

CS4335 Tutorial8 (Dynamic Programming Approach)

Question 1. Consider the interval with maximum sum problem given in week 6. Design a dynamic programming algorithm with running time $O(n)$ to solve the problem.

Input: An array $A[1..n]$ of n integers (positive or negative).

Problem: find an interval $[i, j]$ such that $A[i]+A[i+1]+\dots+A[j]$ is maximized.

Solution:

Let $d[i]$ be the cost of the max sum interval ending at position i .

$$d[i] = \max \{A[i], d[i-1]+A[i]\}.$$

We then choose the biggest $d[i]$ for $d[1], d[2], \dots, d[n]$.

Pseudo codes:

```
for (i=1 to n){
    compute d[i]
    if (d[i] == A[i])
        b[i]=i; // the interval starts at i-th position
    else b[i]=b[i-1];
    // the starting position of the interval is the same as the previous position.
} // end of for
// Backtracking
m is the index such that d[m] is the biggest among d[1], d[2], ..., d[n];
return the interval [b[m], m];
```

Question 2. Maximum sum subsequence problem

Input: An array $A[1..n]$ of n integers (positive or negative).

Problem: find a subsequence of $A[1..n]$ such that the total of the subsequence is maximum and if $A[i]$ is in the subsequence then $A[i-1]$ and $A[i+1]$ cannot be in the subsequence.

Solution:

Let $d[i]$ be the cost of the opt solution for the first i letters.

$$d[0] = d[-1] = 0; \quad // \text{ assume that an empty subsequence has a total } 0$$

$$d[i] = \max \{A[i]+d[i-2], A[i-1]\}.$$

Pseudo codes:

```
for (i=1 to n){
    Compute d[i] using the equation
    If (d[i] == d[i-1])
        B[i]=0;
    Else
        B[i]=1;
} //backtracking
```

```
m=n;
while (m>0){
    if (B[m] ==1){
        print (A[i]);
        m=m-2;
    }
    else m=m-1;
} // end of while
```