρ_i of each sensor $v_i \in V$, and μ is the wireless charging rate of a mobile vehicle. Assume that there are sufficient numbers of fully charged mobile vehicles available in the depot. Then, a mobile vehicle takes $\tau_i = \frac{B_i - RE_i}{\mu}$ time to charge sensor v_i to its full capacity B_i when it approaches sensor v_i. We thus construct a node and edge weighted metric graph $G = (V, E; h, w)$, where V is the set of sensors to be charged in this round. There is an edge in E between any two to-be-charged sensor nodes. For each edge $(u,v) \in E$, its weight is $w(u,v) = \eta \cdot d(u,v)$ which is the amount of energy consumed by a mobile vehicle travelling along the edge, where η is the energy consumption rate of the mobile vehicle traveling per unit length and $d(u,v)$ is the Euclidean distance between nodes u and v. For each node $v_i \in V$, its weight $h(v_i)$ $(= B_i - RE_i = \mu{\cdot}\tau_i)$ is the amount of energy needed to charge sensor v_i from its current energy level to its full capacity. We assume that $IE \ge \max_{v \in V}\{2w(v,r)+h(v)\}$; otherwise, there is no feasible solution. The detailed algorithm is described in `Algorithm 2`. We refer to this algorithm as `NMV`.

Algorithm 2 *finding the optimal number of mobile vehicles and their closed tours (NMV)*

Input: A metric graph $G = (V, E; h, w)$, a root r, and IE.
Output: p-node-disjoint closed r-rooted tours $C_1, C_2, \ldots, C_p$ covering all nodes in V such that $WH(C_i) \le IE$.

```
Let T be an MST of G and WH(T) the total cost of T;
if IE ≥ 2 · WH(T) then
    One mobile vehicle suffices by Lemma 1; EXIT;
end if;
A ← max_{v∈V} {2w(v,r) + h(v)};
if IE/5 ≥ A then
    δ ← IE/5; /* δ is the minimum total cost of each subtree*/
else
    δ ← (ξ−1)A/4, where ξ = IE/A;
end if;
Let C = {C_1, C_2, ..., C_p} be the solution by applying Algorithm 1 for the tree decomposition on T with δ;
```

B. Algorithm analysis

Theorem 2: Given a metric graph $G = (V, E; h, w)$ and the full energy capacity IE of each mobile vehicle, there is an approximation algorithm for the optimal number of mobile vehicles deployment problem in G, and the time complexity of the proposed algorithm is $O(|V|^2)$.

Proof: We first show that `Algorithm 2` can deliver a feasible solution $\mathcal{C} = \{C_1, C_2, \ldots, C_p\}$ to the optimal number of mobile vehicles deployment problem. Let $A = \max_{v \in V}\{2w(v,r) + h(v)\}$. We distinguish into three cases. (1) Case one: if $IE \ge 2 \cdot WH(T)$, then there is a closed tour C including all nodes in V derived from T and the total cost of C, $WH(C)$ $(\le 2 \cdot WH(T) \le IE$ by Lemma 1), is no more than the energy capacity of a mobile vehicle IE. Hence one mobile vehicle suffices for charging all nodes in V. (2) Case two: if $IE/5 \ge A$ with $A = \max_{v \in V}\{2w(v,r)+h(v)\}$, then $\delta = IE/5$, and the total cost of each closed tour in the solution is no more than $5\delta = IE$ by Theorem 1. (3) Case three ($IE/5 < A \le IE$): following `Algorithm 2`, we have $\delta = \frac{(\xi-1)A}{4}$ with $\xi = \frac{IE}{A}$. Clearly, $1 < \xi < 5$ as $A \le IE < 5A$. Also, $w(v,r) \le A/2$ for any node $v \in V$. Then, the total cost of each closed tour C in the solution is $WH(C) \le 2 \cdot (2\delta + w(v_0, r)) \le 4 \cdot \frac{(\xi-1)A}{4} + 2w(v_0,r) \le (\xi-1)A + A = \xi{\cdot}A = IE$, where $w(v_0, r) = \min_{v \in T_i}\{w(v,r)\}$ and T_i is the tree from which C is derived. Thus, the solution is a feasible solution of the problem.

