On View Consistency in Multi-Server Distributed Virtual Environments
Supplementary Material

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Theorem 1: The process of dividing the virtual cells $\{c_i\}$ among the servers (referred to as the *VCtoS problem*) as shown in Eq. (12) of the paper is NP-hard.

Proof: Let us consider the simplified version of the VCtoS problem by neglecting the local VI, i.e., assuming $L_i^m = 0$. Then, Eq. (12) of the paper becomes:

$$\begin{aligned} min \quad & \sum_{i=1}^{N_S}\sum_{j=1}^{N_S}\sum_{m=1}^{N_S}\sum_{n=1}^{N_S} X_{im}X_{jn}T_{m,n}\mu(\mathrm{P}_i,\mathrm{P}_j) \\ s.t. \quad & \textstyle\sum_{j=1}^{N_S} X_{ij} = 1, i \in \{1,\ldots,N_S\} \\ & \textstyle\sum_{i=1}^{N_S} X_{ij} = 1, j \in \{1,\ldots,N_S\} \\ & X_{ij} \in \{0,1\} \end{aligned} \tag{1}$$

where $\mu(\mathrm{P}_i,\mathrm{P}_j) = \sum_{c_k\in \mathrm{P}_i, c_l \in \mathrm{P}_j} \sigma(c_k,c_l)$.

We now transform this simplified VCtoS problem into the quadratic assignment (QAP) problem, which is known to be NP-hard [SG76], as follows. Given n facilities denoted by set NF, n locations denoted by set NL, a flow matrix A, where each entry A_{ij} represents the flow of materials moving from facility i to facility j, and a distance matrix D, where each entry D_{ij} represents the distance between locations i to j, the objective of the QAP problem is to find a one-to-one function π:$NF \longrightarrow NL$, which assigns the facilities to locations such that the cost function, $\sum_{i,j} A_{ij}D_{\pi(i)\pi(j)}$, is minimized.

As π is a one-to-one function, we use a permutation matrix X to realize it, where $\pi(i) = j$ iff $X_{ij} = 1$. Then, the QAP problem can be rewritten as [KB57]:

$$\begin{aligned} min \quad & \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} A_{ij}D_{kl}X_{ik}X_{jl} \\ s.t. \quad & \textstyle\sum_{i=1}^{n} X_{ij} = 1, j \in \{1,\ldots,n\} \\ & \textstyle\sum_{j=1}^{n} X_{ij} = 1, i \in \{1,\ldots,n\} \\ & X_{ij} \in \{0,1\} \end{aligned} \tag{2}$$

Comparing Eq. (2) and (1), if we set $\mu(\mathrm{P}_i,\mathrm{P}_j)$ to A_{ij}, N_S to n, and $T_{m,n}$ to D_{kl}, it is obvious that the VCtoS problem becomes exactly a QAP problem. As a result, it holds that the VCtoS problem is also NP-hard.

In addition, for any $\epsilon > 0$, there does not exist a corresponding ϵ-approximate algorithm for the QAP problem [SG76]. Thus, there does not exist a polynomial time ϵ-approximation algorithm for our VCtoS problem also. ∎

Theorem 2: The virtual cells partitioning problem given in Eq. (13) of the paper is NP-hard.

Proof: Given the VIG, the partitioning problem is now transformed to the problem of how to divide V of VIG into N_S disjoint subsets, $\mathrm{P}_1, \ldots, \mathrm{P}_{N_S}$, so as to meet the requirement presented in Eq. (13) of the paper. To prove that this partitioning problem is NP-hard, we transform it to the *weighted clique partitioning (WCP) problem*, which is known to be NP-hard [GJ79] [JJ05], as follows. Given an undirected complete graph $G = (V_G, E_G)$ with node weight $w(v_i)$ for any $v_i \in V_G$, edge weight $w(e_{ij})$ for any $e_{ij} \in E_G$, and an integer K, the WCP problem is to find a partition $\Gamma = \{\Upsilon_1, \ldots, \Upsilon_K\}$ of V_G that solves:

$$\begin{aligned} max \quad & \sum_{k=1}^{K} \sum_{e_{ij} \in E(\Upsilon_k)} w(e_{ij}) \\ s.t. \quad & w_{min} \leq \textstyle\sum_{v_i \in \Upsilon_k} w(v_i) \leq w_{max}, k = 1, \ldots, K \end{aligned} \tag{3}$$

where $E(\Upsilon_k) = \{(i,j) \in E_G : i, j \in \Upsilon_k\}$.

In other words, the WCP problem is to partition the nodes of G into K cliques such that the sum of the node weights of each partition is bounded between w_{min} and w_{max} while maximizing the sum of the weights on the edges inside the cliques.

The transformation works as follows. In our partitioning problem, since VIG=(V, E) is a complete graph, each partition P_m forms a clique. For any P_m, as it is assigned to one server s_m, the amount of remote VI caused by $T_{m,n}$ among the virtual cells of P_m is zero. Thus, to minimize the amount of VI caused by $T_{m,n}$ among different partitions, we need to maximize the total amount of VI among the virtual cells inside each partition. From the construction of VIG, as $w(e_{ij}) = \sigma(c_i, c_j) + \sigma(c_j, c_i)$, maximizing the total amount of remote VI caused by $T_{m,n}$ among virtual cells inside each partition is equivalent to maximizing the sum of the weights of the edges inside each clique as shown in Eq. (3).

On the other hand, for the constraint of load balancing presented in Eq. (13), we have:

$$0 \leq w(\mathrm{P}_m) \leq (1 + \theta)\overline{w} \tag{4}$$

As w_{min} and w_{max} used in the WCP problem of Eq. (3) can be explicitly set to 0 and $(1+\theta)\overline{w}$, respectively, we have transformed our partitioning problem into the WCP problem.

Since the transformation of our partitioning problem to the weighted clique partitioning problem can be achieved in polynomial time, we conclude that our partitioning problem is also NP-hard. ■

REFERENCES

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