Optimal Auctions For Correlated Private Values: Ex-Post vs. Ex-Interim Individual Rationality

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Background and Motivation (1): Cremer-Mclean

- Single-item auctions with correlated private values: n players have private values $v_1,\ldots,v_n$ that are jointly drawn from some n-dimensional distribution F.
- Expected social welfare SW(F) = $E[\max v_1,\ldots,v_n]$

Cremer-Mclean (1988): Under a certain condition on F, there exists a Dominant-Strategy Incentive-Compatible (DSIC) auction whose expected revenue = SW(F).

- The CM auction satisfies Ex-Interim Individual Rationality (EIIR):
  - EIIR: non-negative ***expected*** utility is guaranteed for a truthful bidder

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- The CM auction satisfies Ex-Interim Individual Rationality (EIIR):
  - EIIR: non-negative *expected* utility is guaranteed for a truthful bidder
- Thus, the maximal-possible revenue among all DSIC and EIIR auctions, $\mathbf{OPT_{EIIR}(F)}$, is equal to SW(F) if F satisfies the Cremer-Mclean (CM) condition

Example

- Two players with values v_1,v_2 that are jointly distributed:

v_1 \ v_2	**1/2**	**1**
1/2	**1/3**	**1/6**
1	**1/6**	**1/3**

- Pr(v_2 = 1) = 1/2
- Pr(v_2 = 1| v_1 = 1) = 2/3, Pr(v_2 = 1/2 | v_1 = 1) = 1/3

- Expected utility of player 1 with v_1 = 1 in a 2nd price auction with random tie-breaking: Pr(v_2 = 1/2 | v_1 = 1) · [1 – 1/2] + Pr(v_2 = 1 | v_1 = 1) · 0.5 · [1 - 1] = 1/6

- (everything is symmetric so the same for player 2)

Example: the Cremer-Mclean (CM) auction

- The CM auction adds 'entry fees' – fixed payment $c_i(v_j)$ that is paid by player i when player j bids v_j regardless of player i's bid and whether she wins or losses.
- After that, run 2nd price.

v_1 \ v_2	**1/2**	**1**
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- Note: truthful regardless of how we fix $c_i(v_j)$.
- Calculation of $c_1(v_2)$, should satisfy two equations:

$$\underbrace{\Pr(v_2 = 1 \mid v_1 = 1) \cdot c_1(v_2 = 1) + \Pr(v_2 = 1/2 \mid v_1 = 1) \cdot c_1(v_2 = 1/2)}_{\text{Expected entry fee when } v_1 = 1} = \text{Expected utility in 2}^{\text{nd}} \text{ price when } v_1 = 1$$

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- Note: truthful regardless of how we fix $c_i(v_j)$.
- Calculation of $c_1(v_2)$, should satisfy two equations:

$$(2/3) \cdot c_1(v_2 = 1) + (1/3) \cdot c_1(v_2 = 1/2) = 1/6$$

$$(1/3) \cdot c_1(v_2 = 1) + (2/3) \cdot c_1(v_2 = 1/2) = 0$$

$$\Rightarrow c_1(v_2 = 1) = 1/3 \quad ; \quad c_1(v_2 = 1/2) = -1/6$$

- When $v_2 = 1$, player 1 <u>pays</u> 1/3; when $v_2 = 1/2$, player 1 <u>receives</u> 1/6. Similarly for player 2. After that, run 2^{nd} price.
- Ex-post utility may be negative; ex-interim utility is always non-negative.

Example: revenue calculation

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- Second-price expected revenue:
  $(1/3) \cdot (1/2) + 2 \cdot (1/6) \cdot (1/2) + (1/3) \cdot 1 = 2/3$

- Expected revenue from entry fees
  [$c_i(v_j = 1) = 1/3$; $c_i(v_j = 0) = -1/6$] :

  $(1/3) \cdot (-1/6 - 1/6) + 2 \cdot (1/6) \cdot (1/3 – 1/6) + (1/3) \cdot (1/3 + 1/3) = (1/3) \cdot 3 \cdot (1/3 – 1/6) = 1/6$

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Disadvantages of the CM auction

- Players may be left with a negative utility
- Well defined only when we can solve the set of inequalities for the entry fees. This is not always true (depending on the distribution F).

