

# Supplementary Material for “HybridNET: a Tool for Constructing Hybridization Networks”

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## 1 CONSTRUCTING A HYBRIDIZATION NETWORK FROM AN MAAF

Let  $T$  and  $T'$  be the two input trees. We augment  $T$  (respectively,  $T'$ ) into a new tree  $\bar{T}$  (respectively,  $\bar{T}'$ ) by first introducing a new root and a dummy leaf and then letting the old root and the dummy leaf be the children of the new root. Assume that  $F$  is an MAAF of  $\bar{T}$  and  $\bar{T}'$ . We present an algorithm to construct a hybridization network from  $\bar{T}$ ,  $\bar{T}'$ , and  $F$ . We proceed as follows.

First, for each vertex  $u$  of  $F$ , we find the lowest vertex  $v$  (respectively,  $v'$ ) in  $\bar{T}$  (respectively,  $\bar{T}'$ ) such that all leaf descendants of  $u$  in  $F$  are also leaf descendants of  $v$  (respectively,  $v'$ ) in  $\bar{T}$  (respectively,  $\bar{T}'$ ). For convenience, we say that  $u$ ,  $v$ , and  $v'$  are *mates* of each other. Moreover, if a vertex of  $\bar{T}$  or  $\bar{T}'$  has a mate in  $F$ , then we call it a *preserved* vertex; otherwise, we call it an *unpreserved* vertex.

We can show that both the root of  $\bar{T}$  and that of  $\bar{T}'$  are preserved vertices. Thus, the reticulate number of two phylogenetic trees is also equal to the number of trees in an MAAF of the two original input trees (instead of their augmented versions) minus one.

We are now ready to construct a network  $N$  of  $\bar{T}$  and  $\bar{T}'$ . Initially, we let  $N$  be a copy of  $\bar{T}$ . Obviously,  $\bar{T}$  fits  $N$ ; we will always maintain this property hereafter. We then add more vertices and edges to  $N$  so that  $\bar{T}'$  also fits  $N$ , by performing the following four steps:

*Step 1:* In this step, we look at each edge  $(u, v)$  in  $F$ . Let  $P'_{u,v}$  denote the path in  $\bar{T}'$  from the mate of  $u$  to the mate of  $v$ . Note that the internal vertices of  $P'_{u,v}$  are unpreserved vertices. In order for  $\bar{T}'$  to fit  $N$ , we embed  $P'_{u,v}$  into the path of  $N$  from the mate  $x$  of  $u$  to the mate  $y$  of  $v$  as follows: If  $P'_{u,v} = u, w'_1, w'_2, \dots, w'_k, v$ , then we find the parent  $z$  of  $y$  in  $N$  and modify  $N$  by splitting the edge  $(z, y)$  into a path  $Q = z, w_1, w_2, \dots, w_k, y$ . For convenience, for each  $i \in \{1, 2, \dots, k\}$ , we call  $w_i$  and  $w'_i$  the *partners* of each other. Roughly speaking, after this step, for each edge  $(u, v) \in F$ , the path in  $N$  from the mate of  $u$  to the mate of  $v$  is an expansion of both  $P'_{u,v}$  and the path of  $\bar{T}$  from the mate of  $u$  to the mate of  $v$ .

*Step 2:* Note that there may exist vertices in  $\bar{T}'$  that have neither mates nor partners in  $N$ . For convenience, we call these vertices of  $\bar{T}'$  *free* vertices. For each free vertex  $v'$  of  $\bar{T}'$ , we add a copy  $v$  of  $v'$  to  $N$  (as an isolated vertex) and again call  $v$  and  $v'$  the *partners* of each other.

*Step 3:* For each edge  $(u', v')$  of  $\bar{T}'$  such that at least one of  $u'$  and  $v'$  is a free vertex, we add edge  $(u, v)$  to  $N$ , where  $u$  and  $v$  are the partners of  $u'$  and  $v'$  in  $N$ , respectively. Note that after this step, the in-degree of each vertex in  $N$  remains to be at most 1.

*Step 4:* For each preserved vertex  $v'$  of  $\bar{T}'$  such that  $v'$  is not the root of  $\bar{T}'$  but its mate in  $F$  is of in-degree 0, we find the parent  $u'$  of  $v'$  in  $\bar{T}'$  and add the edge  $(u, v)$  to  $N$ , where  $u$  (respectively,  $v$ ) is the partner (respectively, mate) of  $u'$  (respectively,  $v'$ ) in  $N$ . Note that after this step, there are exactly  $d - 1$  vertices of in-degree 2 in  $N$ , where  $d$  is the number of connected components in  $F$ . This completes the construction of  $N$ .

