Brief Contributions

Topology Control of *Ad Hoc* Wireless Networks for Energy Efficiency

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Abstract—In *ad hoc* wireless networks, to compute the transmission power of each wireless node such that the resulting network is connected and the total energy consumption is minimized is defined as a Minimum Energy Network Connectivity (*MENC*) problem, which is an NP-complete problem. In this paper, we consider the approximated solutions for the *MENC* problem in *ad hoc* wireless networks. We present a theorem that reveals the relation between the energy consumption of an optimal solution and that of a spanning tree and propose an optimization algorithm that can improve the result of any spanning tree-based topology. Two polynomial time approximation heuristics are provided in the paper that can be used to compute the power assignment of wireless nodes in both static and low mobility *ad hoc* wireless networks. The two heuristics are implemented and the numerical results verify the theoretical analysis.

Index Terms—Multihop, *ad hoc*, wireless networks, energy efficiency, transmission power, topology control.

1 Introduction

An *ad hoc* wireless network consists of a collection of mobile hosts dynamically forming a temporary network without the use of any existing network infrastructure. Unlike wired networks, in which the link topology is fixed at the time a network is deployed, *ad hoc* wireless networks have no fixed link topology. The temporary physical topology of the network is determined by the distribution of the wireless nodes as well as the transmission power of each node. This temporary physical topology is referred to as the *virtual infrastructure* in the literature. If the transmission power of each node is appropriately set, a message from any node can be relayed to every other node in the network and this resulting virtual infrastructure is considered *connected*.

The lack of fixed infrastructure makes it necessary to compute and maintain a connected virtual infrastructure among network nodes, above which the higher-level protocols are implemented. Virtual infrastructure is especially important for proactive routing protocols. A common feature of this category of protocols is that they make significant use of global topology information to make routing decisions. The global topology is obtained by background information exchange, regardless of service requests from applications. Usually, the information dissemination is periodic or, sometimes, event-driven, but there must be a connected network to support the information exchange.

The significance of such a connected network can be seen in [12] and [2]. In [12], a minimum energy broadcast algorithm was proposed that constructs the broadcast tree by selecting a node with minimum incremental cost at each iteration. The minimum incremental cost among all wireless nodes is global information that is obtained by integrating information from all the other nodes. During the construction of the broadcast tree rooted at the source, a master site, which is usually the source itself, needs to know the global information in order to make a routing decision. In [2], global information such as minimum cost edge is used to construct the broadcast tree. Such global information will not be available without the underlying mechanism that efficiently exchanges information among all wireless nodes.

The problem of determining an appropriate topology in *ad hoc* networks, which is also called topology control, has been widely studied. Let V denote the collection of wireless nodes and let $\mathcal{G}(V, E)$ denote the super graph on V that contains all possible edges if each node transmits at its maximum transmission power. The edge set E of $\mathcal{G}$ is constructed in such a way that there is a directed edge from u to v if and only if u can reach v using its maximum transmission power. Graph $\mathcal{G}$ sets an upper bound on the maximum connectivity that a wireless network can have. The topology control algorithm returns a topology $\mathcal{T}$ constructed from $\mathcal{G}$, i.e., $\mathcal{T}$ is a subgraph of $\mathcal{G}$ on V. The evaluation of $\mathcal{T}$ must consider connectivity, energy efficiency, throughput, and robustness to mobility, etc. Among all of these criteria, connectivity is the most basic. In this paper, we consider the energy efficiency of the network topology for the following connectivity requirement: For any pair of nodes u and v, if there is a path from u to v in $\mathcal{G}$, then there is also a path from u to v in $\mathcal{T}$. Especially, if $\mathcal{G}$ is strongly connected, i.e., there is at least one path from each node to any other node, then $\mathcal{T}$ must also be strongly connected.

Since energy efficiency is a critical issue in wireless network protocol design, the energy dissipation in maintaining the network is not negligible. An energy efficient wireless network that uses minimum energy to provide the necessary topological connectivity is highly desired. Most of the wireless routing algorithms addressed the energy efficiency problem only for specific traffic [6], [11], [12] without considering the energy consumption in constructing and maintaining the underlying network topology.