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- Ex-Post Individual Rationality (EPIR) vs. Ex-Interim Individual Rationality (EIIR):
  - EPIR: non-negative utility is guaranteed for a truthful bidder
  - EIIR: non-negative ***expected*** utility is guaranteed for a truthful bidder
  - Thus, $OPT_{EIIR}(F) \geq OPT_{EPIR}(F)$

Ronen (2001): For any F, there exists a DSIC and EPIR auction (the "look-ahead" auction) whose expected revenue $\geq 0.5 \cdot OPT_{EPIR}(F)$.

- Following Ronen, the AGT/CS literature mostly continue to impose EPIR on suggested auctions & benchmark

The look-ahead auction

- (1) all players bid ; (2) highest bidder is offered a take-it-or-leave-it price p^* = optimal price using the conditional distribution of the highest bidder given all other values
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- Expected revenue = $(1/3) \cdot (1/2) + 2 \cdot (1/6) \cdot (1/2) + (1/3) \cdot 1 = 2/3$

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- We saw two **different** benchmarks: $OPT_{EPIR}(F)$ and $OPT_{EIIR}(F)$

Questions / Motivation

- Does there exist a DSIC+EPIR auction the approximates $OPT_{EIIR}(F)$? For all F? Under some condition?

- In particular, does $OPT_{EPIR}(F)$ give some approximation of $OPT_{EIIR}(F)$? For all F? Under some condition?

- Remark: when the CM condition is violated there is an unbounded gap between $OPT_{EIIR}(F)$ and SW(F) – (Albert, Conitzer, Lopomo 2016) – so the latter is not attainable unconditionally.

- Nevertheless, does $OPT_{EPIR}(F)$ and/or $OPT_{EIIR}(F)$ approximate SW(F) under some natural conditions other than the CM-condition?

Main Results (1): An Impossibility

- We study bounded distributions (w.l.o.g. with support in $[0,1]^n$)

THM: $\forall \beta \in (0,1]$ $\exists F_\beta$ such that $OPT_{EPIR}(F_\beta) < \beta \cdot OPT_{EIIR}(F_\beta)$

This holds even if $OPT_{EPIR}(F_\beta)$ optimizes over all truthful-in-expectation auctions

This holds even for n=2 players

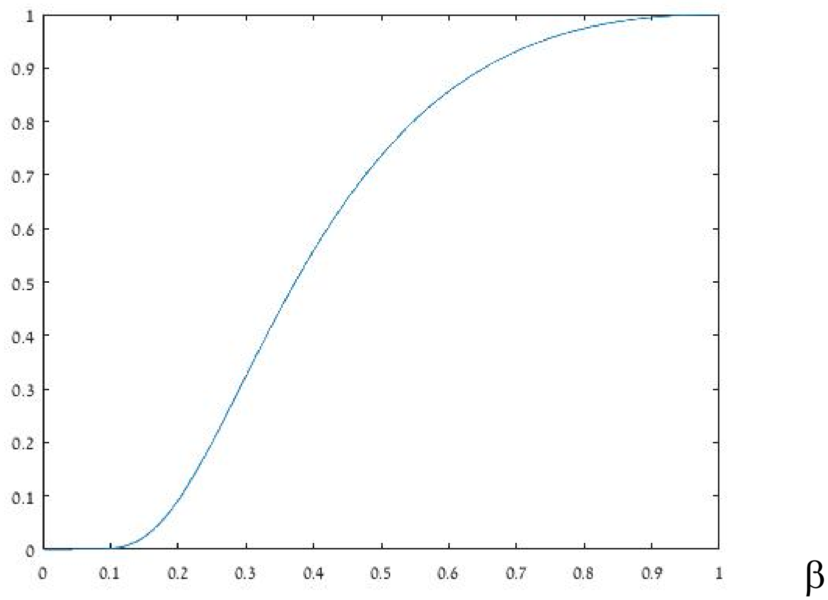
- Thus, in the worst-case, $OPT_{EPIR}(F)$ cannot extract any bounded fraction of $OPT_{EIIR}(F)$

- Proof: a corollary of the second main result, as follows.

