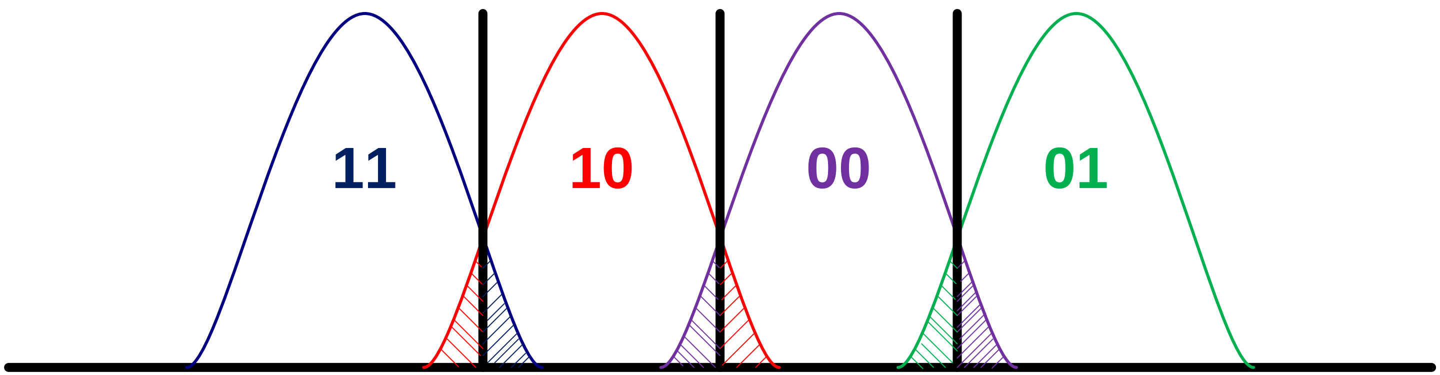
Gaussian Tail or Long Tail: On Error Characterization of MLC NAND Flash

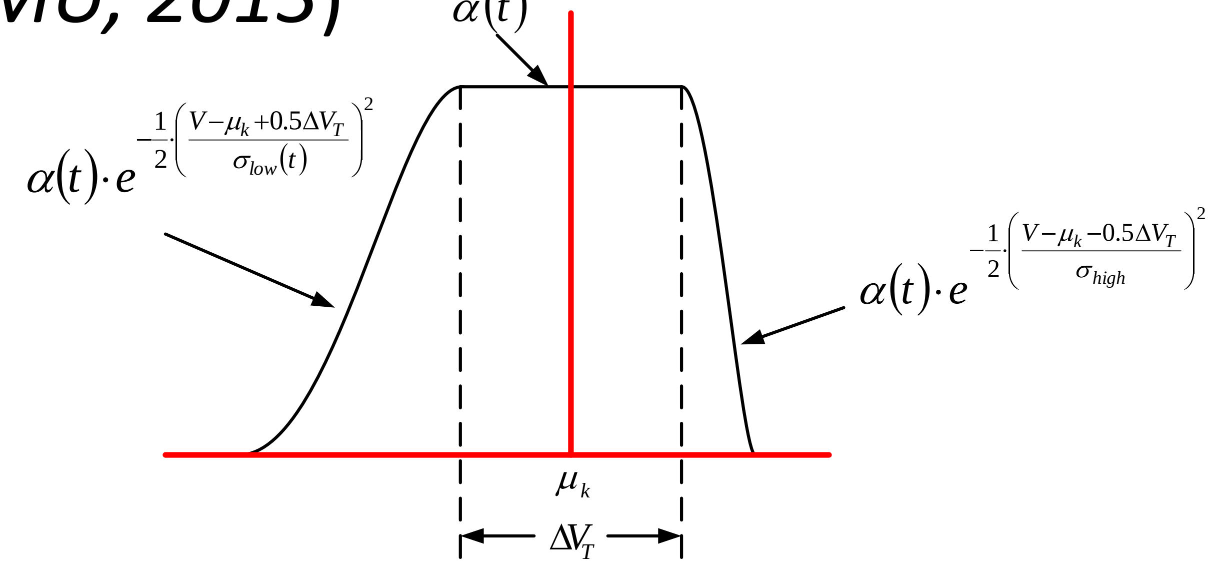
Presented by: Shu-Yi Jack Wong
Computer Engineering
University of Toronto, Ontario, Canada

Importance and Positioning

- A Multi-Level-Cell NAND Flash device can be modeled as a communication channel (*Dong et al., Rensselaer Polytechnic Institute, 2012*)
- Error in the channel is expressible in terms of the Probability Density Function (PDF)



