Analysis of Capacity Improvement by Directional Antennas in Wireless Sensor Networks

Jun Zhang
Hong Kong Polytechnic University
Xiaohua Jia
City University of Hong Kong

and
Yuan Zhou
Huawei Technologies Co. Ltd

In this paper we analyze the capacity improvement by directional antennas over omni-antennas in wireless sensor networks. The capacity in our analysis is the end-to-end per-node throughput. We analyze the typical traffic pattern for sensor networks, where traffics are destined to or originated from the sink. The main results of our analysis are summarized as follows: 1) The network capacity is $O(1/N)$ for both omni and directional antennas, where N is number of sensor nodes in the network. 2) In the case of line deployment, the capacity ratio of directional antennas over omni antennas is bounded by $(2q+3)/(2q-1)$, where q is the ratio of interference radius to transmission radius. 3) In the case of two-dimensional deployment, the capacity of using directional antennas is $O(\frac{1}{\theta})$ for $m = 2$, and $O(\frac{\lg m}{\theta^2 \lg(1/\theta)})$ for $m > 2$, where m is the number of radios (antennas) on each node, and θ is the beamwidth of antennas. 4) When there are $n > 1$ sinks, the capacity has a non-monotonic relationship with the transmission radius. The optimal transmission radius depends on the ratio of n/q. 5) The capacity ratio of directional antennas over omni antennas in multi-channels networks decreases as the increase of channel number/radio number ratio c/m.

Categories and Subject Descriptors: C.4 [**PERFORMANCE OF SYSTEMS**]: Modeling techniques

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1. INTRODUCTION

Directional antennas exploit the spatial reuse to reduce signal interference in wireless networks and thus improve the network throughput. In recent years, the rapid development of

Author's address: Jun Zhang, Department of Computing, Hong Kong Polytechnic University, Hong Kong, email: junalex@alumni.ust.hk; Xiaohua Jia, Department of Computer Science, City University of Hong Kong, Hong Kong; email:jia@cs.cityu.edu.hk; Yuan Zhou, Huawei Technologies Co. Ltd, Shen Zhen, P.R. China, e-mail: michael@huawei.com. This work was supported by a grant from Research Grants Council of Hong Kong [Project No. CityU 114710] and NSF China Grant No. 60970117.

directional antenna technologies has made it possible for sensor nodes to be equipped with directional antennas [Giorgetti et al. 2007]. It is an important research topic to analyze how much performance gain that directional antennas can bring to the sensor networks. Although many studies have been done on the performance improvement by using directional antennas in wireless ad hoc networks, such as [Spyropoulos and Raghavendra 2003; Yi et al. 2003; Li et al. 2005; Tang et al. 2005; Roy et al. 2006; Zhang and Liew 2006; Yang et al. 2007; Dai et al. 2008], the analytical models used by them cannot be applied to sensor networks. This is because the traffic patterns in ad hoc networks are many-to-many, while in sensor networks all sensor nodes send their data to the sink node (which is a many-to-one traffic pattern).

In this paper, we analyze network capacity for wireless sensor networks by using directional antennas. The capacity is defined as the maximal per-node end-to-end throughput. For the purpose of comparison, we study the capacity for both directional and omni antennas. We assume each radio has an antenna attached to it and the multi-radios (with directional antennas) on a node can transmit or receive data simultaneously. We first focus our analysis in a single channel network with only one sink. Then, we extend our discussion to the cases of multiple sinks and multiple channels. Through the analysis, we show that the network capacity is $O(1/N)$, where N is the number of sensor nodes in the network. When there is only one sink, the network capacity can be improved by reducing the number of end-to-end hops k (i.e., the number of hops from the furthest sensor node to the sink in the routing tree), which can be achieved by either increasing the node transmission radius or decreasing the network diameter D. When there are multiple sinks, the network capacity has a non-monotonic relationship with the transmission radius, and the optimal transmission radius depends on the ratio n/q, where n is the number of sinks, and q is the ratio of interference radius to transmission radius. By employing directional antennas, in the case of two-dimensional region deployment, the capacity is $O(\frac{1}{\theta})$ for $m = 2$, and $O(\frac{\lg m}{\theta^2 \lg(1/\theta)})$ for $m > 2$, where m is the number of antennas on each node, and θ is the beamwidth of antennas, when $m\theta < 2\pi$; the capacity is $O((\frac{1}{\theta})^2)$ when $m\theta \geq 2\pi$. The capacity is always upper bounded by $\frac{nmW}{cN}$, where W is the channel bandwidth, and c is the number of channels.

The rest of the paper is organized as follows. The related work on the capacity of the wireless networks is in section 2. The main results of the paper is summarized in section 3. The framework of the analysis of the capacity is presented in section 4. The analysis of the network capacity with omni-antennas is presented in section 5. The analysis of the network capacity with directional antennas is presented in section 6. We extend our discussions to the network with multiple sinks, multiple channels, and same transmission power setting in section 7. Finally we conclude the paper in section 8.

2. RELATED WORK

In the literature of capacity analysis for wireless networks, two metrics of capacity are widely adopted, namely *total end-to-end capacity* and *per-node end-to-end capacity*. In our analysis, we use per-node capacity, which is the maximal per-node end-to-end throughput. For easy comparison, in the following related work, we converted all analytical results of total capacity to per-node capacity and we simply use capacity to refer per-node capacity in the rest of the paper.

The fundamental work by Gupta and Kumar in [Gupta and Kumar 2000] showed that the capacity of a random network of N nodes scales as $\Theta(\sqrt{1/(N \lg N)})$, when nodes are randomly placed and each node randomly chooses its communication pair. The capacity is $O(\sqrt{1/N})$ even with the optimal node placement and traffic pattern. This capacity bound was confirmed by the simulations of the IEEE802.11 in [Li et al. 2001; Li et al. 2002]. In order to improve the network capacity, different techniques are proposed. The work in [Gamal et al. 2004; Grossglauser and Tse 2002; Garetto et al. 2007] analyzed capacity of delay tolerant networks, and showed that capacity can possibly reach $\Theta(1)$ by introduce nodes' mobility. The work in [Liu et al. 2003; Kozat and Tassiulas 2003; Agarwal and Kumar 2004; Liu et al. 2007; Toumpis 2004] analyzed the capacity improvement by adding base stations into the network, where base stations are assumed to be interconnected with each other by wired lines. It was shown in [Kozat and Tassiulas 2003] that capacity scales as $\Theta(1/\lg N)$ by adding $\Theta(N)$ base stations. This capacity bound can be further improved to $\Theta(1)$, as shown in [Liu et al. 2003; Toumpis 2004; Agarwal and Kumar 2004], when $\Omega(\sqrt{N})$ base stations are added or power control is employed.

The analysis of network capacity was extended to multi-radios multi-channels systems in [Kyasanur and Vaidya 2005; 2009]. The analysis was based on the channel model that each channel has a bandwidth of W/c, where W is the total bandwidth, and c is the number of channels. We will employ this channel model in this paper. It was shown in [Kyasanur and Vaidya 2005; 2009] that capacity is $O(W\sqrt{\frac{Nm}{c}})$ when $\frac{c}{m} = O(N)$, and $O(W\frac{Nm}{c})$ when $\frac{c}{m} = \Omega(N)$, when each node has m radios.

As the increasing deployment of directional antennas in recent years, there are some works in the literature about the capacity analysis [Spyropoulos and Raghavendra 2003; Yi et al. 2003; Zhang and Liew 2006; Dai et al. 2008; Li et al. 2005] and capacity improvement [Kumar et al. 2006; Das et al. 2006] by using directional antennas. It was shown in [Yi et al. 2003] that directional antennas can gain $4\pi^2/(\alpha\beta)$ more capacity in random ad hoc networks, compared with omni-antennas, where α and β are the beamwidths of transmitters and receivers, respectively. The analysis was extended to the multi-radios multi-channels wireless ad hoc networks in [Dai et al. 2008]. Both works assumed that directional antennas of a node can cover the same transmission area as a node with omni-antenna. This assumption implies that the network topology by using directional antennas is the same as that by omni-antennas. However, this assumption can not hold for networks with small beamwidth non-steerable directional antennas.

Most of the existing works on capacity analysis are for ad hoc networks where both sources and destinations of traffic are the nodes inside the network. This is different from wireless sensor networks where traffic always destines to or originates from sink nodes. Network capacity is constrained by traffic aggregation around bottlenecks. In [Marco et al. 2003; Melo and Liu 2003], the capacity of a sensor network with a single sink was shown to scale as $\Theta(1/N)$. The work in [Jun and Sichitiu 2003] obtained a similar result with examples in the single-gateway mesh networks. The capacity bounds of networks with such many-to-one traffic pattern was later improved to $\Theta(\frac{lgN}{N})$ in [Gamal 2005] by exploiting data correlation between nearby sensors. The work in [Toumpis 2004] analyzed the capacity in a more general case. It showed that, for a network of N sources and N^d destinations, capacity scales as $\Theta(\sqrt{1/N})$ when $1/2 < d < 1$, and scales as $\Theta(N^{d-1})$ when $0 < d < 1/2$.