We then analyze the approximation ratio of the proposed algorithm. Assume that the the optimal number of mobile vehicles needed is p_{min}. With a similar discussion in Theorem 1, a lower bound on the value of p_{min} is

$$p_{min} \geq \lceil WH(T)/IE \rceil. \quad (2)$$

Let p be the number of vehicles delivered by the proposed algorithm. We show the approximation ratio by the following three cases. (1) When $IE \geq 2 \cdot WH(T)$, only one mobile vehicle suffices, and this is an optimal solution. (2) When $IE/5 \geq A$, we have $\delta = IE/5$. Then, $\frac{p}{p_{min}} \leq \frac{\lfloor WH(T)/\delta \rfloor}{\lceil WH(T)/IE \rceil} \leq \frac{WH(T)/\delta}{WH(T)/IE} = IE/\delta = 5$ by Lemma 2. (3) Otherwise ($IE/5 < A$), we have $\delta = \frac{(\xi-1)A}{4}$ with $\xi = \frac{IE}{A}$ and $1 < \xi < 5$. Then, $\frac{p}{p_{min}} \leq \frac{\lfloor WH(T)/\delta \rfloor}{\lceil WH(T)/IE \rceil} \leq \frac{WH(T)/\delta}{WH(T)/IE} = \frac{4 \cdot IE}{IE-A} = \frac{4}{1-1/\xi}$. Since $1 < \xi < 5$, the approximation ratio of the approximation algorithm is no more than 16/3 when $\xi \geq 4$; 6 when $\xi \geq 3$; 8 when $\xi \geq 2$. Notice that the approximation ratio is $4(1 + 1/\epsilon)$ when $\xi = 1 + \epsilon$ with $0 < \epsilon < 1$, however, in realistic rechargeable sensor networks, it is unlikely that this case will occur; otherwise, a mobile charger with its full-energy capacity can only charge a very few sensors per tour, thus the number of mobile chargers deployed is proportional to the number of sensors to be charged. On the other hand, it is well accepted that in realistic sensor networks $IE >> A$, thus, the number of mobile chargers needed is much less than the numbers of sensors, and the approximation ratio of the proposed algorithm usually is a small value no more than 5.

Since the algorithm invokes `Algorithm` 1 only once, its time complexity analysis is $O(|V|^2)$ by Theorem 1. ■

V. Performance Evaluation

In this section, we evaluate the performance of the proposed approximation algorithm through experimental simulations. We also investigate the impact of several important parameters on algorithm performance including network size n, energy consumption rates of sensors, the variance of energy consumption rates, the energy capacity IE of mobile vehicles, and the critical time interval parameter α.

A. Simulation environment

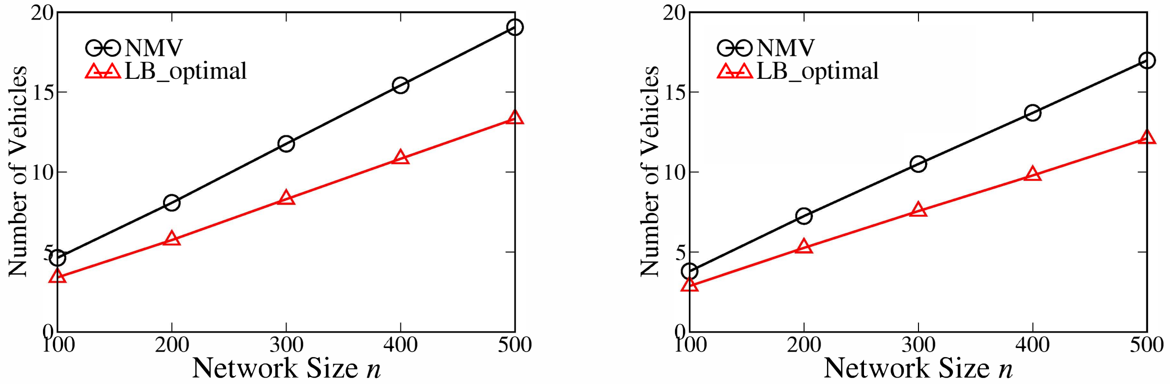
We assume that a wireless rechargeable sensor network consists of from 100 to 500 sensors randomly deployed in a $500m \times 500m$ square area. The battery capacity B_i of each sensor v_i is assumed to be 10.8 $kiloJoules$ (kJ), by referring to a regular $NiMH$ battery [12]. There is a base station located at the center of the square. There is a depot of mobile vehicles which is located at one of the four corners of the square. Each mobile vehicle with energy capacity IE from 100 kJ to 500 kJ travels at a constant speed of $s = 5$ m/s and its energy consumption rate on traveling is $\eta = 30$ J/m, while its energy charging rate is $\mu = 5$ $Watts$ [6]. The default value of the parameter α is 2.