The minimum energy network topology problem has been addressed in [9] in which a distributed protocol is proposed that attains the global minimum energy solution for stationary *ad hoc* networks and the suboptimal solution for mobile networks. The total energy model in [9] is different from this paper. In [9], the total energy model is based on the assumption that a protocol that minimizes the energy consumption of the all-to-*one* traffic with a single sink node can simultaneously minimize the total energy consumption in the all-to-*all* traffic because each node can be independently taken as a sink node and the optimal topologies can be superposed. This is a simplification of the total energy problem in peer-to-peer communication, as the authors pointed out. However, this simplification cannot be justified for the following reason: The union of individual minimum energy sink trees does

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not yield the minimum energy topology for all-to-*all* traffic and the sum of the total energy from each individual sink tree does not yield the total energy consumption in an all-to-*all* traffic either. Therefore, having minimum total energy for each sink tree does not imply a minimum energy network topology. With this model, some transmissions are redundant. For example, a node can be a sink node and it can also be a relay node. It receives the message from each of its neighbors as a sink node for once and as a relay node for zero or more times, depending on if it lies on the paths to other sink nodes. But, actually, it only needs to receive the same message once. Li and Wan [7] improved the results by Rodoplu and Meng [9] and further extended the results to mobile networks. But, the total energy model they used is the same.

The background information exchange for maintaining network connectivity is indeed an all-to-all traffic. But, this doesn't imply that each node has to send the same information to every other node individually, this is because 1) wireless transmission is omnidirectional, therefore the energy saving resulting from omnidirectional transmission should be considered and 2) redundant transmission can be eliminated to be energy efficient.

In this paper, we consider the energy consumption under a different model in which each node only transmits to its direct neighbors periodically. The state information of each node is processed and integrated by its neighbors and further transmitted to other nodes in the network. The totality of this information provides every node with a complete map of the entire network. Without loss of generality, this information exchange is independent of the data traffic that the network is carrying. So, the energy consumption of each node is proportional to the power it uses to transmit, which can be derived from the *minimum energy network topology* in question. The optimal solution for this problem is NP-hard [5], [1]. We propose using topologies derived from spanning trees to support the background information exchange. Specifically, the Minimum Spanning Tree (MST) approximates the optimal solution with a performance ratio 2 in a stationary network [5] and the Minimum Incremental Power (MIP) tree consumes less total energy than MST. They both can be further improved by an optimization algorithm proposed in this paper. The optimization algorithm builds connected topologies on top of spanning trees. It first identifies critical paths that decide the transmission power of the nodes on the paths and then creates short paths for these critical paths to reduce the energy consumption.

We denote the topology returned by the topology control algorithm by $\mathcal{T}$, then each node is connected to the other nodes in the network through $\mathcal{T}$. Each node v has a set of in-edges and out-edges, denoted by $in(v)$ and $out(v)$, respectively. A node v's knowledge about the entire network is kept up to date based on the information received from its direct neighbors connected by $in(v)$. The transmission power of node v is the power needed to reach its farthest neighbor connected by $out(v)$. In spanning trees, each edge is bidirectional, therefore it is in both $in(v)$ and $out(v)$.

In the following sections, we describe in detail the communication model for the topology maintenance, propose two topologies derived from spanning trees, and estimate the energy consumption of these topologies for the background information exchange. We also provide theoretical analysis and simulation results of these topologies. An important assumption that enables us to use transmission power as energy consumption is that the background information exchange is repeated periodically and endlessly as long as the network is functional. We take advantage of this simplification and use *transmission power* in energy calculation without worrying about the transmission time.

2 Communication Model

The most commonly used model for RF systems is the path loss model, in which the received power $P_r = P_t \cdot r^{-\alpha}$, where r is the distance between the transmitter and the receiver and α is the path loss exponent between 2 and 4, depending on the characteristics of the communication medium, and P_t is the transmitting power. Therefore, the transmitting power P_t needed to support a link of length r is $P_{r_{TH}} \cdot r^{\alpha}$, where $P_{r_{TH}}$ is the receiver power threshold for signal detection. When the receivers have the same power threshold, this value becomes a constant and can be further normalized to one and the transmitting power P_t needed to support a link of length r becomes r^{α}. To simplify the presentation, we use r^{α} to represent the cost of an edge. For a wireless network where nodes have different power thresholds, the cost of a directed edge will be replaced by its complete form $P_{r_{TH}} \cdot r^{\alpha}$ in theoretical analysis and numerical simulation and the theorems presented in this paper will still hold.

The energy consumption of a wireless node has two components, i.e., transmitting power and receiving/processing power. The latter is relatively small and usually can be denoted by a constant. Since, in the all-to-all traffic, every node consumes energy in receiving/processing in each period, we ignore this constant and consider only the energy consumption in transmitting.

