

Theorem 1: Considering the source genome A and the destination genome B , A can be transformed into B by translocations if and only if:

- (1) The two genomes contain the same set of genes;
- (2) The two genomes contain the same number (must be at least 2) of chromosomes;
- (3) The two genomes have the same set of ending (either head or tail) genes;
- (4) For any gene g that is an ending gene in A , (a) if g 's sign in A is different from that in B , then g must be a head in one genome and a tail in the other; (b) if g has the same sign in both A and B , then g must be either a head in both genomes or a tail in both genomes.

Proof: (*only if*) Since a translocation never creates or destroys any gene, (1) holds. Since a translocation never changes the number of chromosomes nor the set of ending genes, (2) and (3) hold. Note that, a prefix-prefix translocation never changes the sign of any gene, whereas a prefix-suffix can change the signs of genes. If an ending gene is involved in even number of prefix-suffix translocations, then (a) in (4) holds for such an ending gene. Otherwise, (b) in (4) holds for such an ending gene.

(*if*) Now we show that if conditions (1)–(4) hold, we can transform A into B .

By changing the sign and re-naming the genes, we can assume that all the genes in B are positive and gene $i+1$ is on the right of gene i if gene $i+1$ is not an ending gene on the left of a chromosome. If gene $i+1$ is the ending gene on the left of a chromosome, then gene i is an ending gene on the right of a chromosome. For example, $B = ((1, 2, 3), (4, 5, 6), (7, 8, 9, 10))$.

Our strategy to transform A into B is (1) to put gene 1 in the correct position with positive sign, (2) after having put genes 1 to i to the correct positions with positive sign, we put gene $i+1$ in the correct position with positive sign.

Note that the prefix-prefix and prefix-suffix operations allow us to treat x_1, x_2, \dots, x_n and $-x_n, -x_{n-1}, \dots, -x_1$ as the same chromosome. Thus, we can assume that gene 1 in A is always the left ending gene and the sign is always positive. (Otherwise, we can simply reverse the chromosome to fit our assumption.) Suppose that genes 1 to i have been put in the correct positions with positive sign and there exists a chromosome whose genes are all greater than i . If gene i and gene $i+1$ are not in the same chromosome, we can use one translocation operation to put gene $i+1$ in the correct position with positive sign and keep genes 1 to i "unchanged". If gene i and gene $i+1$ are in the same chromosome, then we can put gene $i+1$ in the correct position with positive sign and keep genes 1 to i "unchanged" by using two translocations. This process can continue until genes 1 to i are in the correct positions with positive sign, gene i and gene $i+1$ are in the same chromosome, say, Y , and there does not exist a chromosome whose genes are all greater than i . Let X be the chromosome $X = x_1, x_2, \dots, x_n$ such that $x_i \leq i$ and x_n is the right ending gene that is the biggest satisfying $x_n \leq i$. In this case, condition (4) ensures that the four ending genes in X and Y are in the correct positions with positive sign.

We decompose X into two segment $X = X_1 X_2$, where $X_1 = x_1, x_2, \dots, x_{n-1}$ and $X_2 = x_n$. Let $Y = Y_1 Y_2$, where $Y_1 = j, j+1, \dots, i$, is the segment of genes in the correct positions with positive sign. Then we can extend the segment Y_1 into $j, j+1, \dots, i, i+1$ by two translocations, where the first translocation acts on X and cuts X into X_1 and X_2 . In the process, we will never further cut X_1 and X_2 . This process can go on until all genes in Y are in the correct positions with positive sign. During the process, even number of prefix-suffix translocations must be used since the right and left ending genes in Y are transformed back as the right ending gene in Y . Thus, X is also recovered. This completes the proof. \square