We consider two different distributions of energy consumption rates: the *linear distribution* and the *random distribution.* In the linear distribution, the energy consumption rate ρ_i of sensor v_i is proportional to its distance to the base station. The nearest sensors to the base station have the maximum energy consumption rates ρ_{max} and the farthest sensors have the minimum energy consumption rates ρ_{min}. While in the random distribution, the energy consumption rate ρ_i of each sensor $v_i \in V_s$ is randomly chosen from an interval $[\rho_{min}, \rho_{max}]$, where $\rho_{min} = 1$ mJ/s and $\rho_{max} = 10$ mJ/s. The linear distribution models the energy consumptions of sensors in most applications of WSNs, where sensor energy is mainly consumed on data transmission. Sensors close to the base station must relay sensing data for other remote sensors, thus consuming their energy much faster. Furthermore, by adjusting the ratio of ρ_{max} to ρ_{min}, it can model data aggregations at relay sensors, i.e., a smaller ratio $\frac{\rho_{max}}{\rho_{min}}$ implies higher data aggregation. On the other hand, the random distribution captures the energy consumptions of heterogeneous sensors. For example, camera sensors in a multimedia sensor network consume plenty of their energy on image processing [1]. Thus, the energy consumption rates of sensors in such sensor networks is not closely correlated with the distance between the sensor and the base station. We further assume that the energy charging rate μ of each mobile vehicle is several orders of magnitude of the energy depletion rate of any sensor, i.e., $\mu >> \max_{v_i \in V}\{\rho_i\}$. A fully charged sensor can survive from 10 days upto 4 months. We put one year as our monitoring period of the sensor network. Each value in figures is the mean of the results by applying each mentioned algorithm to 20 different network topologies of the same network size. To evaluate the performance of the proposed algorithm, we will use a lower bound $LB\_optimal$ on the optimal number of mobile vehicles as the performance benchmark of the proposed algorithm, i.e. $LB\_optimal = \lceil WH(T)/IE \rceil$ by Eq. (2), where $WH(T)$ is the total cost of the MST T of the metric graph G that derives from the to-be-charged sensors in G_s and IE is the energy capacity of each mobile vehicle.

B. Performance evaluation of the proposed algorithm

In this subsection we evaluate the performance of the proposed algorithm. We first evaluate the performance of algorithm `NMV` by varying the network size from 100 to 500 sensors, assuming that energy consumption rates follow the random and linear distributions, respectively. Fig. 3(a) plots the performance curves of algorithm `NMV` and the lower bound on the optimal number of mobile vehicles needed, from which it can be seen that the solution delivered by algorithm `NMV` is fractional of the optimal. In other words, the number of mobile vehicles obtained by algorithm `NMV` is around 40% more than the lower bound. Fig. 3(b) indicates that under both linear and random distributions of energy consumption rates, algorithm `NMV` behaves similarly. Thus, in the following we focus only on the performance evaluation of the proposed algorithm under the random distribution of energy consumption rates.

