Solution for week 3 tutorial

Algorithm: We choose the shortest job first.

Proof:

Let us compare the greedy solution $G=j_1, j_2, \ldots, j_r, j_{r+1}\ldots.$ with an optimum solution $S=j_1, j_2, \ldots, j_r, j'_{r+1}, \ldots., j_{r+1}, \ldots$

Assume that the first r choices of G and S are the same and the (r+1)-th choices is different.
Switch j'_{r+1} with j_{r+1} in S, we obtained a new solution
$S'=j_1, j_2, \ldots, j_r, j_{r+1}, \ldots., j'_{r+1}, \ldots\ldots$

We can see that in S', (1) the waiting time for jobs $j_1, j_2, \ldots, j_r$ and the jobs after j'_{r+1} (blue dots) is not changed and (2) the waiting time for jobs $j_{r+1}, \ldots., j'_{r+1}$ is not increased since the length of j_{r+1} is not larger than that of j'_{r+1}.

So, S' is also an optimal solution and G and S' have r+1 choices in common.
Repeat the process, we can conclude that there exists an optimum solution that is identical to G.