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Proportional Participatory Budgeting

Summer School on Game Theory and Social Choice
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based on joint work with Piotr Skowron and Grzegorz Piercyński
How to Vote

- You’re a member of an academic society which needs to elect a president. Several candidates are running.
- Could ask each member to rank all candidates, and use one of dozens of voting rules to make a decision (no consensus which rule to use)
  - Plurality, Borda, Instant Runoff, Schulze, Kemeny, …
- Or just use Approval Voting: allow each member to approve an arbitrary number of candidates, elect the one with the most approvals.
  - very well behaved
  - easy to use
  - wins if you ask voting theorists to vote for best voting rule
Electing a Council

- Academic society doesn’t only have a president. Also has a council with $k = 8$ members. Many people are running.

- Suppose voters submit approvals. How to select the council?

- Easiest way: select the 8 candidates with highest number of approvals.

- Could be bad: suppose field is split into topic A (60%) and topic B (40%). Rule could select 8 candidates from subfield A.
Rules for Committee Elections

- For a committee $W$ of size $k$, write $u_i(W)$ for the number of committee members that $i$ approves, $|A_i \cap W|$.

- The committee selecting the $k$ candidates with highest approval score is the one maximizing $\sum_i u_i(W)$.

- Idea: to make majorities less overpowering, replace $u_i(W)$ by a concave function.

- Thiele proposed this in 1895 for Sweden.

$$\sum_{i \in N} \frac{1}{1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{|W \cap A_i|}}$$

“Proportional Approval Voting“ (PAV)
PAV works

- Suppose both voters and candidates are partitioned into subfields. Then PAV selects candidates from subfields in proportion to the subfield size. (Thiele showed this in 1895.)

- Above, if $k = 11$, PAV selects 3 green, 2 blue, 5 red, 1 orange committee members.

- Harmonic numbers is the only function $f(|A_i \cap W|)$ that guarantees this (in the sense of following d’Hondt rounding).

Extended Justified Representation (EJR)

• Usually, candidates and voters aren’t partitioned. Approval sets overlap. We still want to be “proportional”. What does this mean formally?

• Consider a group $S$ of voters with $|S| \geq \ell$ $n/k$.  
  -> They are large enough to decide $\ell$ seats.

• Suppose there is a set $T$ of candidates with $|T| = \ell$ such that every voter in $S$ approves all of $T$ (“cohesive group”).

• For a committee $W$ to satisfy EJR, it cannot be that every member of $S$ prefers $T$ to $W$  
  -> thus at least one voter in $S$ must approve at least $\ell$ members of $W$. 
PAV satisfies EJR

• **Theorem: PAV satisfies EJR.**


• Swapping argument.

  • Assume not. Then there is a candidate $c$ in $T$ that is not elected, and all voters in $S$ have utility at most $\ell - 1$.

  • This means adding $c$ to the PAV committee would increase its score by at least \[ \frac{1}{\ell} \cdot |S| = \frac{n}{k}. \]

  • Can check that, on average, removing a candidate from the PAV committee decreases its score by strictly less than $n/k$.

  • So by removing the worst candidate and adding $c$ we get a better committee, contradiction.
The Core

- Can we give a stronger representation guarantee?
- Consider a group $S$ of voters with $|S| \geq \ell \frac{n}{k}$.
  -> They are large enough to decide $\ell$ seats.
- Suppose there is a set $T$ of candidates with $|T| \leq \ell$ but we do not require that everyone in $S$ approves everyone in $T$. (cohesive)
- For a committee $W$ to satisfy the core, it cannot be that every member of $S$ prefers $T$ to $W$.

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Phragmén proposed a rule selecting the green committee in 1894.

- Sequential, poly time, but fails EJR.
- We proposed a new rule that selects the green committee and is EJR, sequential, poly time: the Method of Equal Shares.


- Split $k$ equally among the voters. It costs $1 to elect a candidate. We repeatedly choose a candidate whose approvers have at least $1 left. We spread the $1 as evenly as possible, and if several candidates are available we choose the one with the most even spread.

- (Still fails core.)
Participatory Budgeting

• In Participatory Budgeting, the city government allows residents to vote over how the budget is spent.

— Ballot Paper —

Total available budget: € 3 000 000.

Approve up to 4 projects.

