Exercise for week 3: (Public Holiday)

You got a baking gig on TV, and as you know, the most important thing about TV cooking is looks (taste comes second)! You need to make cookies live on-air. You lay the dough flat and cut out the pieces for each cookie, which you will then place on a baking tray with maximum length L (suppose all the cookies fit in the tray). But, as we know, we need to leave some space in between, since cookies grow when they bake. Suppose you leave a fixed length space between side-by-side cookies. One way to present the trays nicely is to fill the rows of the tray as much as possible, and move to the next row when the current cookie doesn't fit. One way to penalize “ugly" trays is to assign a penalty to each row that is not neat enough. For the sake of simplicity, suppose all the cookies have equal height.

Formally, you have trays of fixed length L, n cookies of width d1; d2, …, ; dn where di≤L for i ∈{1, 2, …, n}. A fixed distance s = 1 that represents the space you leave between cookies. If in row k, you place cookies i to j, then the unoccupied space in this row is computed as follows:

$$L - \sum_{h=i}^{j} d_h - (j - i)$$

Suppose your penalty function is computed as follows:

$$row\_penalty(i,j) = \begin{cases} \infty & \text{if cookies } i \text{ through } j \text{ do not fit in a row} \\ 0 & \text{if } j = n, \text{ last row} \\ L - \sum_{h=i}^{j} d_h - (j - i) & \text{otherwise} \end{cases}$$

Notice that you don't pay any penalty for the last row of cookies.

The total penalty for placing the cookies is the sum of the penalties of all rows. An optimal solution is a placement of the cookies into rows such that your total penalty is minimized. Notice that you do not shuffle the order of the cookies to achieve the best looking trays.

Devise a greedy algorithm to compute the optimal solution with minimum penalty function and show that it is correct.

If the penalty function is changed as follows:

$$row\_penalty(i,j) = \begin{cases} \infty & \text{if cookies } i \text{ through } j \text{ do not fit in a row} \\ 0 & \text{if } j = n, \text{ last row} \\ (L - \sum_{h=i}^{j} d_h - (j - i))^3 & \text{otherwise} \end{cases}$$

Does your algorithm still work?