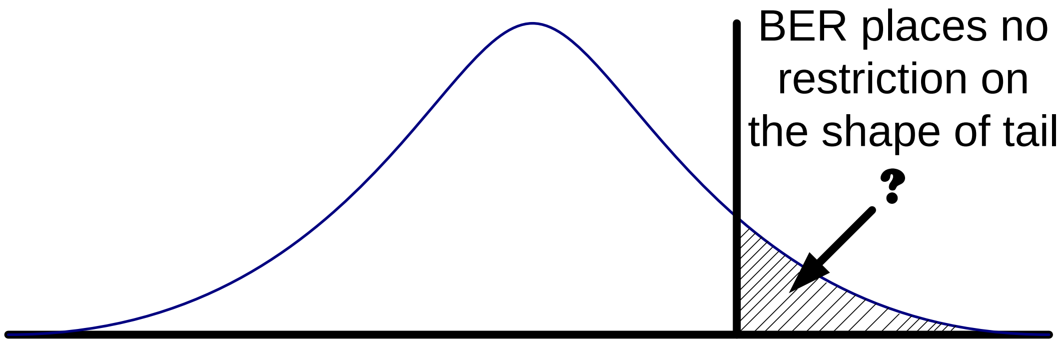
- Gaussian Tail has been used in Error Characterization (*Liu et. al, NTU, 2012*)
- The method bridges the gap between the observable Bit-Error-Rate (BER) and the channel model
- A Gaussian PDF is often deemed sufficient (*Cai et al. CMU, 2013*)



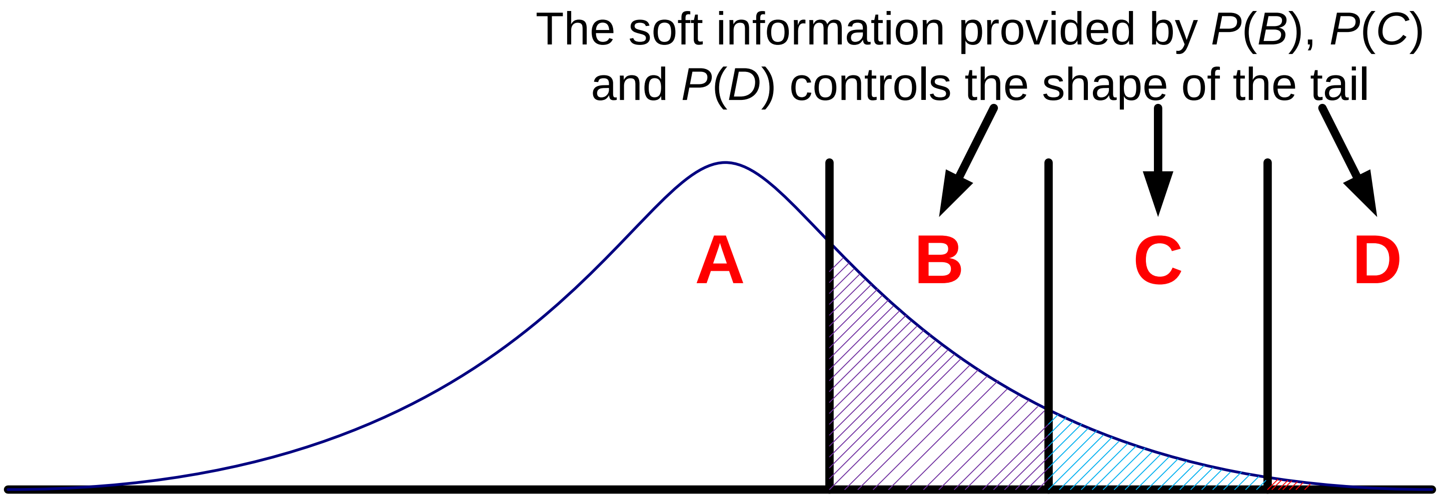
Motivation

- Advanced Forward Error Correction (FEC) needs more than just BER
  - In particular, LDPC's performance can be improved when the probability for more regions of the PDF are known with high precision (*Wang et al., UCLA, 2011*)
- The fine input probabilities could enable:
  - Higher reliability at a given raw error-rate
  - Higher endurance
  - Higher retention span
  - Higher data utilization (less bits for parity)