We analyze the capacity of sensor networks with omni-antennas or directional antennas in this paper. Part of the results on the capacity analysis for single channel networks were presented in [Zhang and Jia 2009]. Our capacity analysis model differs from the previous work in the following aspects:

(1) We take into consideration of the topology change due to insufficient coverage by directional antennas of a node, i.e., when $m\theta < 2\pi$. We show in our analytical model that such a topology change has significant impact on network capacity.
(2) We fully investigate the network capacity for many-to-one type of traffics under various network parameters, such as the number of radios, the number of channels, the number of sinks, transmission radius, and antenna beamwidths. Even though some previous works studied the capacity under some of these parameters, none of them have considered all these important parameters as in our model.
(3) The technique we use to analyze sensor networks with directional antennas is different from the previous works for ad hoc networks whose traffic patterns are many-to-many, particularly our technique for analyzing 2-dimensional deployment when the antennas of a node cannot cover the whole disk area of the node (i.e., $m\theta < 2\pi$).

3. MAIN RESULTS

Following the model used in [Kyasanur and Vaidya 2005], we assume the total bandwidth W is equally divided into c channels. The bandwidth for each channel is W/c. The main results of this paper are as follows:

(1) The capacity is $O(1/N)$ for both omni and directional antennas, where N is the number of sensors.
(2) When nodes are deployed along a line, the capacity ratio of directional antennas over omni-antennas is between

$$[1, (2q+3)/(2q-1)],$$

where q is the ratio of the interference radius to the transmission radius.
(3) When nodes are deployed in a 2-dimensional region, and each node has m radios with beamdwidth θ, the capacity ratio of directional antennas over omni-antennas is between $[1, O((\frac{1}{\theta})^2)]$ when $m\theta \geq 2\pi$, and between 1 and

$$\begin{cases} O(\frac{1}{\theta}) & \text{if } m = 2, \\ O(\frac{\lg m}{\theta^2 \lg(1/\theta)}) & \text{if } m > 2, \end{cases}$$

when $m\theta < 2\pi$.
(4) When the number of sinks $n = 1$, the capacity increases as the increase of transmission radius r_t. When $n > 1$, the capacity has a non-monotonic relationship with transmission radius r_t. The optimal transmission radius to maximize capacity depends on the ratio n/q. When $n \gg q$, the optimal transmission radius is small, in the order of $\frac{D}{(q+1)\sqrt{n}}$; otherwise the optimal transmission radius is large, in the order of $\frac{D}{\sqrt{n}}$, where D is the network diameter.
(5) When θ is sufficiently small, or channel number/radio number ratio c/m exceeds a certain threshold, the capacity is upper bounded by

$$Cap = O(\frac{nmW}{cN}).$$

The capacity ratio of directional antennas over omni-antennas decreases as the increase of c/m, and converges to 1 when c/m is sufficiently large.

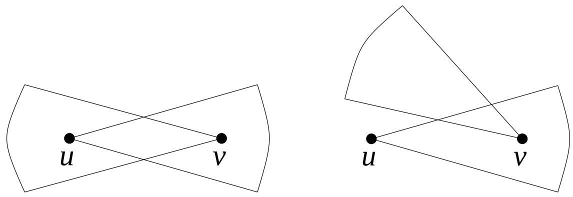
4. THE FRAMEWORK OF CAPACITY ANALYSIS

4.1 System model

We begin our analysis from the model with a single channel and a single sink. The work will be extended to a more general case of multi-sinks and multi-channels in section 7. The wireless sensor network under our analysis consists of one sink node and N sensor nodes. The traffic of nodes destines to or originates from the sink. Nodes are evenly distributed in a region of circle, where the sink node is located in the center of this circle. The diameter of the circular region is D. The system has a single communication channel. Our analysis includes both case of omni-antennas and directional antennas. For the case of directional antennas, we assume each node is equipped with m radios and each radio is associated with a directional antenna. Radios on the same node can be active at the same time. All antennas have the same beamwidth θ ($m\theta \leq 2\pi$) and antennas are not steerable (i.e., they cannot adjust their orientations). For the case of omni-antennas, we assume each node is equipped with a single radio. All nodes have fixed transmission radius r_t and communication links are bi-directional. Since each antenna corresponds to a radio on a node, we use antenna and radio interchangeably in the rest of the paper.

We start with the definitions of interference between nodes. The transmission area of an omni-antenna is a disk with transmission radius r_t, and its interference area is a disk with radius qr_t, where q is the ratio of interference radius to transmission radius, and $q \geq 1$. q is assumed to be an integer in this paper for ease of analysis. When q is not a integer, the network capacity is bounded by the one with the ratio $\lfloor q \rfloor$ and the one with $\lceil q \rceil$, and can be approximately estimated. The transmission area of a directional antenna with beamwidth θ and transmission radius r_t is a sector with angle θ and radius r_t, and its interference area is a sector with angle θ and radius qr_t. Such a radiation model is widely accepted in other previous works on capacity analysis with directional antennas, such as [Yi et al. 2003], [Dai et al. 2008], due to its simplicity in analysis. In practise, the radiation pattern of a directional antenna usually includes a high gain main lobe and low gain side/back lobes, and can be approximated by a main lobe with beamwidth θ plus a side lobe of beamwidth $2\pi - \theta$, as shown in [Ramanathan 2001]. According to the argument in [Sundaresan and Sivakumar 2004], the presence of side lobes contributes to the increase of interference. Nevertheless, as indicated by the discussion in the section 4.2 in [Yi et al. 2003], the capacity follows the same scaling law whenever side lobes are taken into consideration or not. Since the capacity is mainly determined by the size of the interference area of nodes, rather than the shape, there is no substantial difference in the capacity analysis between the circular sector model and a real world interference model for directional antennas that considers side lobes and back lobes. Therefore, in this paper, we do not consider the effect of side and back lobes.

For directional antennas, we assume both transmission and reception are directional, as discussed in paper [Yi et al. 2003]. Directional reception means an antenna can receive signals only from the direction it faces, which is the sector with beamwidth θ. As shown in Fig. 1(a), if antenna u is interfered by antenna v, u and v must face each other (i.e., u and v must be in each other's interference sector). Otherwise, as shown in Fig. 1(b), antenna u does not interfere with antenna v (and v does not interfere with u either) because v does

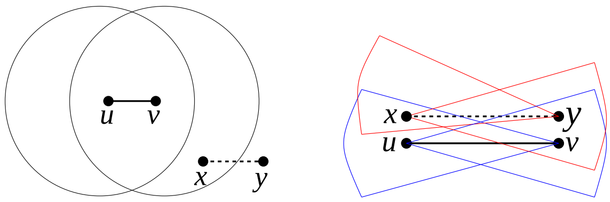


(a) u interferes with v.

(b) u does not interfere with v.

Fig. 1. Directional reception/interference with directional antennas.

not face u even though u faces v. We define that antenna u interferes with antenna v iff u and v are in each other's interference sector.



(a) Omni-antennas.

(b) Directional antennas.

Fig. 2. Interference between links.

The interference between links is defined as follows. Generally, the interference area of a link is the joint area covered by the two interference areas of the end-nodes of the link. A link l_i is said to interfere with link l_j iff one end-antenna of l_i interferes with one end-antenna of l_j. Two cases of link interference by omni-antennas and directional antennas are shown in Fig. 2(a) and Fig. 2(b) respectively. Link uv interferes with xy, because nodes v and x are in each other's interference sector, and interfere with each other. Note that links are bi-directional in our model. According to our definition, the interference between two links is symmetric. That is, if l_i interferes with l_j, l_j must interfere with l_i as well.

4.2 Performance metric

The capacity in our concern the per-node end-to-end capacity. We assume each node has the same end-to-end traffic demand, denoted by α. The capacity is defined as the maximal value of α that the system can support.

In sensor networks, the sink node uses a tree topology to collect data from or disseminate instructions to sensor nodes. We assume the tree is the shortest path tree. The traffic is merged at the tree nodes who further pass the traffic to their parent nodes towards the sink. Let $T(v_i)$ denote the subtree rooted at node v_i, $l(v_i)$ the link connecting subtree $T(v_i)$ to its parent node, and L_{v_i} the traffic on link $l(v_i)$, as show in Fig. 3. We have

$$L_{v_i} = \sum_{v_j \in T(v_i)} \alpha = \alpha |T(v_i)|, \tag{1}$$

where $|T(v_i)|$ is the size of $T(v_i)$, which is the number of nodes in $T(v_i)$, including v_i itself.

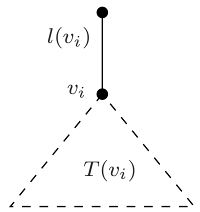


Fig. 3. Node v_i, its corresponding link $l(v_i)$, and its subtree $T(v_i)$.