We also assume omnidirectional antennas are used. This means if a node transmits at power level $P_{r_{TH}} \cdot r^{\alpha}$, any node within a distance r of it can receive the signal if the receiver's power threshold for signal detection is no bigger than $P_{r_{TH}}$. The total energy consumption of the network can be modeled as the sum of each transmitting node. In the total energy model described in Section 1, each node communicates with its direct neighbors, therefore all the nodes are transmitting nodes. This paper focuses on the total energy consumption in maintaining the network connectivity, so the total energy based on this model is

$$total\ energy = \sum_{v_i \in V} P_t(v_i).$$

3 Minimum Energy Network Topology

To support peer-to-peer communications in wireless networks without the use of any existing infrastructure, the network connectivity must be maintained at any time. This requires that, for each node, there must be a route to reach any other node in the network. Such a network is called *strongly connected* [4]. A node v_i can reach a node v_j directly if node v_j is in node v_i's transmission range; node v_i can also use multihop forwarding to reach v_j if node v_j is out of range. For example, a spanning tree with bidirectional links can provide such connectivity.

In this paper, we concern ourselves only with the total energy consumption of all nodes for such a strongly connected network. Our goal is to determine the transmission power $p(v_i)$ of each node v_i such that the network is *strongly connected* and the total transmission power is minimized. Since each node is a potential transmitter and the transmission is periodic, we use the transmission power of each node as a measure of the energy consumption.

We formally define the minimum energy network connectivity (MENC) problem as follows:

Definition 1: Minimum Energy Network Connectivity Problem (MENC). *Given a set of wireless nodes* $V = \{v_0, v_1, v_2, \ldots, v_n\}$, *and the cost function* $F : (V, V) \rightarrow Z$, *determine a power assignment of nodes* $P : V \rightarrow Z$ *such that:*

1. *The induced directed graph* $\mathcal{T}$ *is strongly connected.*
2. $\sum_{v_i \in V} p(v_i)$ *is minimized.*

When we use the path loss model described in Section 2, the cost function is defined as

$$f(v_i, v_j) = |(v_i, v_j)|^\alpha$$

and the power assignment must satisfy

$$p(v_i) \geq |(v_i, v_j)|^\alpha$$

to support a directed edge (v_i, v_j). The induced topology $\mathcal{T}$ must contain paths from each node to every other node in the network.

Note that, in the directed graph $\mathcal{T}$, a node's transmission power only supports the out-edges. This is different from the undirected graph, where each node needs to reach all the neighbors.

It is proven that the MENC problem is NP-complete [5], [1]. Kirousis et al. [5] also proved that MST has 2-approximation ratio. In this paper, we propose two network topologies as approximations to the optimal solution. Both of them outperformed MST, therefore they are within a 2-factor of the optimal solution in terms of total energy consumption.

4 Spanning Tree-Based Network Topologies

In this section, we reveal some important properties of spanning trees. In the following theoretical analysis, we assume the directed graph G is the optimal topology on the set of wireless nodes V for the *MENC* problem.

We define:

$$\begin{aligned} OPT &= \text{total energy consumption of graph } G \\ &= \sum_{v \in V[G]} p(v) \\ &= \sum_{v \in V(G)} \max_{e \in out_G(v)} |e|^\alpha, \end{aligned}$$

where $out_G(v)$ denotes the set of out-edges at v in G.

4.1 Minimum Spanning Tree Has 2-Approximation Ratio

In [5], it is proven that, for a set of wireless nodes V, MST determines a 2-approximation to the Minimum Energy Network Connectivity (MENC) problem:

Theorem 1. *If $T^\star$ is the minimum spanning tree over the set of nodes V, then*

$$2 \cdot OPT \geq \sum_{x \in V[T^\star]} \max_{(x,y) \in E[T^\star]} |(x, y)|^\alpha.$$

The detailed proof is referred to [5]. Next, we'll show that this performance ratio 2 is a tight bound.

The worst-case scenario for minimum spanning tree to approximate the optimal solution is the following case:

Assume there are $2n$ nodes deployed on a circle with separations $\{1, \varepsilon, 1, \varepsilon \ldots\}$ (see Fig. 1). We normalize the edge length to be 1 for the long edges and ε for the short edges and let $\varepsilon \ll 1$.