- BER only provides an aggregate error data

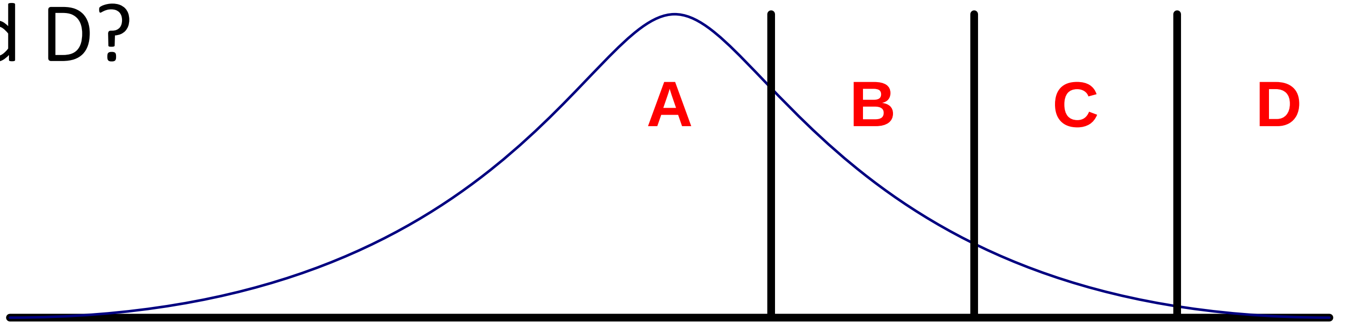


- LDPC requires more



Problems

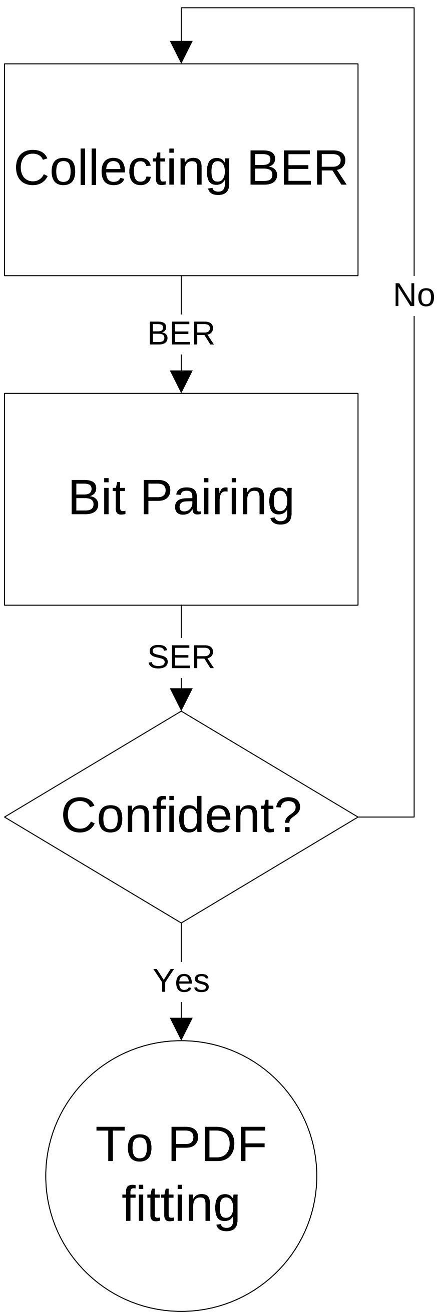
- If the tails of PDF matters, do the Gaussian tails match with real device error?
- How do we observe the error in region B,C and D?



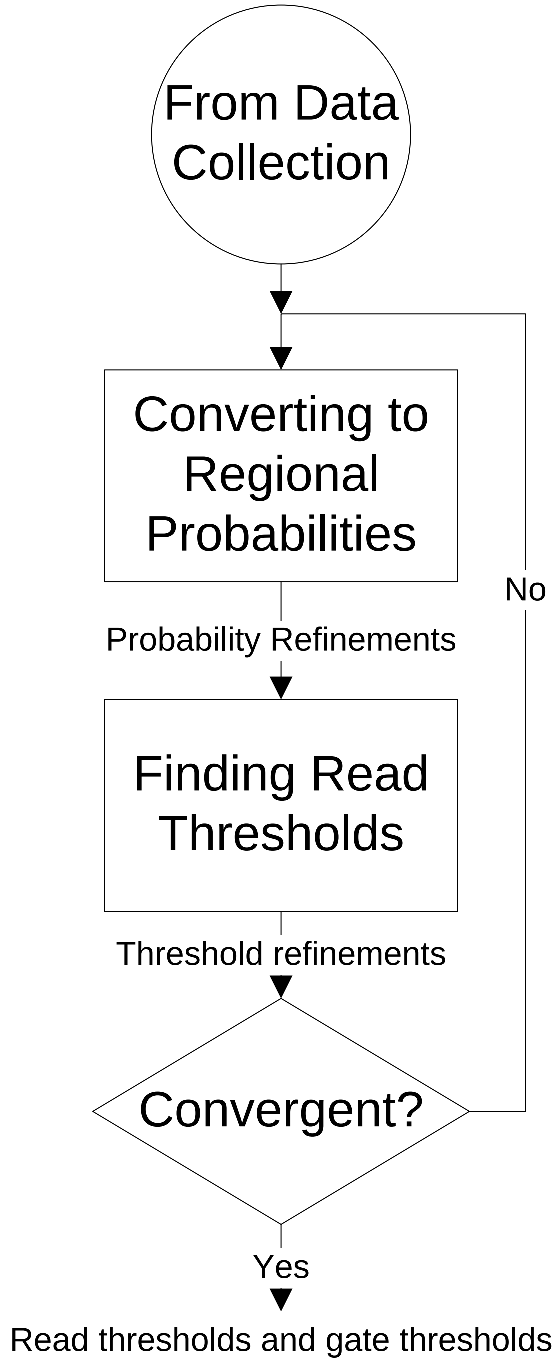
- How do we extract the threshold values from the extra information?

Proposed Methodology

- Collects Symbol Error from real 2Xnm device
  - Not just BER
  - Symbol Error is collected per programmed symbol
- Assumes that Gaussian PDF is correct
  - Link SER to regions of PDF
  - Link regions of PDF to Gaussian Error Functions
  - Solve the Error Functions for the parameters
- Observe the PDF's consistency with real data



Collecting BER
BER
Bit Pairing
SER
Confident?
No
Yes
To PDF fitting



From Data Collection
Converting to Regional Probabilities
Probability Refinements
Finding Read Thresholds
Threshold refinements
Convergent?
No
Yes
Read thresholds and gate thresholds