For link $l(v_i)$, we define its collision set as a set of links that interfere with it, including $l(v_i)$ itself, and denote this collision set as $I(l(v_i))$. We define the (normalized) collision load of $l(v_i)$ as the total number of end-to-end flows of links in the collision set of $l(v_i)$, which is

$$L_{I(l(v_i))} = \sum_{l(v_j)\in I(l(v_i))} |T(v_j)|. \tag{2}$$

In wireless communication, two links that interfere with each other cannot be active at the same time due to the signal interference. We consider the most conservative case that no two links in the same collision set can be active at the same time in order to guarantee successful transmissions. That is, the collision load of any link, say $l(v_i)$, cannot exceed the bandwidth of a channel. Thus, we have:

$$\sum_{l(v_j)\in I(l(v_i))} L_{v_j} = \alpha \sum_{l(v_j)\in I(l(v_i))} |T(v_j)| \leq W, \tag{3}$$

where W is the channel bandwidth.

From ineq. (3), we have

$$\alpha \leq \frac{W}{\sum_{l(v_j)\in I(l(v_i))} |T(v_j)|} = \frac{W}{L_{I(l(v_i))}}. \tag{4}$$

The capacity of our concern is the maximal per-node end-to-end capacity, which is the maximal possible value of α that meets ineq. (4) for any node v_i. According to ineq. (4), we have

$$Cap = \min_{v_i} \frac{W}{L_{I(l(v_i))}} = \frac{W}{\max_{v_i} L_{I(l(v_i))}}. \tag{5}$$

Let L_{max} be the maximal collision load of the system, i.e., $L_{max} = \max_{v_i} L_{I(l(v_i))}$. The capacity can be re-written as

$$Cap = \frac{W}{L_{max}}. \tag{6}$$

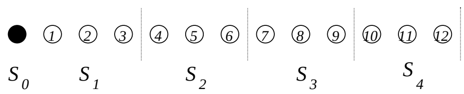
From the above definition of the capacity in eq. (6), we can see that by having the maximal collision load L_{max} of the system, we can compute the capacity. In the next two sections, we first analyze the maximal collision load of the networks with omni-antennas, and compute the capacity. Then, we extend our work to the networks with directional antennas.

5. ANALYSIS OF WIRELESS SENSOR NETWORKS OF OMNI-DIRECTIONAL ANTENNAS

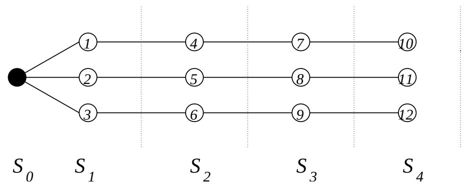
Before studying the general case of 2-dimensional node deployment, we first look at the case where nodes are distributed along a line. This is a typical deployment, when nodes are placed along a line, such as along a bridge or a highway.

5.1 1-dimensional line deployment

In case of line deployment, we assume all N sensor nodes are evenly placed in a line at one side of the sink node. The physical distance of the line is D. Let $k = \lceil D/r_t \rceil$, which represents the minimal number of hops from the furthest nodes to the sink. We divide the line of nodes into segments of length r_t (i.e., the transmission radius). Segment i refers to the section in the deployment line $((i-1)r_t, ir_t]$ from the sink, as shown in Fig. 4(a), where $i \in [1, k]$ and i is an integer. To be consistent, segment 0 contains only the sink node. We denote S_i as the set of nodes in segment i. Since nodes are evenly distributed, the number of nodes in each segment, except the segment 0, is N/k. Following the minimal hop routing, nodes in one segment are always connected to the nodes in the adjacent segments. Each node connects with a node in its left segment, and a node in its right segment. In general, when traffic is evenly distributed, there are $\lceil N/k \rceil$ node-disjoint paths, and each path contains exactly one node in each segment. When N is sufficiently large, $\lceil N/k \rceil$ is approximately N/k. An example of the topology of the line deployment by 12 nodes is illustrated in Fig. 4.



(a) Deployment of nodes in a line.



(b) Topology.

Fig. 4. Examples of line deployment.

For any node $v \in S_i$, its subtree contains all nodes from the segments on its right-hand side, one from each segment j, $j = i, i+1, \ldots, k$. Hence we have

$$\underset{v \in S_i}{|T(v)|} = k - i + 1. \tag{7}$$

As illustrated in Fig. 5, a link between segments $i-1$ and i interfere with all the links between segment $j-1$ and j for all js such that $\max(i-q-1, 1) \leq j \leq \min(i+q+1, k)$, $i \geq 1$, $j \geq 1$, because they have at least one end-node inside the interference area of this

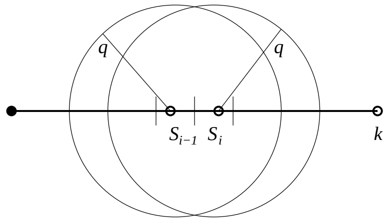


Fig. 5. The interference area of a link between segments $i-1$ and i in a line deployment sensor network.

link between segments $i-1$ and i. This means for any node $v \in S_i$, $I(l(v))$ is

$$\begin{aligned} I(l(v))_{v \in S_i} = \{ & l(u) : u \in S_j, \\ & j \in [\max(i-q-1, 1), \min(i+q+1, k)]\}. \end{aligned} \tag{8}$$

Therefore, the normalized collision load of any link $l(v)$, where $v \in S_i$, $i \geq 1$ is

$$\begin{aligned} L_{\substack{I(l(v)) \\ v \in S_i}} &= \sum_{j=\max(i-q-1,1)}^{\min(i+q+1,k)} \sum_{u \in S_j} |T(u)| \\ &= \sum_{j=\max(i-q-1,1)}^{\min(i+q+1,k)} (k-j+1)(N/k). \end{aligned} \tag{9}$$

Eq. (9) is an increasing function of i when $i \leq q+2$ and a decreasing function of i when $i \geq q+2$. Therefore, when $v \in S_{\min(q+2,k)}$, the normalized collision load reaches the maximum, which is

$$\max_v L_{I(l(v))} = \begin{cases} \frac{k+1}{2} N & \text{if } k \leq 2q+3 \\ \frac{(2q+3)(k-q-1)}{k} N & \text{otherwise.} \end{cases} \tag{10}$$

According to eq. (6), we have

$$Cap = \begin{cases} \frac{2W}{(k+1)N} & \text{if } k \leq 2q+3 \\ \frac{kW}{(2q+3)(k-q-1)N} & \text{otherwise.} \end{cases} \tag{11}$$

From eq. (11) we have the following observations:

a) When $k \leq 2q+3$, the capacity is independent from q, because the interference area of the link that has the largest collision load covers the whole network area. When $k > 2q+3$, the capacity decreases as the increase of q, due to the larger interference area.

b) The capacity is proportional to $1/N$. It is a decreasing function of k, but does not decrease to zero when $k \to \infty$. $k = 2q+3$ is a critical point of the curve of the capacity by k. When $k \leq 2q+3$, the capacity decreases as the increase of k, because the interference area of the link that has the largest collision load already covers the whole network area in this case and the increase of k will increase the number of segments inside that interference area. Note $k = \lceil D/r_t \rceil$. When D and N are fixed, increasing k means reducing node transmission radius and making per-hop distance shorter. When $k > 2q+3$, the decrease of the capacity as the increase of k is not significant, because the number of segments in the collision set of the maximal collision load becomes constant at $2q+3$, and load of each link in the maximal collision set, which is near the sink, converges to N. This result tells us that in sensor networks reducing node transmission power (for the purpose of energy saving or reducing interference) would decrease the capacity in general. But, when the network size is large enough (i.e., $k \geq 2q+3$), reducing transmission power has little impact on the capacity decrease.

c) When k = 1, the network has the highest capacity, which is W/N; when $k \to \infty$,

$$\lim_{k\to\infty} \frac{kW}{(2q+3)(k-q-1)N} = W/((2q+3)N).$$

Therefore, the capacity is in the range of $[W/((2q+3)N), W/N]$.

5.2 2-dimensional region deployment

In 2-dimensional region deployment, all N sensor nodes are assumed to be evenly distributed in a region of circle of diameter D and the sink node is located at the center of the circle. Let $k = \lceil D/(2r_t)\rceil|$, which represents the minimal number of hops from the furthest node to the sink. We divide the region into rings by circles centered at the sink and with radius from r_t, $2r_t$, $3r_t$,..., to kr_t. We denote ring i as the area between circles with radius $(i-1)r_t$ and ir_t, $i \in [1,k]$. To be consistent, ring 0 contains only the sink node. Let R_i denote the set of nodes in ring i. Since nodes are evenly distributed, the number of nodes in ring i, except the ring 0, is

$$|R_i| = \frac{\pi(i^2-(i-1)^2)}{\pi k^2}N = N(2i-1)/k^2. \quad (12)$$

In order to minimize the end-to-end hops, we assume that links always connects nodes in the adjacent rings, as shown in Fig. 6. The subtrees rooted from nodes in R_i contain all the nodes outside ring i, including R_i itself. Thus, we have for $i \geq 1$,

$$\sum_{v\in R_i} |T(v)| = \sum_{j=i}^{j=k} |R_j| = \sum_{j=i}^{j=k} N(2j-1)/k^2 = N(1-\frac{(i-1)^2}{k^2}). \quad (13)$$

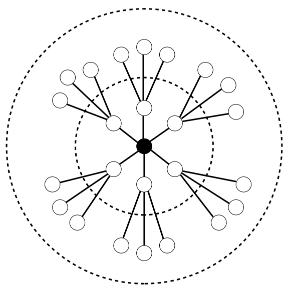


Fig. 6. Examples of the topology of 2-dimensional region deployment.