- Extension of the Public Library
  Cost: € 200 000

- Photovoltaic Panels on City Buildings
  Cost: € 150 000

- Bicycle Racks on Main Street
  Cost: € 20 000

- Sports Equipment in the Park
  Cost: € 15 000

- Renovate Fountain in Market Square
  Cost: € 65 000

- Additional Public Toilets
  Cost: € 340 000

- Digital White Boards in Classrooms
  Cost: € 250 000

- Improve Accessibility of Town Hall
  Cost: € 600 000

- Beautiful Night Lighting of Town Hall
  Cost: € 40 000

- Resurface Broad Street
  Cost: € 205 000
Participatory Budgeting

- Participatory Budgeting now happens in 100s of cities
- During 2016-2021, largest in Paris (€100 million per year)
- Better theory could improve practice around the world
- Same or similar models for other important applications:
  ➔ Research grant funding
  ➔ Scheduling
The diagram illustrates population data and cost figures for different districts in Circleville. Each district is labeled with its population and cost. The table on the right side provides a summary of votes and costs for various options:

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Proportionality

Proportional Representation requires that a group of 30% of voters with similar interests should be represented by spending of about 30% of the budget.

A voter could be part of several interest groups!

Research Question:
Can we design a rule that on its own finds all interest groups and represents all of them proportionally?
Reduction to Committee Voting

• Suppose our budget is $k, and each project costs $1 (the “unit cost assumption”).

• Then PB voting turns into a committee election!

• So maybe we can generalize known rules to work for the knapsack constraint.

• For example, maximize the PAV objective over all feasible knapsacks. —> Fails badly.
Every voting rule that only depends on voters’ utility functions and the collection of budget-feasible sets must fail proportionality, even on instances with a district structure.

Theorem 1.
Method of Equal Shares

- Split the city budget evenly among residents.
- Put each resident’s share in a virtual bank account.
- Repeatedly, until the budget runs out:
  - identify a project whose supporters have enough money left to afford it
  - divide the cost among supporters as evenly as possible, and charge them

Q: How to choose between affordable projects?
A: Take the one where max payment is smallest.
=> cheaper better
=> wealthier supporters better
=> more supporters better

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EJR for Budgeting

• Consider a group $S$ of voters with $|S| \geq \alpha B/n$.

• Suppose there is a set $|T| = \ell$ of candidates with $\text{cost}(T) \leq \alpha B$ such that every voter in $S$ approves all of $T$ (“cohesive group”).

• For a committee $W$ to satisfy EJR, it cannot be that every member of $S$ prefers $T$ to $W$
  -> thus at least one voter in $S$ must approve at least $\ell$ members of $W$.

• Theorem: The Method of Equal Shares satisfies EJR.
Extending to Additive Utilities

- Equal Shares: can extend using following idea: a voter’s payment for a candidate should be proportional to the voter’s utility for the candidate.
  - this rule satisfies EJR “up to one project”.
- Consider case $n = 1$. Then EJR means we must solve the knapsack problem. So no strongly polynomial time rule can satisfy EJR.
- We also have a rule satisfying stronger guarantees — but extremely hard to compute!

Example:
2019, Paris, 16th arrondissement
€560k: refurbish sports facility — 775 votes
€3k: materials for classroom project — 670 votes
— 1.15x as popular, 186x the cost!

We can still use approval voting, but instead of using 0/1 utilities, we can use “0/cost” utilities:
- Approved: utility = cost of project
- Not approved: utility = 0
Discussion

• Idea of proportionality and fairness can be applied to all kinds of decision making situations (scheduling, design, recommendations, rankings)

• Can we implement sophisticated voting rules in public applications? What about computational complexity?

• Fairness over time: participatory budgeting happens every year.
Some Directions
Mutually Exclusive Projects

• Assume there is an empty plot of land, and several ideas what to build there. We can only choose one.

• Easy to adapt Equal Shares. Easy to adapt EJR. But Equal Shares doesn’t satisfy EJR.

• Can EJR be satisfied?

• Related to “Public Decision Making”, committee elections with variable number of winners.

• Existing very recent work on this question: very special cases, or without proportionality.
Divisible Projects

- Some projects can take an arbitrary amount of funding (e.g. how much should we spend fixing potholes?)
- Can easily incorporate by introducing lots of projects, $1 each.
- This fixes exhaustiveness issue: entire budget will be used.
- But: we know PAV is best for “copyable” projects (see party-approval).
- Equal Shares behaves weirdly: two parties A and B. 20% of voters approve both. Of the rest, 60% approve A and 40% approve B. Then Equal Shares will give 68% to A and 32% to B.
Projects with Milestones

• Suppose a project comes in three possible sizes: $100k, $150k, or $170k.

• Voters can indicate up to which size they approve the project.

• Similarly: divisible project where voters indicate the maximum amount of spending they approve.