Collecting Symbol Error

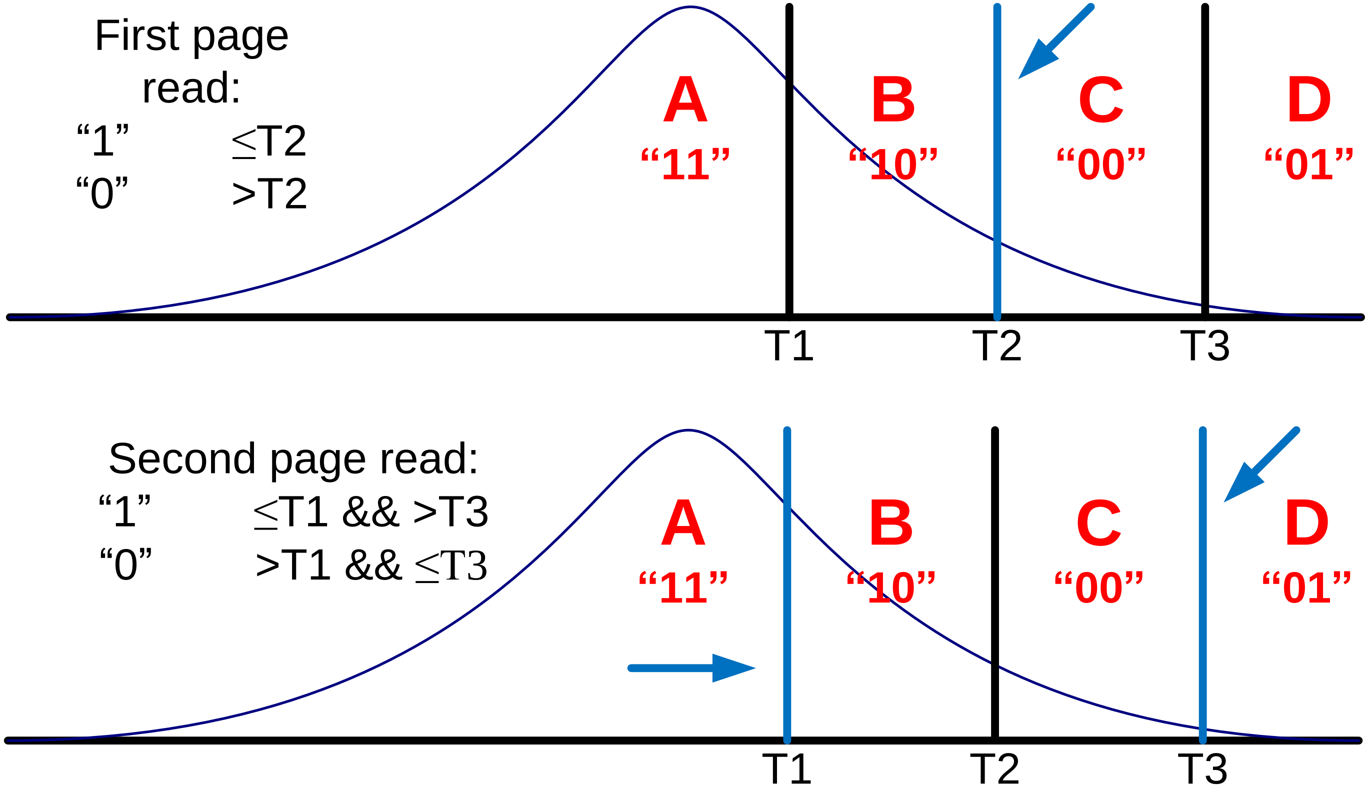
- Bits of a cell are accessed on different pages
- In MLC, for example, after programming of both the first and the second page with “1”
  - We pair the read value from the first page with that of the corresponding second page
  - The symbol error may be “10”, “00” or “01”
- To estimate the error rate for each error:
  - How many symbols should we sample?
  - We apply the method of Monte Carlo Estimation

- How confident? $1-\alpha$
- How precise? $\delta_{mc} = \frac{\bar{\sigma}}{\sqrt{n}} \phi^{-1}\left(\frac{1-\alpha}{2}\right)$
- $\phi(X) = \int_0^X N(0,1)dx,\quad \bar{\sigma} = \sqrt{\bar{p}\cdot(1-\bar{p})}$

Estimated Symbol Error Rate	Precision (δ)
$(1-\overline{p_1})\cdot\overline{p_2}$	$\pm 3\sqrt{\frac{\overline{p_2}}{n}}$
$\overline{p_1}\cdot(1-\overline{p_2})$	$\pm 3\sqrt{\frac{\overline{p_1}}{n}}$
$\overline{p_1}\cdot\overline{p_2}$	$\pm 3\sqrt{\frac{\overline{p_1}\cdot\overline{p_2}}{n}}\cdot\left(\sqrt{\overline{p_1}}+\sqrt{\overline{p_2}}+\frac{3}{\sqrt{n}}\right)$