Since nodes are evenly distributed, the number of links falling into the interference area of a given link $l(v)$ is independent from the location of v, i.e., the number of links in the collision set of link $l(v)$ is a constant when the distance between two end-nodes is fixed. Considering the collision load defined in eq. (2), since the collision set size of a link is independent from its location, the maximal collision load of link $l(v)$ depends on the total size of subtrees whose parent links are in $I(l(v))$. Since the size of a subtree increases as its parent link gets closer to the sink, the collision load reaches the maximum for a link that is directly connected to the sink (i.e., a link between ring 1 and ring 0). Let $l(v^\star)$ be such a link between ring 1 and ring 0. The interference area of $l(v^\star)$ is the joint area of two circles with radius qr_t and centered at the two end-nodes of $l(v^\star)$. The interference area of $l(v^\star)$ is larger than the circle with radius qr_t but smaller than the circle with radius $(q+1)r_t$ all centered at the sink. Thus, we use the circle of radius qr_t centered at the sink as a close lower bound to approximate the interference area of $l(v^\star)$. The collision set of $l(v^\star)$ contains all the links between rings $i-1$ and i, for $i = 1, \ldots, \min(q+1, k)$. That is

$$I(l(v^\star)) \supseteq \{l(u) : u \in R_i, i \in [1, \min(q+1, k)]\}. \tag{14}$$

Therefore, we have

$$\begin{aligned} \max_v L_{I(l(v))} &\geq \sum_{i=1}^{\min(q+1,k)} \sum_{u \in R_i} |T(u)| \\ &= \sum_{i=1}^{\min(q+1,k)} N(1 - \frac{(i-1)^2}{k^2}.) \end{aligned} \tag{15}$$

According to eq. (6), we have

$$Cap \leq \begin{cases} \frac{W}{kN - N\sum_{i=1}^{i=k-1} i^2/k^2} & \text{if } k \leq q+1 \\ \frac{W}{(q+1)N - N\sum_{i=1}^{i=q} i^2/k^2} & \text{otherwise,} \end{cases} \tag{16}$$

and

$$Cap = O(\frac{W}{N\min(k, q+1)}). \tag{17}$$

From eq. (16) and eq.(17)we have the following observations:

a) When $k \leq q+1$, the capacity is independent from q, because the link of the maximal collision load interferes all links in the network. When $k > q+1$, the capacity decreases as the increase of q.

b) Similar to the line deployment, the capacity of 2-dimensional region deployment is $O(1/N)$, and decreases as the increase of k, i.e., increases as the increase of transmission radius r_t. $k = q+1$ is a critical point for the capacity as a function of k. When $k < q+1$, the capacity is almost inversely proportional to k; when $k \geq q+1$, the capacity remains nearly a constant, with respect to k. This shows that power control is an efficient way to improve the capacity of wireless sensor network, when the transmission power is so large such that all nodes can be interfered by the sink; otherwise, adjusting transmission power has marginal effect on the capacity.

c) When k = 1, the network has the highest capacity, which is W/N; when $k \to \infty$,

$$\lim_{k\to\infty} \frac{W}{(q+1)N - N\sum_{i=1}^{i=q} i^2/k^2} = W/((q+1)N). \tag{18}$$

The capacity is in the range of $[W/((q+1)N), W/N]$.

From the above analysis, we can see that, given a set of nodes N and deployment area D, the network with larger end-end hops (i.e., k) would have less capacity. But, when k is large enough, the capacity becomes at least $1/(\xi q)$ times of the upper bound of the capacity (i.e., $1/N$), where ξ is a constant depending on the network deployment shape. This result also tells us when constructing the topology for a large network, there is no need to pay attention to how far away nodes are connected, because the connection of those nodes has little impact on maximal collision load of the whole system (and thus the network capacity). The topology control algorithm should focus on how to connect the nodes that are close to the sink to effectively reduce the maximal collision load of the system and to increase the capacity.

6. ANALYSIS OF WIRELESS SENSOR NETWORKS OF DIRECTIONAL ANTENNAS

In the analysis in this section, each node is equipped with m directional antennas with beamwidth θ, and $m \geq 2$. All the other network parameters are the same as the section for omni antennas. We start with the case of line deployment.

6.1 Line deployment

When nodes are distributed along a line, two antennas on each node are sufficient to maintain the network connectivity. The directions of two antennas on each node is one for connecting left neighbors and one for right neighbors. We can see that an antenna on a node u pointing to its right direction interferes with an antenna on another node v pointing to its left direction iff the distance between u and v is less than qr_t and v is at the right hand side of u. Similar to the analysis for the line deployment with omni-antennas, we divide the line into segments with length r_t. A link between segment $i-1$ and i interferes with another link between segment $j-1$ and j, iff $|j-i| \leq q-1$, i.e., $i-q+1 \leq j \leq i+q-1$. Hence the collision set, $I(l(v))$, $v \in S_i$, is

$$\begin{aligned} \underset{v\in S_i}{I(l(v))} &= \{l(u) : u \in S_j, \\ & j \in [\max(i-q+1, 1), \min(i+q-1, k)]\}. \end{aligned} \tag{19}$$

The size of the subtree of each node v, $T(v)$, is the same as that in eq. (7). Following the similar derivation of the capacity as that in omni-antennas' case from eq. (7) to eq. (11), we obtain the capacity in a line-deployment network with directional antennas, which is

$$Cap = \begin{cases} \frac{2W}{(k+1)N} & \text{if } k \leq 2q-1 \\ \frac{kW}{(2q-1)(k-q+1)N} & \text{otherwise.} \end{cases} \tag{20}$$

From eq. (20) we can observe:

a) When $k \leq 2q-1$, the capacity is independent from q. Notice that this constraint of k is smaller than that of the omni-antennas, because directional antennas have a smaller interference area.

b) The network with directional antennas achieves higher capacity than (at least the same as) the network with omni-antennas. The capacity ratio of directional antennas over omni-antennas increases as the increase of k. The links with the largest collision set in directional antennas is between segment $q-1$ and q, while in omni-antennas these links appear between segment $q+1$ and $q+2$ (assuming $q+1 < k$). Therefore, the capacity bottleneck in directional antennas appears closer to the sink than that in omni-antennas.

c) Comparing the results in eq. (20) with eq. (11), when $k \leq 2q-1$, the network with directional antennas has the same capacity as omni-antennas; when $k \to \infty$, the capacity ratio of directional antennas over omni-antennas is

$$(2q+3)/(2q-1). \tag{21}$$

Thus, the capacity ratio of directional antennas over omni-antennas is in the range of $[1,(2q+3)/(2q-1)]$. Considering that omni-antennas has only 1 radio, and directional has two radios, the capacity is not necessary doubled by directional antennas, because the two radios on a node may not be able to be active at the same time due to the interference with the neighboring nodes. Notice that the capacity is inversely proportional to the maximal collision load, and the latter is approximately proportional to the interference area when both N and k are sufficiently large. Thus the capacity ratio decreases as the increase of q, due to the increase in the ratio of interference area by a link with directional antennas over a link with omni antennas (which is $\frac{2q-1}{2q+3}$). This suggests that there is more capacity gain of directional antennas over omni antennas when the interference ratio is small.

6.2 2-dimensional region deployment

For directional antennas deployed in a 2-dimensional region, we assume one antenna can serve several links (in a time sharing fashion) so long as they are within its transmission sector. We also assume that antennas on each node have fixed orientations, and transmission sectors of antennas on the same node do not overlap with each other, so that antennas on a node can be active at the same time. The whole transmission area of a node with m antennas of beamwidth θ is the collection of m sectors of angle θ and radius r_t. When $m\theta < 2\pi$, the transmission area of a node is not equivalent to a disk with radius r_t from this node. Some nodes may be outside this node's transmission area even when their distance to this node is within r_t, and they need to use multi-hops to connect to this node.

The analysis of the capacity of the 2-dimensional sensor network of directional antennas is different from that of omni-antennas, due to the following reasons: 1) Nodes may need more than $\lceil D/(2r_t)\rceil$ hops to reach the sink node, since the number of antennas on each node may not be sufficient to maintain the same topology as the omni-antennas. 2) The collision set of a link with directional antennas is different from that with omni-antennas.