The optimal solution for this scenario is a directed cycle, as shown in Fig. 1a. So, the optimal energy consumption is

$$OPT = n(1 + \varepsilon^\alpha) \doteq n.$$

We construct an MST by removing the longest edge from the cycle and making every edge undirected (Fig. 1b). Now, the MST $T^\star$ consumes energy

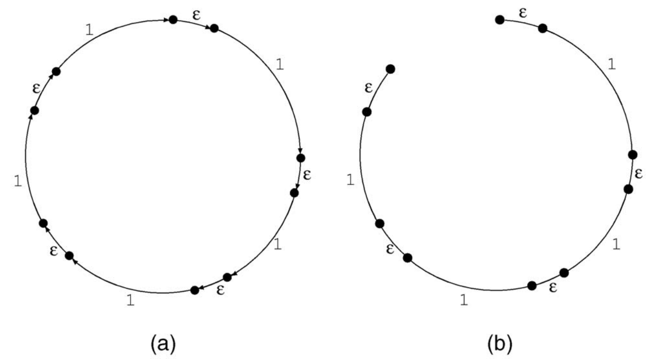


Fig. 1. Worst-case scenario of MST. (a) Optimal solution. (b) Minimum spanning tree.

$$\begin{aligned} Energy(T^\star) &= \sum_{x \in V[T^\star]} (\max_{(x,y) \in E[T^\star]} |(x, y)|)^\alpha \\ &= 2 \cdot \varepsilon^\alpha + (2n - 2) * (1) \\ &\doteq 2n - 2. \end{aligned}$$

Since

$$\lim_{n \to \infty} \frac{2n - 2}{n} = 2,$$

this verifies that the performance ratio 2 is a tight bound.

4.2 A New Property of Spanning Trees

Given a set of wireless nodes V, choose a vertex $r \in V$ and construct a spanning tree. For every node $v \in V - r$, there must be a path from v to r. First, assign each edge on these paths a direction toward the root r and let T_s denote this sink tree rooted at r. We define $e(v)$ as the unique out-edge at v in T_s. So,

$$\max_{e \in out_{T_s}(v)} |e|^\alpha = \begin{cases} 0, & if\ v = r \\ |e(v)|^\alpha, & if\ v \neq r. \end{cases}$$

Then, we assign each edge a direction away from the root and denote this spanning tree by T. For each vertex v, let $m(v)$ denote the longest out-edge at v in T such that

$$|m(v)| = max_{e \in out_T(v)} |e|.$$

All $m(v)$s form disjoint paths from internal vertices to leaves. Let us call these paths *critical paths*. We denote the set of **E**dges in **C**ritical paths as *EC*. In Fig. 2c, the thick-lined edges are in critical paths.

Now, we are ready to introduce our theorem:

Theorem 2. *The optimal solution OPT is upper bounded by the sum of the costs of all the edges in a spanning tree and the edges in its critical paths.*

$$OPT \leq \sum_{e \in E(T)} |e|^\alpha + \sum_{e \in EC} |e|^\alpha.$$

Proof. Note that

$$OPT \leq \sum_{v \in V(T)} \max_{e \in edge(v)} |e|^\alpha,$$

where $edge(v)$ denotes the set of undirected edges at v.

Moreover,

$$\max_{e \in edge(v)} |e|^\alpha \leq |m(v)|^\alpha + |e(v)|^\alpha.$$

Since edge costs are symmetrical based on assumptions in Section 2, we have

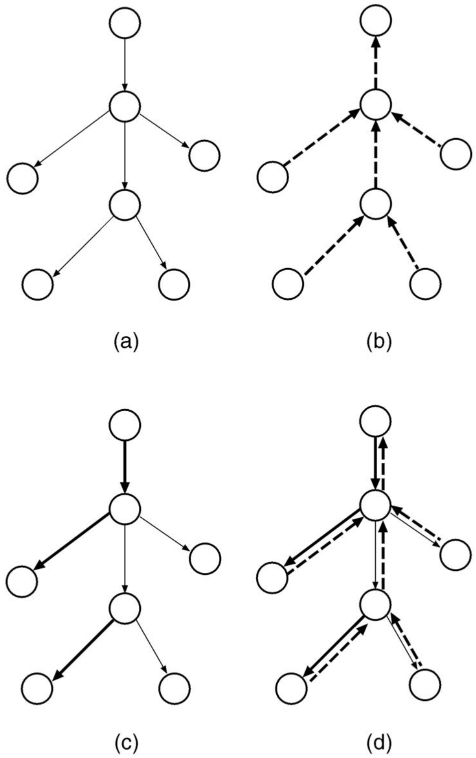


Fig. 2. (a) Spanning tree T. (b) $e(v)$s are shown in dashed lines in sink tree T_s. (c) $m(v)$s are shown in thick lines in spanning tree T. (d) $max_{e\in edge(v)}|e|^\alpha \leq |m(v)|^\alpha + |e(v)|^\alpha$.