Impact of Circuit to SER

- Observed SER ≠ Regional probabilities



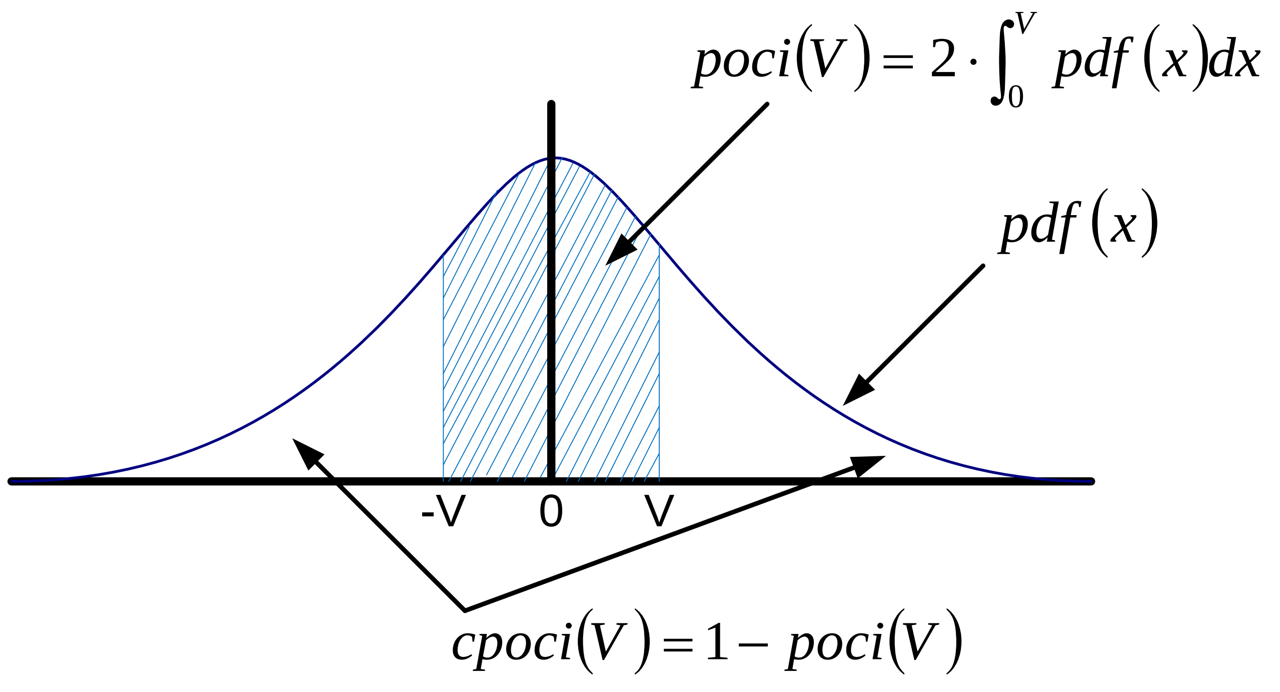
- $$\begin{aligned} P(\text{"11"}) &= P(A) - P(\theta) \\ P(\text{"10"}) &= P(B) + P(\theta) \\ P(\text{"00"}) &= P(C) + P(\theta) \\ P(\text{"01"}) &= P(D) + P(\theta) \end{aligned}$$

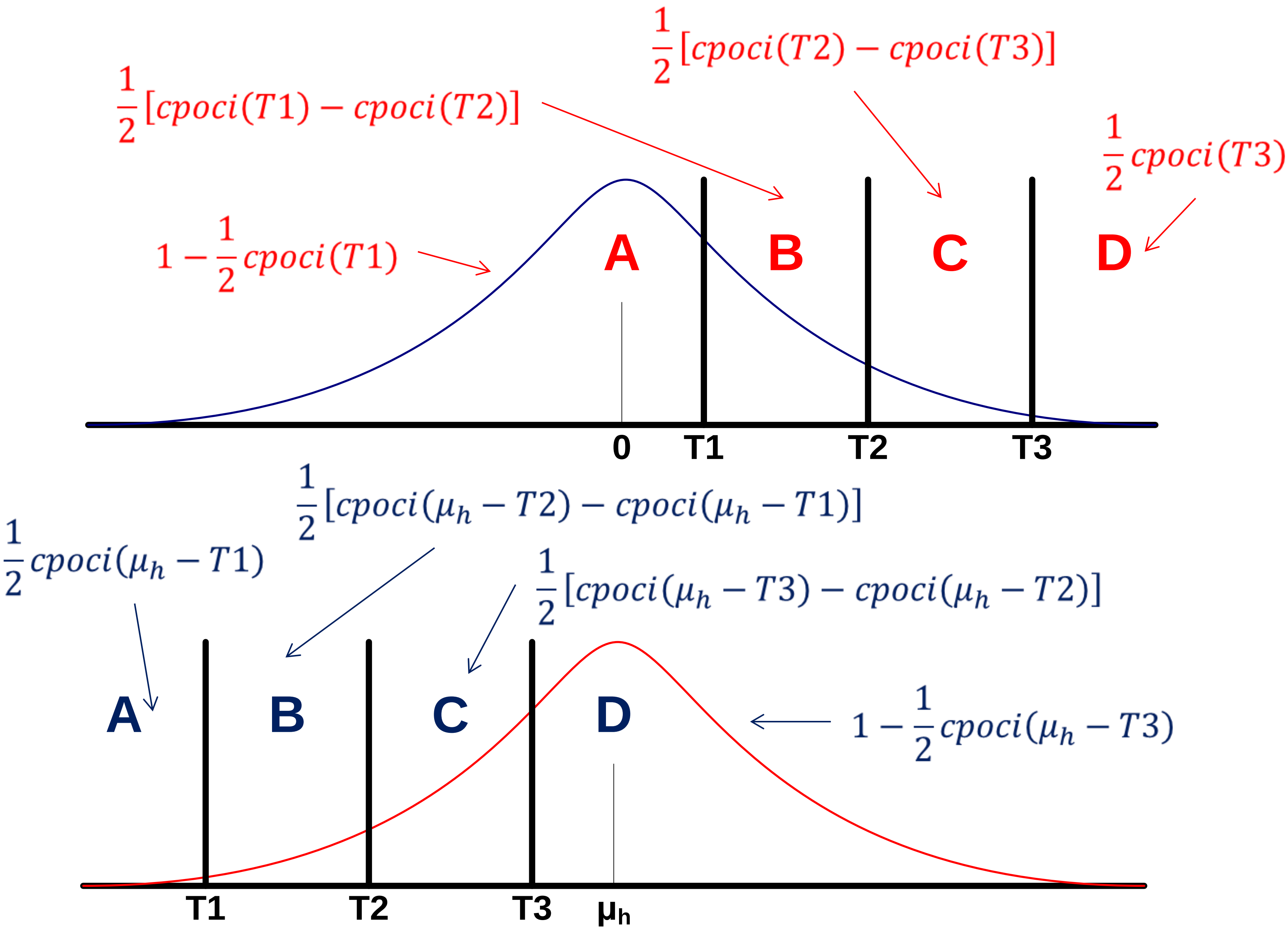
- $P(\theta) = P(A) \cdot P(C) - P(B) \cdot P(D)$