Similar to the analysis for omni-antennas, the region is divided by circles centered at the sink and with radius from r_t, $2r_t$, $3r_t$,..., to kr_t. Now we look at the maximal collision load of links in the system. Similar to the argument in section 5.2, the number of links falling into the interference area of a link is independent from its location. The collision load of a link depends on the total size of subtrees whose parent links are in the collision set of this link, as defined in eq. (2). Thus, the collision load of links increases as they are closer to the sink. However, since the interference area of a link using directional antennas is the joint area of two sectors, the collision set of a link depends on the orientation of antennas of this link and the orientation of other nearby links. The link with the maximal collision load may not appear on the links that are directly connected to the sink. Nevertheless, the collision load of a link that directly connects to the sink is a close approximation of the maximal collision load in the system. Let $v^\star \in R_1$ be a node directly connecting to the sink and $l(v^\star)$ is the link connecting $v^\star$ to the sink. The interference area of link $l(v^\star)$ is fully covered by a circle from the sink with radius qr_t. To compute the collision load of $l(v^\star)$, we use the average size of subtrees whose parent links have at least one end-node in this circle as an approximation. Let $L_q = \sum_{i=1}^{\min(q+1,k)} \sum_{v\in R_i} |T(v)|$, which is the total size of all the subtrees whose parent links have at least one end-node falling into the circle with radius $\min(q, k)$ and centered at the sink. L_q is actually the total load of all the links that have at least one end-node in the circle. The total load of the links that have at least one end-node in the interference area of $l(v^\star)$ is $L_q\phi$, where ϕ is the ratio of the size of the interference area of $l(v^\star)$ over the size of this circle. According to the link interference definition of antennas, not all the links that have an end-node in the interference area of $l(v^\star)$ interfere with $l(v^\star)$. Let ρ_0 be the probability that a link interferes with $l(v^\star)$, given that an end-node of this link is in the interference area of $l(v^\star)$. The average total load of links in the collision set of $l(v^\star)$ is

$$L_{I(l(v^\star))} = L_q\phi\rho_0. \tag{22}$$

We use this average collision load as a close lower bound of the maximal collision load in the system. That is

$$\max_v L_{I(l(v))} \geq L_q\phi\rho_0. \tag{23}$$

On the other hand, the sink has N nodes in its tree and it has m antennas for use. Thus, we have

$$\max_v L_{I(l(v))} \geq N/m. \tag{24}$$

In the following, we will show how to obtain L_q, ϕ and ρ_0.

First, we consider computing L_q. In 2-dimensional region deployment, when $m\theta < 2\pi$, directional antennas on each node is not sufficient to cover the whole disk of radius r_t centered at that node. Thus the sink node needs multiple hops to connect nodes in the ring 1, and this multiple hop connection may also happen between other adjacent rings. Since not all the nodes in ring i are the parents of nodes in ring $i+1$ due to this multi-hop connection, we cannot compute L_q as we did in eq. (15). We divide the region into two parts, the area inside ring 1 and the area outside ring 1. Let L_{R_1} denote the total size of all the subtrees whose roots are inside ring 1, and $L_{R_{2*}}$ the total size of all the subtrees whose roots are in ring 2 or beyond. That is:

$$L_{R_1} = \sum_{v \in R_1} |T(v)|, \tag{25}$$

$$L_{R_{2*}} = \sum_{i=2}^{\min(q+1,k)} \sum_{v \in R_i} |T(v)|. \tag{26}$$

We have

$$L_q = L_{R_1} + L_{R_{2*}}. \tag{27}$$

We first calculate L_{R_1} as follows. Let h denote the number of hops necessary to cover all the nodes in ring 1. At least $\frac{\theta}{2\pi}\frac{mN}{k^2}$ nodes in ring 1 can be covered by the m antennas from the sink. For those nodes that are directly connected to the sink, each of them uses 1 antenna for the uplink to the sink, and $m-1$ antennas to link children. They can establish links to cover at least $\frac{\theta}{2\pi}\frac{mN}{k^2}(m-1)$ child nodes, which can further link $\frac{\theta}{2\pi}\frac{mN}{k^2}(m-1)^2$ child nodes. We assume nodes in ring 1 have higher priority to be linked than nodes outside ring 1. That is, no node outside ring 1 can be linked to the tree before the nodes inside ring 1. Repeat the above operation. Let H_i denote the set of nodes that are i hops to the sink.

We have

$$|H_i| = \frac{Nm\theta}{2\pi k^2}(m-1)^{i-1}. \tag{28}$$

Since

$$\sum_{i=1}^{i=h-1} |H_i| \le |R_1| \le \sum_{i=1}^{i=h} |H_i|, \tag{29}$$

we have

$$\frac{Nm\theta}{2\pi k^2}\sum_{i=0}^{h-2}(m-1)^i \le N/k^2 \le \frac{Nm\theta}{2\pi k^2}\sum_{i=0}^{h-1}(m-1)^i. \tag{30}$$

Thus, we have

$$\begin{cases} \frac{\lg(\frac{2\pi(m-2)}{\theta m}+1)}{\lg(m-1)} \le h \le \frac{\lg(\frac{2\pi(m-2)}{\theta m}+1)}{\lg(m-1)} + 1 & \text{if } m > 2 \\ \pi/\theta \le h \le \pi/\theta + 1 & \text{if } m = 2. \end{cases} \tag{31}$$

.

Since the total number of nodes in the subtrees whose roots are in H_i plus the number of the nodes that are less than i hops to the sink is N, we have, for each $i \le h$,

$$\sum_{v\in H_i} |T(v)| = N - \sum_{j=1}^{j=i-1}\sum_{v\in H_j} 1 = N - \frac{N\theta m}{2\pi k^2}\sum_{j=0}^{i-2}(m-1)^j. \tag{32}$$

Summing up the size of the subtrees whose roots are in H_i for all $1 \le i \le h$, we have

$$\begin{aligned} L_{R_1} &= \sum_{i=1}^{i=h}\sum_{v\in H_i} |T(v)| \\ &= \begin{cases} Nh - N\frac{\theta m[(\frac{(m-1)^h-1}{m-2})-h]}{2\pi(m-2)k^2} & \text{if } m > 2 \\ Nh - Nh(h-1)\frac{\theta}{2\pi k^2} & \text{if } m = 2. \end{cases} \end{aligned} \tag{33}$$

Next we calculate $L_{R_{2*}}$. $L_{R_{2*}}$ reaches its minimum when all the nodes in ring $i+1$ are directly connected to the nodes in ring i, for all $i > 1$. This is because, given the size of a tree, the total size of all subtrees for a shorter balanced tree is greater than that for a taller balanced tree. In fact, when N is large enough, all the nodes in ring $i+1$ can be directly connected to the nodes in ring i, for all $i > 1$. Thus, a close lower bound of $L_{R_{2*}}$ is the case when all the nodes in ring $i+1$ are directly connected to the nodes in ring i, $i > 1$. With the similar technique as in section 5.2, we have

$$L_{R_{2*}} \ge \begin{cases} N\sum_{i=1}^{i=k-1}(1 - i^2/k^2) & \text{if } k \le q+1 \\ N\sum_{i=1}^{i=q}(1 - i^2/k^2) & \text{otherwise.} \end{cases} \tag{34}$$

We calculate ϕ as follows. Let a and b be the two end-nodes of a link which is incident to the sink, and A and B be the two interference sectors centered at a and b, respectively. We define $z = \min(q, k)$. The joint area of the two sectors must be inside the circle of radius zr_t, centered at the sink. As shown in Fig. 7, the two sectors are with angle θ and radius zr_t. Since the distance between a and b is less than r_t, we can always find a sector of angle θ and radius $(z-1)r_t$, inside sector A and outside sector B. Let us denote this sector as A'. The size of the joint area of sector A and B is at least

$$\frac{\theta 1}{2}(zr_t)^2 + \frac{\theta}{2}((z-1)r_t)^2. \tag{35}$$

Therefore, we have

$$\phi \geq \frac{2z^2 - 2z + 1}{2z^2}\frac{\theta}{\pi}. \tag{36}$$

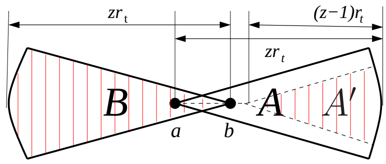


Fig. 7. The joint area of two sectors A and B of angle θ and radius q. $||ab|| \leq zr_t$.