$$\sum_{v\in V(T_s)} |e(v)|^\alpha = \sum_{e\in E(T)} |e|^\alpha,$$

moreover,

$$\sum_{v\in V(T)} |m(v)|^\alpha = \sum_{e\in EC} |e|^\alpha.$$

Therefore, the theorem holds. □

Theorem 2 holds for networks where every node has the same power threshold $P_{r_{TH}}$, therefore edge costs are symmetrical. In networks where $P_{r_{TH}}$ varies, the theorem should be presented in its complete form:

$$OPT \leq \sum_{e\in E(T_s)} P_{r_{TH}} \cdot |e|^\alpha + \sum_{e\in EC} P_{r_{TH}} \cdot |e|^\alpha,$$

where $P_{r_{TH}}$ is the receiving node's power threshold. Inspired by Theorem 2, we design heuristics to reduce the cost of critical paths in the following section.

5 Energy Efficient Topology Control Algorithms

5.1 Heuristic 1

We first consider a Minimum spanning tree (MST). MST has the property that the longest edge in the tree is the minimum among all spanning trees [2]. When MST is used as the virtual infrastructure for maintaining the network connectivity, it has the advantage that the maximum energy expenditure is minimum and, hence, the disparity of the energy levels is least severe among all spanning trees. It is very important to minimize the maximum transmission power because the battery energy is not shared among wireless nodes, it is, rather, a local resource supplied to each individual node. Saving total energy consumption of all wireless nodes doesn't necessarily reduce the energy consumption of each individual node. Some nodes may run out of energy and become disconnected even though other nodes still have plenty of energy. Shah and Rabaey [10] described the *network survivability* as the capability to maintain the network for as long as possible. One of the key factors is the energy health should be of the same order. Even though MST is not the best candidate for minimizing total energy, it is still widely used for its quality of energy health.

If we use spanning tree as the virtual infrastructure, each edge in the spanning tree must be bidirectional to maintain the network connectivity. The minimum spanning tree $T^\star$ uses total energy of

$$\begin{aligned}&\sum_{v\in V(T^\star)} \max_{e\in out_{T^\star}(v)} |e|^\alpha\\ &= \sum_{v\in V(T^\star)} \max(|e(v)|^\alpha, |m(v)|^\alpha)\\ &= \sum_{e\in E(T^\star)} |e|^\alpha + \sum_{v\in V(T^\star)} \max(|e(v)|^\alpha - |m(v)|^\alpha, 0).\end{aligned}$$

Therefore, to reduce the total energy consumed by MST, we should try to reduce the cost on the critical paths.

We assume graph $\mathcal{G}(V,E)$ is the induced connected graph by a given set of wireless nodes V if each node is using its maximum transmission power. $\forall(x,y)\in E$, the cost of edge (x,y) is defined by power function $f(x,y) = |(x,y)|^\alpha$. We propose a heuristic *MST-Reduced* for minimum energy topology construction as follows:

Algorithm MST-Reduced$(\mathcal{G}(V,E))$
```
BEGIN
    sort edges in E by cost in nondecreasing order
    for each vertex v ∈ V do
        makeSet(v)
    end for
    initialize T_r ← φ
    while |T_r| < |V| − 1 do
        for each edge (u, v) ∈ E in the sorted order, do
            if set(u) ≠ set(v) then
                T_r ← T_r ∪ (u, v)
                union(u,v)
            end if
        end for
    end while
    randomly choose a vertex r as the root of spanning tree T_r
    criticalPathReduction(T_r)
END
```

In algorithm MST-Reduced$(\mathcal{G}(V,E))$, makeSet(v) and union(u,v) are standard set operations as described in [4].