Main Results (2): A Possibility

- We utilize a certain function φ: $(0,1] \rightarrow (0,1]$
  - It is strictly monotone, $\lim_{\beta \rightarrow 0} \varphi(\beta) = 0$, $\varphi(1) = 1$

$$\varphi(\beta) = \frac{e^{1-\frac{1}{\beta}}}{\beta}$$



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- REV(LK,F) denotes the expected revenue of the look-ahead auction for F

THM: $\forall$F, if SW(F) $\geq \varphi(\beta)$ then REV(LK,F) $\geq \beta \cdot \mathrm{OPT}_{\mathrm{EIIR}}(F)$

In fact, $\forall$F, if SW(F) $\geq \varphi(\beta)$ then REV(LK,F) $\geq \beta \cdot$ SW(F)

Corollaries

(**Reminder**: $\forall$F, if SW(F) $\geq \varphi(\beta)$ then REV(LK,F) $\geq \beta \cdot$ SW(F))

- We can obtain specific bounds, for example:

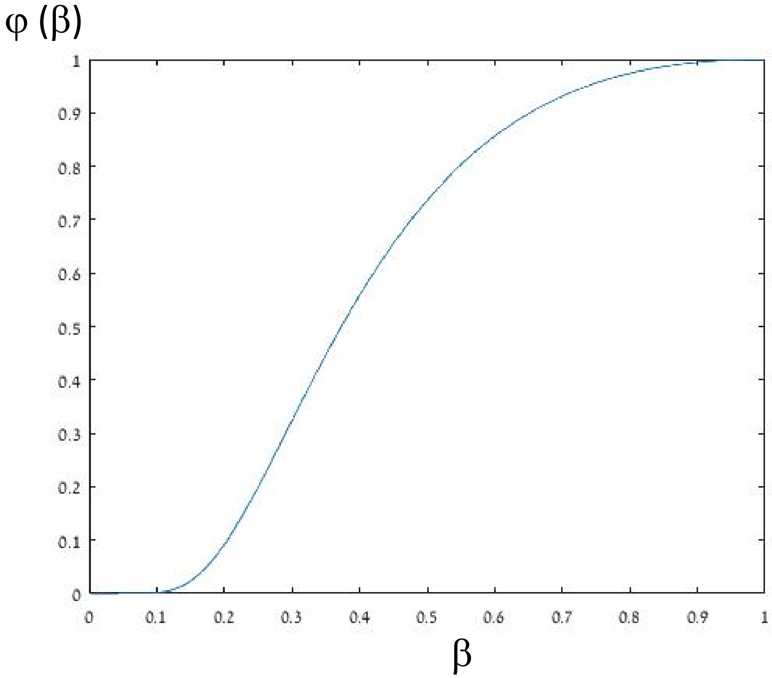
Corr: SW(F) $\geq$ 0.0013 $\Rightarrow$ REV(LK,F) $\geq 0.1 \cdot$ SW(F)

[since $\varphi(0.1) \approx 0.0013$]

Corr: For any 'marginal-value-symmetric' F,

REV(LK,F) $\geq 0.37 \cdot$ SW(F)

[since SW(F) = 0.5 $\approx \varphi(0.37)$]



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In fact, $\forall$F, if SW(F) $\geq \varphi(\beta)$ then REV(LK,F) $\geq \beta \cdot$ SW(F)

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- Obviously, SW(F) $\geq \varphi(\beta)$ is sufficient but not necessary (there are distributions F with SW(F) $< \varphi(\beta)$ and a high look-ahead revenue)
- But, the analysis is tight, this bound is the best possible for this criterion:

THM: $\forall \beta \in (0,1]$, $\varepsilon > 0$ $\exists F_{\beta,\varepsilon}$ s.t. $\mathrm{SW}(F_{\beta,\varepsilon}) \geq \varphi(F_{\beta,\varepsilon})$ but $\mathrm{OPT}_{\mathrm{EPIR}}(F_{\beta,\varepsilon}) < \beta \cdot \mathrm{OPT}_{\mathrm{EIIR}}(F_{\beta,\varepsilon}) + \varepsilon$

This holds even for n=2 players, and implies our main negative result.