We calculate ρ_0 as follows. Let $P_I(u,v)$ be the probability that a node u is in the interference sector of an antenna at a node v with beamwidth θ, $||uv|| \leq qr_t$. We have

$$P_I(u,v) = \theta/(2\pi). \tag{37}$$

Suppose that link $l = (s,t)$ has at least one end-node, say node s, inside the interference area of link $l(u) = (u,v)$, $||su|| \leq qr_t$ and $||sv|| \leq qr_t$. The probability ρ_0 that link l interferes with link $l(u)$ is at least the probability that u or v falling into the interference sector of s. Thus, we have

$$\rho_0 \geq 1 - (1 - P_I(u,s))(1 - P_I(v,s)) = \theta/\pi - (\theta/(2\pi))^2. \tag{38}$$

Substitute L_q, ϕ, ρ_0 into eq. (23) from eq.s (27), (36), (38) and consider the constraint in eq. (24), the capacity according to eq. (6) is

$$Cap \leq \frac{W}{\max(N/m, \frac{2z^2-2z+1}{2z^2}((\frac{\theta}{\pi})^2 - (\frac{1}{4}(\frac{\theta}{\pi}))^3)(L_{R_1} + L_{R_{2*}}))}. \tag{39}$$

Notice that $L_{R_1} = O(Nh)$, and $L_{R_{2*}} = O(\min(k-1,q)N)$. We have

$$Cap = \min(O(\frac{W}{(\frac{\theta}{\pi})^2 N(h + \min(k-1,q))}), \frac{mW}{N}), \tag{40}$$

and

$$h = \begin{cases} O(\frac{\pi}{\theta}) & \text{if } m = 2, m\theta < 2\pi, \\ O(\frac{\lg \frac{2\pi}{\theta}}{\lg m}) & \text{if } m > 2, m\theta < 2\pi, \\ 1 & \text{if } m\theta = 2\pi. \end{cases} \tag{41}$$

From eq. (39) and eq. (40) we have the following observations:

a) Directional antennas achieve a higher capacity than omni-antennas, and this capacity increases as the decrease of beamwidth θ, and increases as the increase of number of antennas m, when $m\theta < 2\pi$. The reason is as follows. The collision load depends on two factors: 1) the size of the collision set (which is proportional to $\frac{2z^2-2z+1}{2z^2}((\frac{\theta}{\pi})^2 - (\frac{1}{4}(\frac{\theta}{\pi}))^3)$), and 2) the total size of all the subtrees whose roots are in the collision set. The former is $O(\theta^2)$. The latter is proportional to $L_{R_1}+L_{R_{2*}}$. According to eq. (33), L_{R_1} is $O(h)$, which is $O(1/\theta)$ for $m = 2$ or $O(\frac{\lg(1/\theta)}{\lg m})$ for $m > 2$ (see eq. (31)). $L_{R_{2*}}$ is independent from θ (see eq. (34)). Thus, the collision load is $O(\theta)$ for $m = 2$ or $O(\frac{\theta^2 \lg(1/\theta)}{\lg m})$ for $m > 2$. Hence, the capacity is

$$\begin{cases} O(\frac{1}{\theta}) & \text{if } m = 2, \\ O(\frac{\lg m}{\theta^2 \lg(1/\theta)}) & \text{if } m > 2, \end{cases} \tag{42}$$

when $m\theta < 2\pi$. Different from capacity scaling law for ad hoc networks with directional antennas in [Yi et al. 2003], the capacity improvement by directional antennas in sensor networks is less than $\frac{1}{\theta^2}$ when $m\theta < 2\pi$, due to more end-to-end hops.

When $m\theta = 2\pi$, the network with directional antennas has the same topology as that with omni antennas. Thus the capacity improvement by directional antennas in such a case only depends on the reduction of size of the collision set. The capacity is

$$O((\frac{1}{\theta})^2), \tag{43}$$

when $m\theta = 2\pi$.

However, the capacity does not increase to infinity with the decrease of θ. It is upper bounded by

$$mW/N, \tag{44}$$

according to the eq. (24).

We can observe that, when the capacity is under the upper bound mW/N, the capacity does not improve linearly by adding more antennas on each node, because the total size of all the subtrees whose roots are in the collision set, i.e. $(L_{R_1} + L_{R_{2*}})$, depends on the total number of hops to cover those nodes from the sink, i.e. $q + h$, while $q + h$ is $O(\frac{1}{\lg m})$.

b) The capacity ratio of directional antennas over omni-antennas is

$$\min[O(\frac{(\frac{\pi}{\theta})^2 \min(k, q+1)}{(h - 1 + \min(k, q+1))}), m \min(k, q+1)]. \tag{45}$$

When $k \to \infty$, the capacity ratio is upper bounded by

$$\min[O(\frac{(\frac{\pi}{\theta})^2 (q+1)}{q+h}), m(q+1)]. \tag{46}$$

This ratio reaches the maximal value $m(q + 1)$ when beamwidth θ is sufficiently small.

c) Since the number of nodes in the ring 1 is N/k^2, and an antenna will not be used when it does not cover any node in its transmission sector, the number of used antennas on the sink is at most N/k^2. When $m = \Theta(N/k^2)$ and θ is sufficiently small, the capacity can reach a upper bound as $O(W/k^2)$, regardless of the number of nodes N.

7. DISCUSSIONS

In the above analysis, we assume that there is only one sink node, and one channel in the system. In the following, we will extend our analysis to a more general case of multiple sinks and multi-channels. What is more, we will analyze how the capacity of networks with directional antennas changes with the beamwidth when the transmission power, rather than the transmission radius is a fixed.

7.1 Network capacity for multiple sinks

When multiple sinks are deployed in a sensor network, the capacity will be improved, compared with a single-sink network, because the end-to-end hops to the sinks are reduced, and the traffic load aggregated at each sink decreases. As what we will show in this subsection, the relationship between the capacity and the number of sinks is not necessarily linear. It depends on transmission radius, the number of sinks, and even beamwidth of antennas. Furthermore, different from the single-sink case, the capacity of network with multiple sinks has a non-monotonic relationship with transmission radius r_t.

The network model for multiple sinks is as follows. Suppose that there are n sinks and N sensor nodes evenly distributed in a 2-dimensional region, inside the disk of diameter $D = 2kr_t$, where r_t is the transmission radius. k reflects the maximal number of hops from nodes to the center of the region. Let each sensor node connect to its nearest sink through the path with the minimal number of hops. The sensor network consists of n disjoint subtrees, one for each sink. We denote k_n as the maximal number of hops from each sink to sensor nodes in this subtree. According to the Lemma 1 in [Toumpis 2004], when nodes are randomly distributed, each subtree has $O(N/n)$ sensors with high probability, when N is sufficiently large. We assume that each sensor only selects its closest sink. Thus the sensors for each sink are distributed in the disk of radius $k_n r_t$ centered at the sink.

We have the following theorem on the capacity of a network with multiple sinks.

THEOREM 1. *The capacity of a network with multiple sinks is as follows:*
1) When sinks are sparsely distributed, i.e., $k_n \geq \Omega(1)$ $(n \leq k^2 = O(\frac{D}{2r_t})^2)$*), the network capacity is*

$$Cap = \begin{cases} O(\frac{r_t}{D}\frac{\sqrt{n}W}{N}) & \text{if } \frac{D}{2r_t} \leq O(q+1), \\ O(\frac{D}{r_t}\frac{\sqrt{n}W}{(q+1)^2 N}) & \text{if } \Omega(q+1) \leq \frac{D}{2r_t} \leq O((q+1)\sqrt{n}), \\ O(\frac{nW}{(q+1)N}) & \text{if } \frac{D}{2r_t} \geq \Omega((q+1)\sqrt{n}). \end{cases} \quad (47)$$

2) When sinks are densely distributed, i.e., $k_n = 1$ $(n \geq k^2 = \Omega(\frac{D}{2r_t})^2)$*), there is no capacity improvement by adding more sinks. The capacity in this case is*

$$Cap = \begin{cases} O(\frac{W}{N}) & \text{if } \frac{D}{2r_t} \leq O(q+1), \\ O((\frac{D}{r_t})^2\frac{W}{(q+1)^2 N}) & \text{if } \Omega(q+1) \leq \frac{D}{2r_t} \leq O(\sqrt{n}). \end{cases} \quad (48)$$

Proof. For a single-sink sensor network of N sensor nodes and of radius kr_t, the capacity is $O(W/(kN))$ when k is small ($k \leq q+1$), and $O(W/[(q+1)N])$ when k is large ($k > q+1$), according to the capacity with omni-antennas by eq. (16), and with directional antennas by eq. (39).

Now, we consider the capacity of this multi-sinks network. When k_n is large ($k_n \geq q+1$), the sink nodes are quite far away from each other (in terms of hops) and they do not interfere with each other. Notice that

$$k_n = \Theta(k/\sqrt{n}), \tag{49}$$

so $k \geq \Omega((q+1)\sqrt{n})$. In this case, the capacity of the whole network is equivalent to the capacity of a subtree network with one sink and $O(N/n)$ nodes. Thus the network capacity is

$$Cap = O(\frac{W}{(q+1)N/n}) = O(\frac{nW}{(q+1)N}). \tag{50}$$

When k_n is small ($k_n \leq q+1$, i.e., $k \leq O((q+1)\sqrt{n})$), sinks may interfere with each other. The link that has the maximal collision load in a subtree may interfere other sinks, which results in a much larger maximal collision load for the whole system. The maximal collision load in the whole system is at most the collision load of a subtree with one sink node and N/n sensors, times the number of interfered sinks. Notice that the density of sinks is $\frac{n}{\pi(kr_t)^2}$, and the interference area of one link is $\Theta(\pi((q+1)r_t)^2)$, so the number of interfered sinks by one link is

$$min(n, \Theta(\frac{n\pi((q+1)r_t)^2}{\pi(kr_t)^2})). \tag{51}$$

Thus, when $k_n \geq 1$, i.e., $k \geq \Omega(\sqrt{n})$, the network capacity is

$$Cap = O(\frac{W}{(N/n)(k/\sqrt{n})\min(n, n\frac{(q+1)^2}{k^2})}). \tag{52}$$

When $k_n = 1$, i.e., $k \leq O(\sqrt{n})$, sinks are densely distributed, i.e., each sensor can find a sink within its transmission radius. The network capacity in this case becomes

$$O(\frac{W}{(N/n)\min(n, n\frac{(q+1)^2}{k^2})}). \tag{53}$$

Combining the results from eq. (50), eq. (52), and eq. (53), and noticing that $r_t = D/(2k)$, we can derive the capacity of a network with multiple sinks, which is shown in eq. (47), and eq. (48). □