So, basically, the MST-Reduced algorithm is a two-phase algorithm. It first builds a minimum spanning tree T_r by iteratively connecting two disconnected components until all components are connected into one. The edge that is selected as a bridge edge at each step is a minimum cost edge. The second phase is a postprocessing procedure that is used to further reduce the cost of the minimum spanning tree T_r. The basic idea of it is to create cycles on critical paths rather than to use the backtracking edges $e(v)$ if doing this can reduce the cost.

Algorithm criticalPathReduction(T_r) BEGIN
```
    initialize T ← T_r
    findCriticalPaths(T_r)
    for each critical path (v_0, ..., v_k) do
        if Σ_{i=1}^{k} max(|e(v_i)|^α − |m(v_i)|^α, 0) > |d(v_0, v_k)|^α then
```

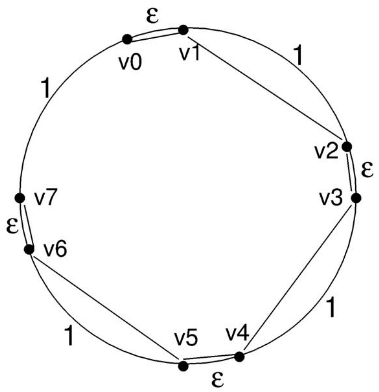


Fig. 3. An example.

$T \leftarrow T \cup (v_k, v_0)$
$T \leftarrow T - \cup_{i=1}^{k} e(v_i)$
update the power of v_i, $\forall i = 1, k$
end if
end for
return topology T and power assignment P
END

The procedure findCriticalPaths(T_r) is provided below:

Algorithm findCriticalPaths(T_r)
BEGIN
$r \leftarrow root(T_r)$
initialize $path(r) \leftarrow \phi$
for each child x of r **do**
if x is r's farthest child **then**
$path(r) \leftarrow path(r) \cup (r, x)$
else
$T_x \leftarrow$ the subtree rooted at x
findCriticalPaths(T_x)
end if
end for
END

The same operation can also apply to the subpath of each critical path. If $(v_0, \ldots, v_k)$ is a subpath of a critical path, especially when v_k is not a leaf node, and if $\sum_{i=1}^{k} \max(|e(v_i)|^\alpha - |m(v_i)|^\alpha, 0) > |d(v_0, v_k)|^\alpha - |m(v_k)|^\alpha$, then increase power at v_k to reach v_0.

We will discuss this idea for the worst-case example in which MST reaches its bound.

Let us review this example here:

Consider a cycle $(v_0, v_1, \ldots, v_7)$ with $d(v_0, v_1) = \varepsilon = d(v_2, v_3) = d(v_4, v_5) = d(v_6, v_7)$ and

$$d(v_1, v_2) = 1 = d(v_3, v_4) = d(v_5, v_6) = d(v_7, v_0).$$

Construct a minimum spanning tree rooted at v_0, which is a path $(v_0, \ldots, v_7)$ (Fig. 3). Then,

$$|e(v_0)| = 0, |m(v_0)| = \varepsilon, |e(v_1)| = \varepsilon, |m(v_1)| = 1, \ldots, |e(v_7)| = \varepsilon$$
$$|m(v_7)| = 0,$$
$$|e(v_0)|^\alpha - |m(v_0)|^\alpha = 0 - \varepsilon^\alpha < 0,$$
$$|e(v_1)|^\alpha - |m(v_1)|^\alpha = \varepsilon^\alpha - 1 < 0,$$
$$|e(v_2)|^\alpha - |m(v_2)|^\alpha = 1 - \varepsilon^\alpha,$$
$$\ldots,$$
$$|e(v_7)|^\alpha - |m(v_7)|^\alpha = \varepsilon^\alpha.$$

TABLE 1
Total Energy Consumption with $\alpha = 2$

Number of nodes	MST-Reduced	Minimum Spanning Tree
3	14219	14219
4	35974	39266
5	23057	24965
6	42399	44994
7	36291	38199
8	41064	43286
9	49066	52358
10	48602	51894

Sum of the positive values $= 3(1 - \varepsilon^\alpha) + \varepsilon^\alpha > |d(v_7, v_0)|^\alpha$.

So, we increase power at v_7 to reach v_0.

With the optimization algorithm, the minimum spanning tree is easily improved to the optimal solution, which is a single cycle, as shown in Fig. 1a.