Obviously,  $\bar{T}$  fits  $N$  because initially  $\bar{T}$  fits  $N$  and Steps 1 through 4 do not invalidate this property. Moreover,  $\bar{T}'$  fits  $N$  because each edge of  $\bar{T}'$  is either embedded in  $N$  or copied to  $N$ . Now, since  $N$  is a hybridization network with exactly  $d - 1$  reticulate vertices, it is optimal.

If we want a hybridization network of  $T$  and  $T'$  (instead of their augmented versions), it suffices to modify the above  $N$  by removing the root and its dummy child. The whole algorithm for constructing the optimal hybridization network  $N$  of  $T$  and  $T'$  runs in linear time of the input tree size.

In the above construction of  $N$ , when we embed a path  $P$  of  $\bar{T}'$  into  $N$ , we may have multiple choices to do so. That is, it may be possible to construct more than one optimal hybridization network from  $T$ ,  $T'$ , and  $F$ .

## 2 EXPERIMENTAL RESULTS

To compare the efficiency of *HybridNet* with the previously best exact programs (namely, *SPRDist* by Wu (2009) and *HybridInterleave* by Collins *et al.* (2009)), we have run *HybridNet*, *SPRDist*, and *HybridInterleave* on both simulated data and biological data. We omit the comparison with non-exact programs such as *EEEP*, *HorizStory*, *DarkHorse*, *RIATA-HGT*, *LatTrans*. The experiment was performed on a 3.33 GHz Linux PC. Note that *SPRDist* computes the rSPR distance of two phylogenetic trees while *HybridInterleave* computes the hybridization number of two phylogenetic trees.

**Table 1.** Computing the rSPR distance and the hybridization number on simulated data from Beiko and Hamilton (2006). Columns  $d$  and  $h$  show the rSPR distance and the hybridization number, respectively. Columns *HybridNet*, *SPRDist*, and *HybridInterleave* show the running times of *HybridNet*, *SPRDist*, and *HybridInterleave*, respectively. Time is measured in seconds (s), minutes (m), hours (h), and days (d). When a program crashes, we use symbol ‘-’ to show its running time. When a program did not stop after one day, we simply stopped it and use ‘> 1d’ to show its running time.

| $d$ | <i>HybridNet</i> | <i>SPRDist</i> | $h$ | <i>HybridNet</i> | <i>HybridInterleave</i> |
|-----|------------------|----------------|-----|------------------|-------------------------|
| 10  | <1s              | -              | 10  | <1s              | >1d                     |
| 10  | <1s              | -              | 10  | <1s              | >1d                     |
| 9   | <1s              | 9.1m           | 9   | <1s              | >1d                     |
| 9   | <1s              | 13m            | 9   | <1s              | >1d                     |
| 10  | <1s              | -              | 10  | 1s               | >1d                     |
| 9   | <1s              | 12s            | 9   | <1s              | >1d                     |
| 10  | <1s              | -              | 10  | <1s              | >1d                     |
| 10  | <1s              | -              | 10  | 3s               | >1d                     |
| 10  | <1s              | -              | 10  | <1s              | >1d                     |
| 10  | <1s              | -              | 10  | <1s              | >1d                     |
| 8   | <1s              | 9s             | 8   | <1s              | >1d                     |
| 7   | <1s              | -              | 7   | <1s              | 10.4h                   |
| 8   | <1s              | 9.6m           | 8   | <1s              | >1d                     |
| 8   | <1s              | -              | 8   | <1s              | >1d                     |
| 8   | <1s              | 9.8m           | 8   | <1s              | >1d                     |
| 8   | <1s              | -              | 8   | <1s              | >1d                     |
| 7   | <1s              | 14s            | 7   | <1s              | 6.7m                    |
| 8   | <1s              | 8s             | 8   | <1s              | >1d                     |
| 7   | <1s              | 25s            | 7   | <1s              | >1d                     |
| 8   | <1s              | 29s            | 8   | <1s              | >1d                     |

## 2.1 Simulated Data

We use the benchmark dataset provided by Beiko and Hamilton (2006). To obtain a pair  $(T, T')$  of trees, Beiko and Hamilton (2006) first generate  $T$  randomly and then obtain  $T'$  from  $T$  by performing a specified number  $\tilde{d}$  (say, 10) of random rSPR operations on  $T$ . So, the actual rSPR distance of  $T$  and  $T'$  is at most  $\tilde{d}$ . Moreover, the hybridization number of  $T$  and  $T'$  can be  $\tilde{d}$ , smaller than  $\tilde{d}$ , or larger than  $\tilde{d}$ . In this way, they obtain a lot of benchmark tree pairs. To compare the efficiency of our program with *SPRDist* and *HybridInterleave*, we only pick the 10 tree pairs with the largest size (100 leaves) and the most random rSPR operations performed (10). See Table 1 for the experimental results.