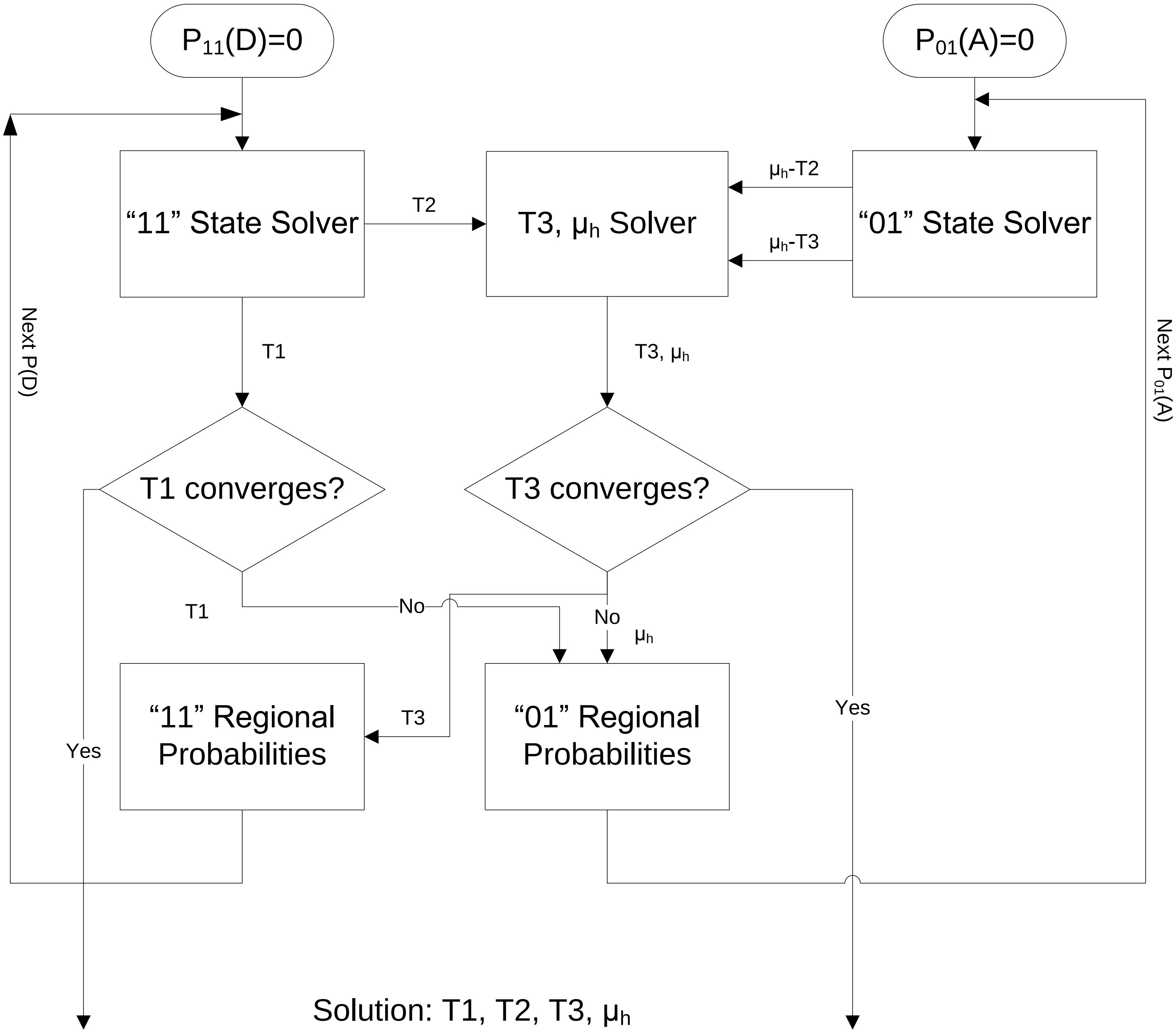
- $$\begin{bmatrix} P(A) \\ P(B) \\ P(C) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 \\ -1 & 0 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} P(\text{"11"}) + P(\text{"10"}) \\ P(\text{"00"}) + P(\text{"01"}) \\ P(\text{"11"}) + P(\text{"01"}) \\ P(\text{"10"}) + P(\text{"00"}) \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} P(D)$$