From Theorem 1, we have the following observations:

a) When multiple sinks are sparsely distributed, the capacity has a non-monotonic relationship with transmission radius. When transmission radius r_t is so small that sinks do not interfere with each other, the capacity remains nearly as a constant; when r_t is large enough for sinks to interfere with nearby sinks, the capacity decreases as the increase of r_t, due to the increase of interference between nearby sinks (for networks with one single sink, such a stage does not appear as there is no interference from other sinks); when r_t increases to the level such that the collision set of each link becomes saturated, the capacity increases as the increase of r_t, due to the reduction of end-to-end hops between nodes and sinks. When sinks are densely distributed, its capacity is equivalent to the capacity of the network with $O(k^2)$ sinks. The capacity with dense distribution of sinks quadratically decreases as the increase of r_t, due to the quadratical increase of interference area by the increase of r_t.

b) Still in the case of sparse deployment of sinks, the capacity has two local maximums at $r_t = \Theta(\frac{D}{(q+1)\sqrt{n}})$, and $r_t = \Theta(\frac{D}{q+1})$, with value $O(\frac{Wn}{(q+1)N})$ and $O(W/N)$ respectively, according to Theorem 1. When $r_t = \Theta(\frac{D}{(q+1)\sqrt{n}})$, the furthest sensor for each sink is at hops $\Theta(q+1)$ away, and sinks do not interfere with each other. When $r_t = \Theta(\frac{D}{q+1})$, each sink is interfered by all other sinks, and each sensor is connected with a sink within one hop. The optimal transmission radius depends on the relationship between sink number n and ratio of interference radius to transmission radius q. When $n \gg q$, the highest capacity of a sensor network is $O(\frac{nW}{(q+1)N})$, achieved by a small transmission radius; otherwise, the highest capacity of a sensor network is only $O(W/N)$, achieved by a large transmission radius such that all nodes connect to sinks within one hop. This implies that reducing transmission power can improve the network capacity only if the number of sinks are sufficiently large, otherwise transmitting packets at the highest power level is the optimal choice to maximize capacity.

c) The capacity increases as the increase of the number of sinks n, and the increasing rate decreases as more sinks are added to the network. When sinks do not interfere with each other, the capacity increasing rate is $O(n)$; when sinks interfere neighboring sinks, but can not connect its corresponding sensors within one hop, the capacity increasing rate is $O(\sqrt{n})$; when the number of sinks keeps increasing and all sensors have at least one sink within one hop, the capacity does not increase with the number of sinks. The relationship between the capacity and sinks number n can be expressed as

$$Cap = \begin{cases} O(n) & \text{if } n \leq O((\frac{D}{(q+1)r_t})^2), \\ O(\sqrt{n}) & \text{if } \Omega((\frac{D}{(q+1)r_t})^2) \leq n \leq O((\frac{D}{r_t})^2), \\ O(1) & \text{if } n \geq \Omega((\frac{D}{r_t})^2). \end{cases} \tag{54}$$

When directional antennas are employed, the capacity following the same scaling law in eq. (54) when the beamwidth is not small, and the capacity ratio of directional antennas over omni-antennas is the same as the single-sink case. When beamwidth is small, capacity can even super-linearly increase as the increase of n, if $1 \leq k_n \leq q+1$ ($\Omega(\sqrt{n}) \leq k \leq O((q+1)\sqrt{n})$). Similar to eq. (51), the number of interfered sinks by each directional link is

$$\min(n, \Theta(\frac{\theta}{2\pi}\frac{n\pi((q+1)r_t)^2}{\pi(kr_t)^2})), \tag{55}$$

which is $O(1)$ when θ is enough small. Similar to the argument in Theorem 1, the capacity is

$$Cap = O(\frac{W}{(N/n)(k/\sqrt{n})}) = O(\frac{r_t n^{1.5} W}{DN}). \tag{56}$$

In such a case, the capacity ratio of directional antennas over omni-antennas is $O(\frac{(q+1)^2 n}{k^2})$, which increases as more sinks are placed. Nevertheless, when beamwidth keeps decreasing, the network capacity is limited by the number of radios on each node rather than interference, and scales as

$$Cap = O(\frac{mW}{N/n}) = O(\frac{mnW}{N}). \tag{57}$$

The capacity ratio of directional antennas over omni-antennas in this case is in the range of $[m(q+1), mn]$ if $q+1 \leq n$, and $[mn, m(q+1)]$ if $n \leq q+1$.

7.2 Network capacity for multiple channels

The network model for multiple channels is similar to that in Section 7.1. In addition, we adopt the channel model in [Gupta and Kumar 2000] and [Dai et al. 2008], which is the channel model 1 in [Kyasanur and Vaidya 2005]. The total bandwidth W is equally divided among c channels, and therefore the bandwidth of each channel is W/c. We assume that radio number m does not exceed c for omni-antennas, and does not exceed $2\pi c/\theta$ for directional antennas. Because otherwise there are redundant antennas on the same channel, which cannot be active simultaneously on the same node. We will show that, when a sensor network has multiple channels, the capacity scaling laws with the number of nodes remains as $O(1/N)$, and directional antennas still outperform omni-antennas in such a network.

Network capacity is limited by three constraints: *sink bottleneck constraint*, *arbitrary interference constraint*, and *maximal collision load constraint*.

(1) *Sink interface constraint*: since all flows destine to sinks, capacity is restricted by the number of radios (interfaces) at sinks. As each sink has m radios and each radio can support at most W/c bits/sec, the *sink interface constraint* capacity bound λ_S is expressed as

$$\lambda_S = O(\frac{nmW}{cN}). \tag{58}$$

(2) *Arbitrary interference constraint*: according to eq. (7) of appendix I in [Kyasanur and Vaidya 2005], and eq. (9) of appendix A in [Dai et al. 2008], the per-node capacity of arbitrary wireless networks with directional antennas is limited by interference as

$$Cap \times N \times D_n = O(\frac{W}{\theta}\sqrt{\frac{Nm}{c}}),$$

where D_n is average distance between nodes and sinks. Notice that $D_n = \Theta(1/\sqrt{n})$, the *arbitrary interference constraint* capacity bound λ_A is expressed as

$$\lambda_A = O(\frac{W}{\theta}\sqrt{\frac{nm}{cN}}). \tag{59}$$

(3) *Maximal collision load constraint*: the capacity of a sensor network is constrained by maximal collision load. When there are multiple channels, collision load will be distributed among different channels, resulting in a smaller collision load per link. For each link, the number of different channels used by links in its collision set is at most c. There exists at least one channel, such that its collision load is at least $1/c$ of the original collision load for a single channel. Thus the capacity is

$$Cap \le \frac{W/c}{L^s_{max}/c} = \frac{W}{L^s_{max}}, \tag{60}$$

where L^s_{max} is the maximal collision load for a network with a single channel. It means that the capacity upper bound by maximal collision load constraint is not relevant to the number of channels c. Let λ_M^{omni}, and λ_M^{dir} denote the single channel *maximal collision load constraint* capacity bound with omni-antennas and directional antennas, respectively. The former, λ_M^{omni}, can be referred to eq. (47), assuming that sinks are sparsely distributed. When sinks are densely distributed, the capacity bound is equivalent to case with $\Theta(k^2)$ sparsely distributed sinks, according to Theorem 1.

The latter, λ_M^{dir}, can be expressed as

$$\lambda_M^{dir} = f_{omni}^{dir} \lambda_M^{omni}, \tag{61}$$

where f_{omni}^{dir} is the capacity gain of directional antennas over omni-antennas in a single channel network. Similar to the observation (a) for eq. (39), the value of f_{omni}^{dir} for different m and θ can be found in eq. (42) and eq. (43).

Combining the three constraints, the capacity is $O(\min(\lambda_S, \lambda_A, \lambda_M^{omni}))$ for omni-antennas, and $O(\min(\lambda_S, \lambda_A, \lambda_M^{dir}))$ for directional antennas. We have the following theorems on the capacity of multi-channels networks. Notice that since $n \leq N$ and $k \leq N$, the *arbitrary interference constraint* is always either dominated by *sink interface constraint*, or *maximal collision load constraint*.

THEOREM 2. *Let Γ_{omni} denote the c/m ratio threshold for omni-antennas, which is*

$$\Gamma_{omni} = \begin{cases} \frac{D}{2r_t}\sqrt{n} & \text{if } \frac{D}{2r_t} \leq O(q+1), \\ \frac{2(q+1)^2 r_t \sqrt{n}}{D}) & \text{if } \Omega(q+1) \leq \frac{D}{2r_t} \leq O(q+1)\sqrt{n}, \\ q+1 & \text{if } \frac{D}{2r_t} \geq \Omega((q+1)\sqrt{n}). \end{cases} \tag{62}$$

The capacity of a network with omni-antennas and multiple channel is as follows:
1) when $c/m \leq \Gamma_{omni}$, the capacity is

$$\begin{cases} O(\frac{W r_t \sqrt{n}}{DN}) & \text{if } \frac{D}{2r_t} \leq O(q+1), \\ O(\frac{WD\sqrt{n}}{(q+1)^2 N r_t}) & \text{if } \Omega(q+1) \leq \frac{D}{2r_t} \leq O(q+1)\sqrt{n}, \\ O(\frac{Wn}{(q+1)N}) & \text{if } \frac{D}{2r_t} \geq \Omega((q+1)\sqrt{n}); \end{cases} \tag{63}$$

2) when $c/m \geq \Gamma_{omni}$, the capacity is $O(\frac{nmW}{cN})$.