We then investigate the impact of the energy capacity of mobile vehicles IE on the performance of algorithm `NMV` by varying IE from 100 kJ to 500 kJ. Fig. 4 shows that with the growth of the energy capacity IE, the number of mobile vehicles delivered by algorithm `NMV` decreases, and the gap between the solution delivered by the algorithm and the lower



(a) random distribution

(b) linear distribution

Fig. 3. Performance of the proposed approximation algorithm NMV by varying network size under different distributions of energy consumption rates when $IE = 100\ kJ$, $\rho_{min} = 1\ mJ/s$, and $\rho_{max} = 10\ mJ/s$.

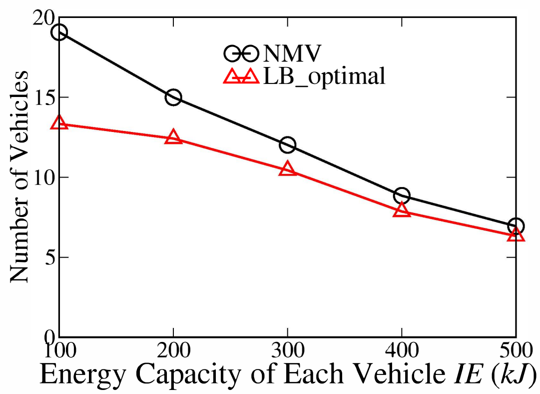


Fig. 4. Performance of algorithm NMV by varying energy capacity IE when $n = 500$, $\rho_{min} = 1\ mJ/s$, and $\rho_{max} = 10\ mJ/s$.

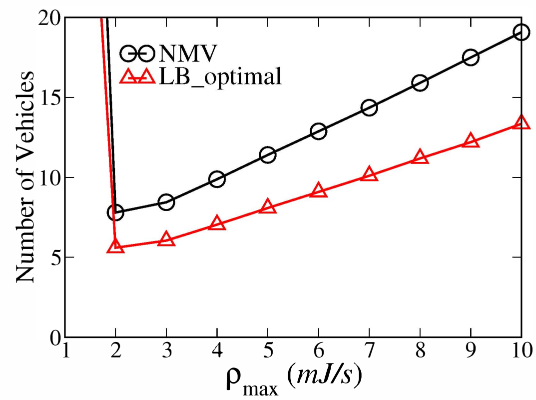
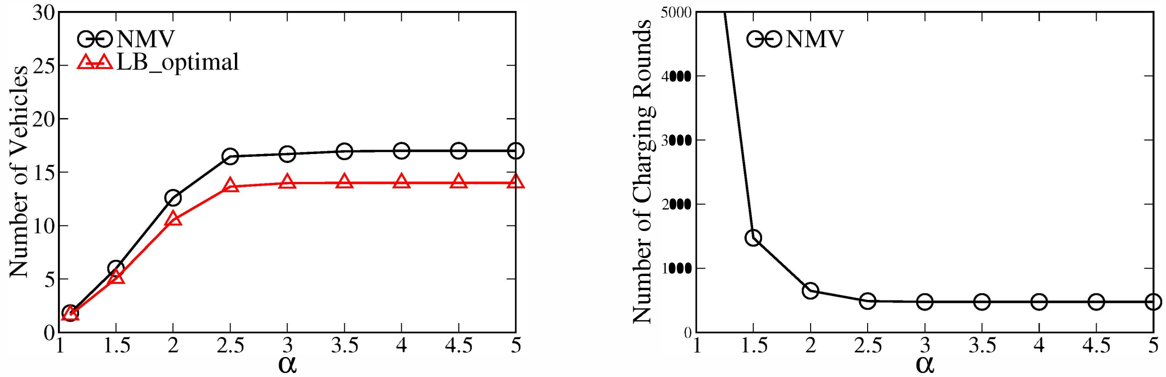


Fig. 5. Performance of algorithm NMV by varying the maximum energy consumption rate ρ_{max} when $n = 500$, $IE = 100\ kJ$, and $\rho_{min} = 1\ mJ/s$.

bound on the optimal one becomes smaller and smaller, which implies that the performance of algorithm NMV is near-optimal.