5.2 Heuristic 2

Inspired by the Broadcast Incremental Power (BIP) algorithm [12], we designed the Minimum Incremental Power (MIP) tree algorithm. The main idea of MIP has been described in [3]. To give readers a complete view of the heuristic, we include it in the following algorithm description. So far, among all the tree topologies, the MIP tree is known to be the most energy efficient in terms of the total energy consumption. Since the total energy is proportional to the total transmission power used, it is related to the total interferences in the system, so it is suitable for situations where interference must be considered [8].

Now, we propose a topology that is based on the Minimum Incremental Power(MIP) tree and optimized by the `criticalPathReduction` algorithm.

Algorithm MIP-Reduced($\mathcal{G}(V, E)$)
BEGIN
randomly choose a vertex r from V
initialize $T_r \leftarrow \phi$, vertex set $S \leftarrow \{r\}$

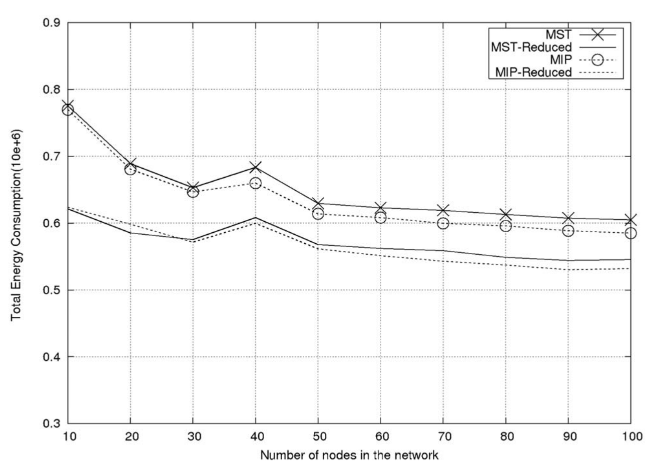
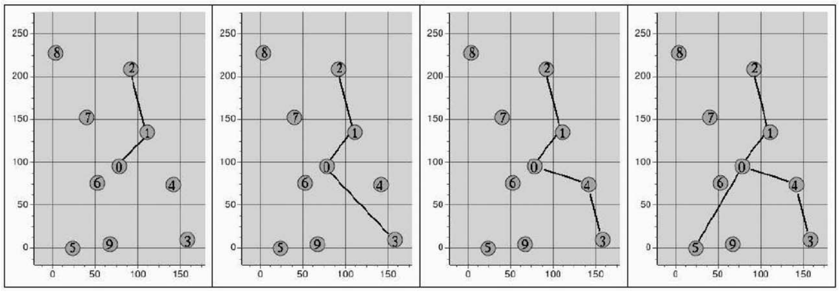
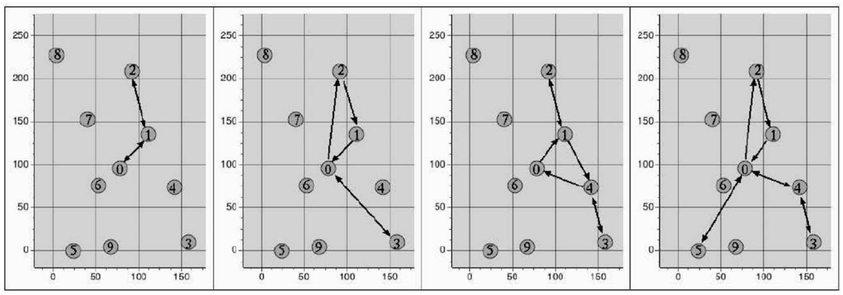


Fig. 4. Total energy consumption using MST, MST-Reduced, MIP, and MIP-Reduced.

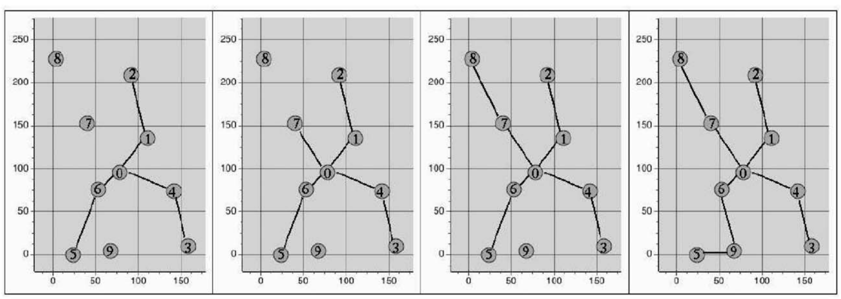


(a)