The experimental results in Table 1 indicate that *HybridNet* can give the exact solutions within a second. *SPRDist* takes 9 seconds to 14.5 minutes for some easy cases. However, when the number of leaves or the rSPR distance is large, *SPRDist* often crashes. *HybridInterleave* is quite slow for simulated data and it takes more than one day to finish for many cases. Therefore, *HybridNet* is more efficient and stable than *SPRDist* and *HybridInterleave*.

## 2.2 Biological Data

We use the Poaceae dataset from the Grass Phylogeny Working Group (Grass PWG, 2001). The dataset contains sequences for six loci: internal transcribed spacer of ribosomal DNA (ITS); NADH dehydrogenase, subunit F (ndhF); phytochrome B (phyB); ribulose

1,5-biphosphate carboxylase/oxygenase, large subunit (rbcL); RNA polymerase II, subunit  $\beta''$  (rpoC2); and granule bound starch synthase I (waxy). The Poaceae dataset was previously analyzed by Schmidt (2003), who generated the inferred rooted binary trees for these loci. See Table 2 for the experimental results.

As can be seen from Table 2, *HybridNet* is generally more efficient and stable than *SPRDist* and *HybridInterleave*. In more details, *HybridNet* is always faster than *SPRDist*; this is particularly obvious for the tree pair (ndhf,ITS). *HybridNet* compares well with *HybridInterleave*; in particular, for the tree pair (rbcL,ITS), it runs much faster. Of special interest is that even when we turn on the option MAAFs or MAFs to find all solutions, *HybridNet* runs faster than *HybridInterleave* and *SPRDist* which find only a single solution. So, one can conclude that *HybridNet* runs faster not because of luck.

**Table 2.** Computing the rSPR distance and the hybridization number on 15 pairs of trees for the Poaceae data. Column *pair* shows the tree pairs. Column *#taxa* shows the number of leaves in an input tree, while columns  $d$  and  $h$  show the rSPR distance and the hybridization number, respectively. Columns *HybridNet*, *SPRDist*, and *HybridInterleave* show the running times of *HybridNet*, *SPRDist*, and *HybridInterleave*, respectively. Time is measured in seconds (s), minutes (m), and hours (h).

| <i>pair</i> | <i>#taxa</i> | $d$ | <i>HybridNet</i> | <i>SPRDist</i> |
|-------------|--------------|-----|------------------|----------------|
| ndhF,phyB   | 40           | 12  | <1s              | 2m9s           |
| ndhF,rbcL   | 36           | 10  | <1s              | 29s            |
| ndhF,rpoC2  | 34           | 11  | <1s              | 50s            |
| ndhF,waxy   | 19           | 7   | <1s              | 12s            |
| ndhF,ITS    | 46           | 19  | 20s              | <b>14h</b>     |
| phyB,rbcL   | 21           | 4   | <1s              | 5s             |
| phyB,rpoC2  | 21           | 6   | <1s              | 4s             |
| phyB,waxy   | 14           | 3   | <1s              | 2s             |
| phyB,ITS    | 30           | 8   | <1s              | 22s            |
| rbcL,rpoC2  | 26           | 11  | <1s              | 1.3m           |
| rbcL,waxy   | 12           | 6   | <1s              | 3s             |
| rbcL,ITS    | 29           | 13  | <1s              | 8.5m           |
| rpoC2,waxy  | 10           | 1   | <1s              | <1s            |
| rpoC2,ITS   | 31           | 14  | <1s              | 27m            |
| waxy,ITS    | 15           | 7   | <1s              | 8s             |

  

| <i>pair</i> | <i>#taxa</i> | $h$ | <i>HybridNet</i> | <i>HybridInterleave</i> |
|-------------|--------------|-----|------------------|-------------------------|
| ndhF,phyB   | 40           | 14  | 14s              | 8s                      |
| ndhF,rbcL   | 36           | 13  | 2s               | 2s                      |
| ndhF,rpoC2  | 34           | 12  | <1s              | 7s                      |
| ndhF,waxy   | 19           | 9   | <1s              | 1s                      |
| ndhF,ITS    | 46           | 19  | 3.75m            | 4m                      |
| phyB,rbcL   | 21           | 4   | <1s              | <1s                     |
| phyB,rpoC2  | 21           | 7   | <1s              | <1s                     |
| phyB,waxy   | 14           | 3   | <1s              | <1s                     |
| phyB,ITS    | 30           | 8   | <1s              | <1s                     |
| rbcL,rpoC2  | 26           | 13  | <1s              | 7s                      |
| rbcL,waxy   | 12           | 7   | <1s              | 1s                      |
| rbcL,ITS    | 29           | 14  | 2s               | <b>1.5h</b>             |
| rpoC2,waxy  | 10           | 1   | <1s              | <1s                     |
| rpoC2,ITS   | 31           | 15  | 4s               | 14.5m                   |
| waxy,ITS    | 15           | 8   | <1s              | <1s                     |

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