We thirdly study the variance among energy consumption rates of sensors on the performance of algorithm NMV by varying ρ_{max} from $1\ mJ/s$ to $10\ mJ/s$ while keeping ρ_{min} fixed at $1\ mJ/s$. Fig. 5 indicates that the number of mobile vehicle delivered by algorithm NMV deceases, followed by going up slowly. The rationale behind is that when the variance among the energy consumption rates is quite low (i.e., the gap between ρ_{max} and ρ_{min} is small), the solution delivered by algorithm NMV will include almost all sensors in each charging round, thus, a large number of mobile vehicles are required to charge these sensors. With the increase on the variance, the number of sensors to be charged in each charging round decreases significantly. On the other hand, when the maximum energy consumption rate ρ_{max} is large, the average energy depletion rate among the sensors becomes faster, the solution by algorithm NMV includes more sensors per charging round as more sensors are within their critical lifetimes, e.g., each sensor $v_i \in V_s$ at time t with residual lifetime $\gamma_i(t) = \frac{RE_i(t)}{\rho_i(t)}$ is more likely in the critical lifetime interval $[\gamma_{max},\ \alpha{\cdot}\gamma_{max}]$, which can increase the number of vehicles deployed by algorithm NMV.

We finally evaluate the impact of critical time interval parameter α on the performance of the proposed algorithms by varying the value of α from 1 to 5. A smaller α implies that more frequent scheduling is needed, and fewer number of mobile vehicles are employed per charging round. With the growth of α, more and more sensors will be included in V, and more sensors will be charged by mobile vehicles per charging round. Fig. 6(a) implies that with the growth of α, more vehicles delivered by algorithm NMV are needed in each charging round as more sensor nodes fall in the defined critical lifetime interval. However, it can also be seen that no further more vehicles are required when the value of α is larger than 3, since all sensors in this extreme case will be charged in each charging round. On the other hand, Fig. 6(b) implies that the number of charging rounds for the entire monitoring period will decrease with the growth of α.



(a) the number of vehicles deployed per charging round

(b) the number of charging rounds for the entire monitoring period

Fig. 6. Performance of algorithm NMV by varying α when $n = 500$, $IE = 200\ kJ$, $\rho_{min} = 50\ mJ/s$, and $\rho_{max} = 100\ mJ/s$.

VI. Conclusion

In this paper we studied the use of minimum number of wireless mobile vehicles to charge sensors in a wireless rechargeable sensor network so that none of the sensors will run out of its energy, subject to the energy capacity constraint imposed one each mobile vehicle. We first proposed a new wireless energy charging paradigm. We then formulated the optimal number of mobile vehicles deployment problem. Since the problem is NP-hard, we instead devised an approximation algorithm for it with a provable performance guarantee. To evaluate the performance of the proposed algorithm, we finally conducted extensive experiments by simulations. Experimental results demonstrate that the proposed algorithm is promising, and the solution obtained is fractional of the optimal.

Acknowledgment

This work is partially supported by a research grant funded by the Actew/ActewAGL Endowment Fund, the ACT Government of Australia. The research of Wenzheng Xu is also partially supported by China Scholarship Council (CSC).

References

[1] I. F. Akyildiz, T. Melodia, and K. R. Chowdury. Wireless multimedia sensor networks: a survey. *IEEE Wireless Communications*, Vol.14, pp.32-39, 2007.

[2] I. F. Akyildiz, W. Su, Y. Sankarasubramaniam, and E. Cayirci. Wireless sensor networks: a survey. *Computer Networks*, Vol.38, pp.393-422, 2002.

[3] X. Jiang, J. Polastre, and D. Culler. Perpetual environmentally powered sensor networks. *Proc. of IPSN*, IEEE, 2005.

[4] A. Kansal, J. Hsu, S. Zahedi, and M. B. Srivastava. Power management in energy harvesting sensor networks. *ACM Trans. Embedded Computing Systems*, Vol.6, 2007.

[5] B. Krishnamachari, D. Estrin, and S. Wicker. Impact of data aggregation in wireless sensor networks. *Proc. of ICDCS Workshops*, IEEE, 2002.