Relating SER and Error Functions

- Regions of PDF can also be described in terms of the Error Functions of its PDF



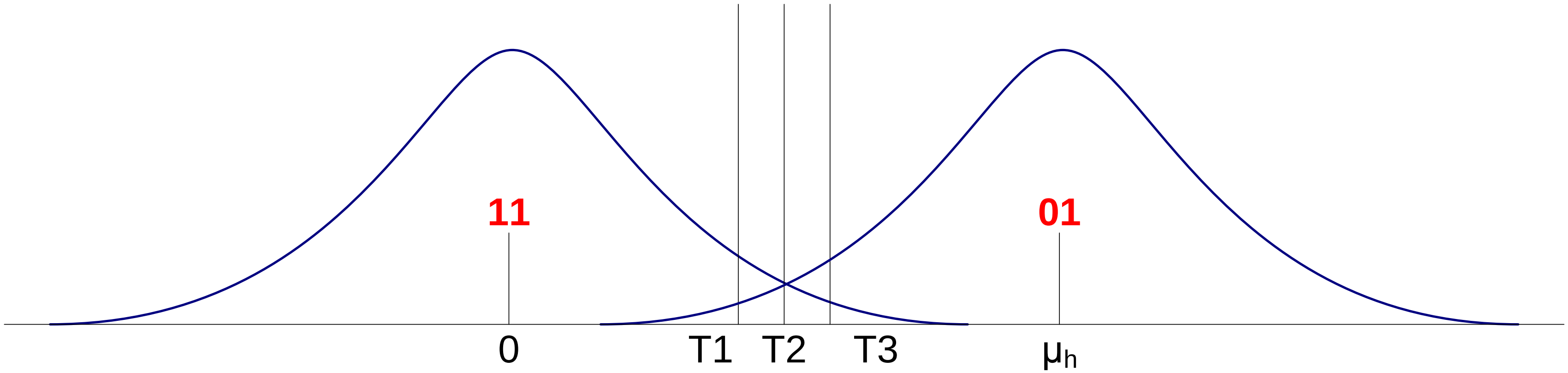

$\frac{1}{2}[cpoci(T2) - cpoci(T3)]$
$\frac{1}{2}[cpoci(T1) - cpoci(T2)]$
$\frac{1}{2}cpoci(T3)$
$1 - \frac{1}{2}cpoci(T1)$
A
B
C
D
0
T1
T2
T3
$\frac{1}{2}[cpoci(\mu_h - T2) - cpoci(\mu_h - T1)]$
$\frac{1}{2}cpoci(\mu_h - T1)$
$\frac{1}{2}[cpoci(\mu_h - T3) - cpoci(\mu_h - T2)]$
A
B
C
D
$1 - \frac{1}{2}cpoci(\mu_h - T3)$
T1
T2
T3
μ_h


P11(D)=0
P01(A)=0
"11" State Solver
T2
T3, μh Solver
μh-T2
μh-T3
"01" State Solver
T1
T3, μh
Next P(D)
Next P01(A)
T1 converges?
T3 converges?
T1
No
No
μh
"11" Regional Probabilities
T3
"01" Regional Probabilities
Yes
Yes
Solution: T1, T2, T3, μh

Result

T1	T2	T3	Mean of "01"
2.98 V	3.53 V	4.15 V	7.02 V

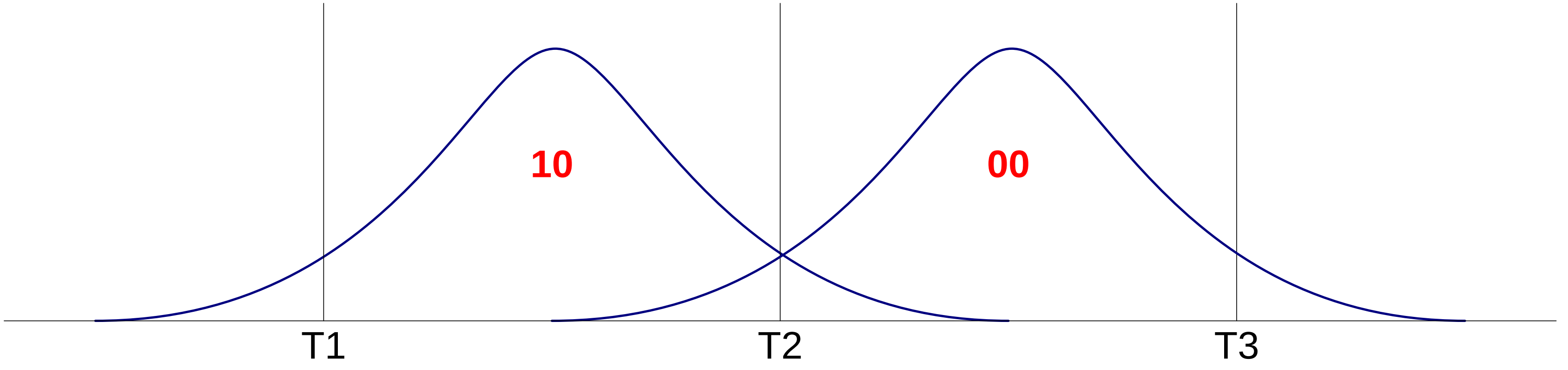
*voltages are normalized by the spread of the PDF



- By solving the data for symbol "11" and "01"
  - we obtain separation of read thresholds, 0.5-0.6V
  - mean threshold to the closest read threshold, 3V

Consistency

- If we were solving for the symbol *"10"* and *"00"*, we would not get a consistent solution
- i.e. by solving for "11" and "10", we get one set of value; by solving *"00"* & *"01",* we get another.