THEOREM 3. *Let Γ_{dir} denote the c/m ratio threshold for directional antennas, which is $\Gamma_{omni}/f_{omni}^{dir}$. The capacity ratio of directional antennas over omni-antennas is:*
1) when $c/m \leq \Gamma_{dir}$, the ratio is f_{omni}^{dir};
2) when $\Gamma_{dir} \leq c/m \leq \Gamma_{omni}$, the ratio is

$$\max(1, f_{omni}^{dir} \times \begin{cases} \Theta(\frac{m}{c}\frac{D\sqrt{n}}{r_t}) & \text{if } \frac{D}{2r_t} \leq O(q+1) \\ \Theta(\frac{m}{c}\frac{(q+1)^2 r_t \sqrt{n}}{D}) & \text{if } \Omega(q+1) \leq \frac{D}{2r_t} \leq O(q+1)\sqrt{n}); \\ \Theta(\frac{m}{c}(q+1)) & \text{if } \frac{D}{2r_t} \geq \Omega((q+1)\sqrt{n}) \end{cases} \tag{64}$$

3) when $c/m \geq \Gamma_{omni}$, the ratio is 1.

From Theorem 2 and Theorem 3, we have the following observations:

a) The capacity is always $O(1/N)$, in regardless of c/m ratio. This shows that a sensor network has different capacity scaling rule to that of a wireless multi-radios multi-channels ad hoc network in [Kyasanur and Vaidya 2009] and [Dai et al. 2008], because of its special many-to-one traffic pattern.

b) The capacity depends on c/m ratio. When c/m is below the threshold Γ_{omni} for omni-antennas or Γ_{dir} for directional antennas, the capacity is only constrained by interference and not relevant to c/m; when c/m exceeds the threshold, the capacity is constrained by the number of radios rather than interference, and decreases as the increase of c/m. In addition, different from the ad hoc networks of many-to-many traffic pattern, the c/m

thresholds for a sensor network only depend on the number of sinks n and the transmission radius r_t, rather than the number of nodes N.

c) Directional antennas have higher capacity than omni-antennas in multi-channels sensor networks. The ratio of directional antennas over omni-antennas decreases and converges to 1 when c/m increases. This suggests to use directional antennas in the scenario where there are not many channels. When the number of channels are sufficiently large, directional antennas have no gain in capacity than omni-antennas, if they have the same transmission radius. When there are sufficient channels, and when the transmission power is same for omni-antennas and directional antennas, the directional antenna achieves more capacity only because it has a larger transmission radius.

7.3 Network capacity with the same transmission power setting

In previous sections of the paper, we focus on the capacity analysis when the transmission radius is same for both omni-antennas and directional antennas. In the case that the transmission power is same for both omni-antennas and directional antennas, their transmission radii are different, due to different antenna gain, as shown by the results in [Ramanathan 2001]. In general, directional antennas improve the network connectivity [li et al. 2009], [Bettstetter et al. 2005] and reduce hop counts [Vilzmann et al. 2005] on average. Given the path loss exponent γ, according to the calculation in the eq. (7) of [Dai 2009], the transmission radius of a directional antenna with beamwidth θ, denoted by $t_{r,dir}$, can be expressed as follows:

$$t_{r,d} = (\frac{2}{\tan(\theta/2)})^{\frac{2}{\gamma}} t_{r,o}, \tag{65}$$

where $t_{r,o}$ is the transmission radius of the an omni-antenna with the same transmission power. Let $k_o = \lceil D/r_{t,o} \rceil$, be the minimal number of hops from the furthest nodes to the sink by omni antennas, and $k_d = \lceil D/r_{t,d} \rceil$ be that by directional antennas. k_d can be represented as

$$k_d = (\frac{2}{\tan(\theta/2)})^{-\frac{2}{\gamma}} k_o. \tag{66}$$

The maximal number of hops from the furthest nodes to the sink by directional antennas, denoted by $k_{d,max}$, is composed by the number of hops inside and outside the first ring, which is

$$k_{d,max} = h + k_d - 1 = h + [(\frac{2}{\tan(\theta/2)})^{-\frac{2}{\gamma}} k_o - 1]. \tag{67}$$

It is larger than k_o when sensors are distributed in a small area, due to more hops inside the first ring; and is smaller than k_o when sensors are distributed in a large area, due to less hops outside the first ring. According the the capacity formulation for omni-antennas eq. (17), and for directional antennas eq. (40), we have the ratio of capacity of directional antennas over omni antennas under the same transmission power setting, which is

$$\min[O(\frac{(\frac{\pi}{\theta})^2 \min(k_o, q+1)}{(h - 1 + \min(k_d, q+1))}), m \min(k_o, q+1)]. \tag{68}$$

Let $\theta_1, \theta_{q+1}, \theta_{k_o}$ be the beamwidth such that the the corresponding k_d is 1, $q+1$ and k_o. The directional/omni capacity ratio changes with beamwidth as follows: a)When $k_o \leq q+1$, the ratio is $O(\frac{k_o \pi^2}{h\theta^2})$ if $\theta \leq \theta_1 (k_d = 1)$, and decreases to $O(\frac{k_o \pi^2}{(h+k_o)\theta^2})$ as the beamwidth increase from θ_1 to θ_{k_o} (k_d increases from 1 to k_o); b) When $k_o \geq q+1$, the ratio is $O(\frac{(q+1)\pi^2}{h\theta^2})$ if $\theta \leq \theta_1 (k_d = 1)$, decreases to $O(\frac{(q+1)\pi^2}{(q+h)\theta^2})$ as the beamwidth increases from θ_1 to θ_{q+1} (k_d increases from 1 to $q+1$), and remains at $O(\frac{(q+1)\pi^2}{(h+q)\theta^2})$ as the beamwidth is larger than θ_{q+1} ($k_d \geq q+1$). The directional/omni capacity ratio is larger than 1, and decreases as the increase of beamwidth, as both hop count and interference increase with the beamwidth. Compared with the case that both omni-antennas and directional antennas have the same transmission radius, the directional/omni capacity ratio with the same transmission power is larger, because the transmission radius by directional antennas is larger than that of omni-antennas. In all cases, the directional/omni capacity ratio does not increase to infinity when θ approaches zero, and it is always upper bounded by $m \min(k_o, q+1)$.

8. CONCLUSION

In this paper, we analyzed the per-node end-to-end capacity of wireless sensor networks that use omni or directional antennas. For the use of directional antennas, we showed that they affect the network capacity not only by reducing interference, but also by changing network topology. From our analytical results we found that the network capacity for N sensors' network is $O(1/N)$ for both omni and directional antennas. By using directional antennas, the network capacity can be improved, compared with omni-antennas. This capacity gain is bounded by $(2q+3)/(2q-1)$ for line deployment. The directional antennas with smaller beamwidth have better capacity due to less interference they cause. However, for the two-dimensional deployment, the capacity is bounded by $\frac{nmW}{cN}$, no matter how small the beamwidth can be, where n is sink number, m is radio number, c is channel number, and W is the channel bandwidth. Furthermore, for the two-dimensional deployment and $m\theta < 2\pi$, the capacity is $O(\frac{1}{\theta})$ for $m = 2$, and $O(\frac{\lg m}{\theta^2 \lg(1/\theta)})$ for $m > 2$, where θ is antennas' beamwidth. From our analytical results, we also found that using more radios with directional antennas can increase the network capacity. But, the capacity increase is not linear to the increase of the number of radios due to the interference.

We also investigated the impact of multi-sinks and multi-channels to the capacity. We showed that, when there is only one sink, the capacity increases as the increase of transmission radius; when there are $n > 1$ sinks, the capacity has a non-monotonic relationship with transmission radius. A small transmission radius can produce a higher capacity only when the number of sinks is greater than the ratio of interference radius to transmission radius; otherwise, the network capacity reaches the maximum when transmission radius is so large that all nodes can directly connect to sinks. In addition, we showed that, adding more sinks to the network will generally improve the capacity. However, the capacity improvement decreases from $O(n)$ to $O(1)$ as the number of sinks increases. For a special case, when directional antennas are used and the beamwidth is very small, the capacity can even super-linearly increases as the increase of number of sinks n.

When there are multiple channels, the capacity remains as $O(1/N)$. When c/m exceeds a certain threshold, the capacity decreases as the increase of c/m. The ratio of directional antennas over omni-antennas decreases as the increase of c/m, and converges to 1 when c/m is sufficiently large.

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