Proof Structure

1. **The single-dimensional case**: worst distribution is an equal-revenue distribution (requires a short but well-known proof), a short calculation gives our bound.

- The "Equal Revenue distribution": For some revenue $\alpha,\quad \forall t \geq \alpha\text{:}\ \ t \cdot P(v \geq t) = \alpha$

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4. **Negative result, second step**: slightly perturb this first-step distribution so that it will satisfy the CM-condition. This does not significantly change SW(F), OPT_{EPIR}(F), REV(LK,F) but ensures that OPT_{EIIR}(F) = SW(F).

Some Open Questions – EPIR vs. EIIR

- We showed an approximation bound for marginal-value symmetric distributions. Are there other interesting classes of distributions with a good approximation bound?

- Find other parameters\features of the distribution (besides $SW(F)$) that can indicate the relation between the optimal revenue of an EPIR vs. EIIR auction.

What is the Optimal EIIR auction (either DSIC or BIC)?

- DSIC and EPIR auction – Papadimitriou and Pierrakos (2011)
- DSIC and EIIR auction - **partially** solved by Cremer and Mclean (1988)
- BIC and EIIR auction - **partially** solved by Albert, Conitzer, and Lopomo (2016)

- What about approximating the optimal EIIR revenue unconditionally?
  - DSIC and EPIR approximations: Ronen (2001), Dobzinski et al. (2011), Chen et al. (2011)
  - Nothing is known about EIIR

OPT DSIC+EIIR = An EPIR auction + Entry Fees

- Cremer and Mclean add 'entry fees' to a second price auction:
  - Player i's entry fee, $c_i(v_{-i})$, depends on others' reports; always charged (win or lose)
  - For distributions that satisfy the CM condition, the expected revenue of a second price auction with optimal entry fees is the expected social welfare (thus optimal)

THM: For any distribution F, there exists a DSIC+EPIR auction and optimal entry fees that together extract the optimal DSIC+EIIR expected revenue.

- Given any DSIC+EPIR auction A, one can compute optimal entry fees for A using a simple and concise linear program. The question is, which A to use?

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The second price auction? **NO** (as we show)

The look-ahead auction? **NO** (as we show)

The optimal EPIR+DSIC auction? **NO** (as we show)

A simple approximation

- Let Ai denote the auction that always gives the item for free to player i and charges optimal entry fees from i. Then,

THM: For any F, $OPT_{EIIR}(F) < (n+1) \max \{ OPT_{EPIR}(F), A1, \ldots, An \}$

- For n=2, $OPT_{EPIR}(F)$ is computationally efficient. For larger n, we can use the look-ahead auction instead.

- Weaknesses:
  - Not interesting for large n (but even for n=2 nothing was previously known)
  - $OPT_{EIIR}(F)$ requires DSIC, does this make sense? (BIC is implied by EIIR)

Summary

- Study single-item auctions with correlated private values; compare optimal revenue with ex-post individually rational vs. ex-interim individually rational auctions ($OPT_{EPIR}(F)$ vs. $OPT_{EIIR}(F)$)
- Most of the AGT/CS literature uses $OPT_{EPIR}(F)$ as the benchmark for optimal revenue with correlated values
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- Most of the AGT/CS literature uses $OPT_{EPIR}(F)$ as the benchmark for optimal revenue with correlated values
- However our main result shows that $OPT_{EPIR}(F)$ might only give an unboundedly small fraction of $OPT_{EIIR}(F)$
- But some good news, if the expected social welfare is high enough, the expected revenue of the look-ahead auction gives a bounded approximation of $OPT_{EIIR}(F)$
  - If the expected social welfare (max. value) is at least 0.0013 times the maximal element in the support, then the expected revenue of the look-ahead auction is at least 1/10 of $OPT_{EIIR}(F)$
- We're missing a better understanding of the structure of optimal EIIR auctions; entry fees play a key role