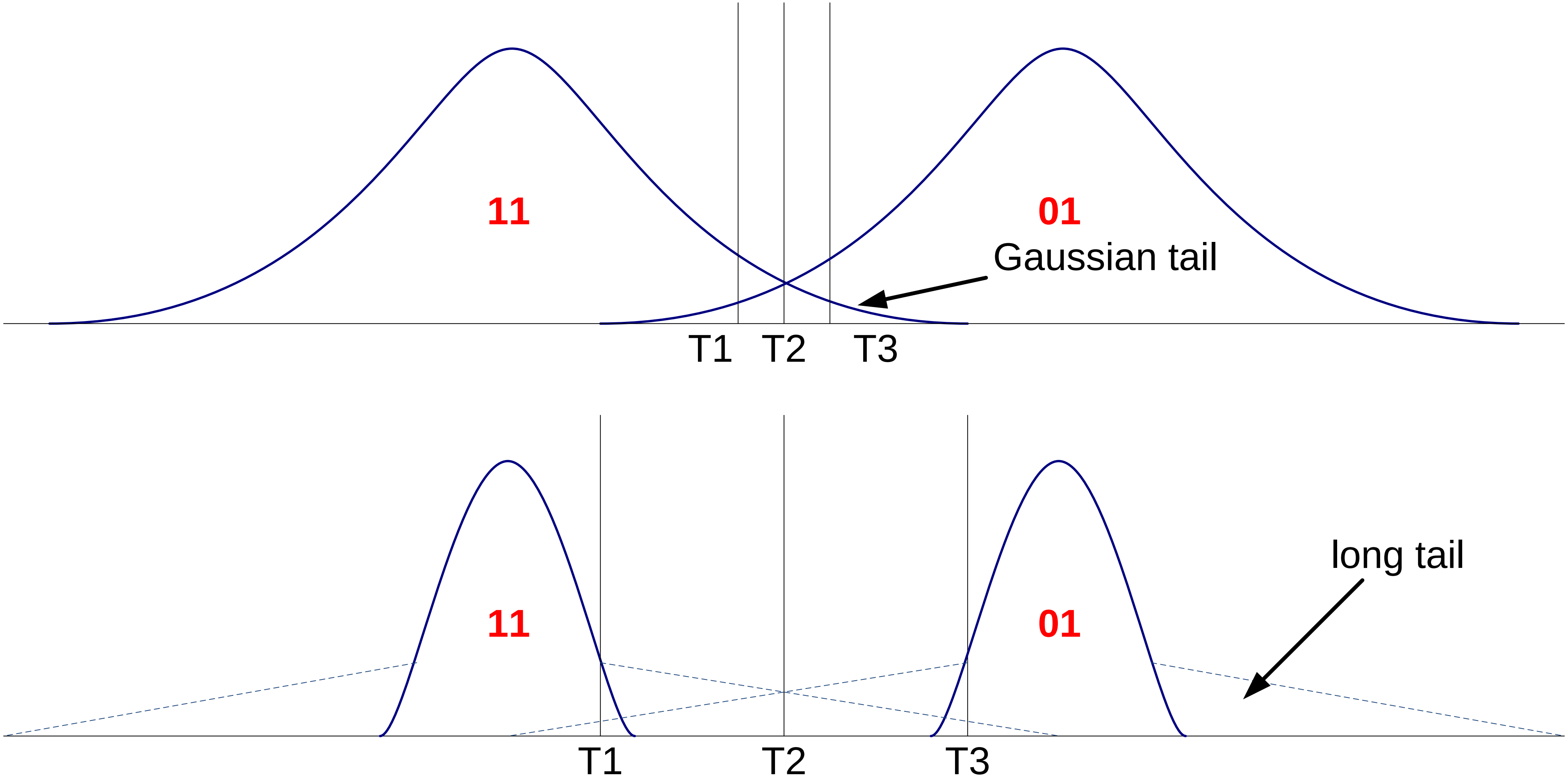
Symbol Probabilities for MLC NAND Flash

Program Value\Read Value	"11"	"10"	"00"	"01"
"11"	≈1	1.26e-5	3.53e-9	2.86e-7
"10"	3.63e-5	≈1	6.52e-7	5.30e-9
"00"	3.53e-9	4.06e-7	≈1	2.69e-5
"01"	4.15e-7	5.30e-9	2.53e-5	≈1

With the Gaussian PDF assumption, no consistent solution is possible for the regions highlighted in yellow

Conclusion

- SER contains information from the tail regions of a device PDF, which is not captured in BER
- The information is useful for optimizing the performance of LDPC
- By assuming a Gaussian PDF, we try extracting read thresholds using the extra SER data but fail to get a consistent solution
- The result is due to the nature of SER data, which cannot be modeled by the “short tails” of Gaussian model


11
01
Gaussian tail
T1
T2
T3
11
01
long tail
T1
T2
T3