[6] A. Kurs, A. Karalis, R. Moffatt, J. D. Joannopoulos, P. Fisher, and M. Soljacic. Wireless power transfer via strongly coupled magnetic resonances. *Science*, Vol.317, pp.83-86, 2007.

[7] A. Kurs, R. Moffatt, and M. Soljacic. Simultaneous mid-range power transfer to multiple devices. *Applied Physics Letters*, Vol.96, pp.044102-044102-3, 2010.

[8] W. Liang, X. Ren, X. Jia, and X. Xu. Monitoring quality maximization through fair rate allocation in harvesting sensor networks. *IEEE Transactions on Parallel and Distributed System*. Vol. 24, pp.1827–1840, 2013.

[9] M. Rahimi, H. Shah, G. Sukhatme, J. Heideman, and D. Estrin. Studying the feasibility of energy harvesting in a mobile sensor network. *Proc. of ICRA*, IEEE, 2003.

[10] X. Ren, W. Liang, and W. Xu. Maximizing charging throughput in rechargeable sensor networks. To appear in *Proc of 23rd International Conference on Computer Communications and Networks* (ICCCN), IEEE, 2014.

[11] X. Ren, W. Liang, and W. Xu. Use of a mobile sink for maximizing data collection in energy harvesting sensor networks. *Proc of 42nd International Conference on Parallel Processing* (ICPP), IEEE, 2013.

[12] Y. Shi, L. Xie, Y. T. Hou, and H. D. Sherali. On renewable sensor networks with wireless energy transfer. *Proc. of INFOCOM*, IEEE, 2011.

[13] B. Tong, G. Wang, W. Zhang, and C. Wang. Node reclamation and replacement for long-lived sensor networks. *IEEE Trans. Parallel and Distributed Syst.*, Vol.22, pp.1550-1563, 2011.

[14] L. Xie, Y. Shi, Y. T. Hou, W. Lou, and H. D. Sherali. On traveling path and related problems for a mobile station in a rechargeable sensor network. *Proc. of MobiHoc*, ACM, 2013.

[15] L. Xie, Y. Shi, Y. T. Hou, W. Lou, H. D. Sherali, and S. F. Midkiff. On renewable sensor networks with wireless energy transfer: the multi-node case. *Proc. of SECON*, IEEE, 2012.

[16] L. Xie, Y. Shi, Y. T. Hou, W. Lou, H. D. Sherali, and S. F. Midkiff. Bundling mobile base station and wireless energy transfer: modeling and optimization. *Proc. of INFOCOM*, IEEE, 2013.

[17] W. Xu, W. Liang, and X. Lin. Approximation algorithms for min-max cycle cover problems. *IEEE Transactions on Computers*, preprint, 2013.

[18] W. Xu, W. Liang, X. Lin, G. Mao, and X. Ren. Towards perpetual sensor networks vis deploying multiple mobile wireless chargers. To appear in *Proc of 43nd International Conference on Parallel Processing* (ICPP), IEEE, 2014.

[19] X. Xu and W. Liang. Placing optimal number of sinks in sensor networks for network lifetime maximization. *Proc. of ICC*, IEEE, 2011.

[20] Z. Xu, W. Liang, and Y. Xu. Network lifetime maximization in delay-tolerant sensor networks with a mobile sink. *Proc. of DCOSS*, IEEE, 2012.

[21] J. Yick, B. Mukherjee, and D. Ghosal. Wireless sensor network survey. *Computer Networks*, Vol.52, pp.2292-2330, 2008.

[22] Z. Yuan, R. Tan, G. Xing, C. Lu, Y. Chen, and J. Wang. Fast sensor placement algorithms for fusion-based target detection. *Proc. of RTSS*, IEEE, 2008.

[23] J. Zhao and R. Govindan. Understanding packet delivery performance in dense wireless sensor networks. *Proc. of SenSys*, ACM, 2003.

[24] M. Zhao, J. Li, and Y. Yang. Joint mobile energy replenishment and data gathering in wireless rechargeable sensor networks. *Proc. of 23rd Intl Teletraffic Congress* (ITC), IEEE, 2011.