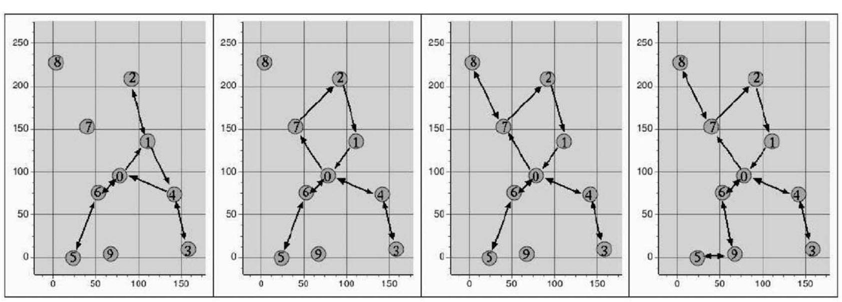


(b)

Fig. 5. The solution from MST and MST-Reduced for each network instance. The number on each node is the sequence of the node joining the network. (a) Minimum spanning trees. (b) Solutions from MST-Reduced (same as optimal solutions).



(a)



(b)

Fig. 6. The solution from MST and MST-Reduced for each network instance. The number on each node is the sequence of the node joining the network. (a) Minimum spanning trees. (b) Solutions from MST-Reduced.

while $|T_r| < |V| - 1$ **do**
  for each edge $(u, v) \in E$ **do**
    if $u \in S$ and $v \in V - S$ **then**
      calculate the power increment
      $p_{u,v} = |(u, v)|^\alpha + \max\{0, |(u, v)|^\alpha - p(u)\}$, where
        $p(u) = \max_{(u,x)\in T_r} |(u, x)|^\alpha$
    end if
  end for
  choose the node pair (u, v) with the minimum power
    increment $p_{u,v}$

$T_r \leftarrow T_r \cup (u, v)$
$S \leftarrow S \cup v$
end while
criticalPathReduction(T_r)
END

We will show next how much improvement the `criticalPathReduction` procedure can bring to the minimum spanning tree and MIP tree.

6 Experimental Results

The purpose of this simulation study is to show the energy efficiency of the two proposed topologies in maintaining network connectivity. We simulate stationary networks with node positions randomly deployed. For small networks, we use exhaustive search to find the optimal solution and compare the energy consumption of MST, MST-Reduced, and the optimal solution. MST-Reduced shows significant energy reduction and sometimes it approaches the optimal solution (for $n = 3 - 6$). It is also verified that the MST is within 2-approximation of the optimal solution. Table 1 shows the total energy resulting from MST and MST-Reduced. Figs. 5 and 6 show the topologies for $n = 3 - 6$ and $n = 7 - 10$, respectively.

For the network with a large number of nodes, we compare the solutions by MST, MST-Reduced, MIP, and MIP-Reduced as the node density increases. We run the topology control algorithms for networks of different sizes and, for each network size, 100 network instances are randomly generated and the average results are plotted in Fig. 4. Note that the total energy doesn't increase as the nodes number increases from 10 to 100. This is due to the fact that the nodes density increases correspondingly and each node can reduce its transmission power to reach other nodes. It is observed that MIP consumes 3-4 percent less total energy than MST and the optimization algorithm improved both MST and MIP by 10-25 percent for all the network instances.

7 Conclusion and Future Work

In this paper, we considered the power assignment of nodes in an *ad hoc* wireless network such that the induced topology is strongly connected and it consumes minimum total energy. The optimal solution for this problem is NP-hard. We explored the relation between the optimal solution and spanning tree-based solutions. From the theoretical analysis, we can draw a conclusion that, in spanning trees, the key factors that determine the total energy consumption are *the critical paths* as we called them. We provided an algorithm to identify the critical paths and apply an optimization algorithm to create short paths on critical paths if the resulting graph can reduce the energy consumption. Two heuristics based on MST and MIP trees are provided and the experiment results showed that the optimization algorithm improved the results from MST and MIP by 10-25 percent.

In this study, we focus only on the energy efficiency of maintaining the network connectivity. Distributed implementation of these algorithms will be studied in our future work. Especially when they are applied in mobile environment, the network reconfiguration under high mobility is a challenge task. It is also our interest to further investigate the performance of the proposed algorithms in terms of throughput and message complexity, etc., in static networks and the packet drop rate when they are used in mobile environment. Another future research topic is to provide a scalability analysis for this type of topology when network size increases or the nodes mobility increases.

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