Myerson's Lemma and VCG mechanism in Mechanism Design

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Outline

In this tutorial, we will introduce

- What is mechanism design
- Two useful tools to design truthful mechanism
  - Myerson's lemma
  - VCG mechanism

In this tutorial, we

- assume no prior knowledge about mechanism design
- go through the detailed proof
- do not mention recent results

Warm up example - Single item auction

- An auction house wants to auction a valuable item among n bidders
- Each bidder i has a private value v_i for the item.
- Auction process:
  - Auctioneer asks the bidders for their private value.
  - For each bidder i, he/she claims b_i as the private value simultaneously. We call b_i the bid of bidder i.
  - Auctioneer decides winner and the price p according to $\{b_1, \cdots, b_n\}$ and a specific mechanism.
- The utility for bidder i is $v_i - p$ if he/she is the winner and 0 otherwise.
- Question: how to design a "good" mechanism?

Second Price Auction Mechanism

- First Price Auction Mechanism
  - Winner = bidder with highest bid; price = his/her bid
  - Bidder has the incentive to cheat

3 bidders.

private value: 3, 5, 10.

bids: 3, 5, 10.

winner = 3rd bidder
price = 10
$u_3 = 10 - 10 = 0$.

bid. 3, 5, 6 ← cheating bid.

winner = 3rd bidder
price = 6
$u_3 = 10 - 6 = 4 > 0$.

Second Price Auction Mechanism

- First Price Auction Mechanism
  - Winner = bidder with highest bid; price = his/her bid
  - Bidder has the incentive to cheat
- Second Price Auction Mechanism (Vickrey's Auction)
  - Winner = bidder with highest bid; price = second highest bid
  - Bidder has NO incentive to cheat

3 bidder.

private value: 3, 5, 10

3rd bidder: reports honestly. utility = 5.

1°. reports $b_3 > 5$. winner = 3rd bidder, price = 5. utility: $10 - 5 = 5$.

2°. reports $b_3 < 5$. winner = 2nd bidder. price = $\max(b_3, 3)$. utility for 3rd bidder = 0.

2nd bidder: reports honestly. utility = 0.

1°. reports $b_2 < 10$. utility = 0.

2°. reports $b_2 > 10$. winner = 2nd bidder, price = 10, utility = $5 - 10 = -5 < 0$

Second Price Auction Mechanism

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  - Bidder has the incentive to cheat
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  - Bidder has NO incentive to cheat

Solution concept

- Truthfulness (Strategy-Proofness, Incentive-Compatibility): whatever the others do, if the agent acts truthfully (that is, reveal the true private information, or act according to true preferences), it maximizes the agent's utility.

Mechanism Design

- Design mechanism
- Private information VS. public information
- Objective of designer: truthful + some other objectives
  - social welfare maximization ✓
  - revenue maximization
  - fairness
- Need a game perspective

Trivial truthful mechanism for single item auction.

winner : 1 st bidder.

price = 0

Mechanism Design

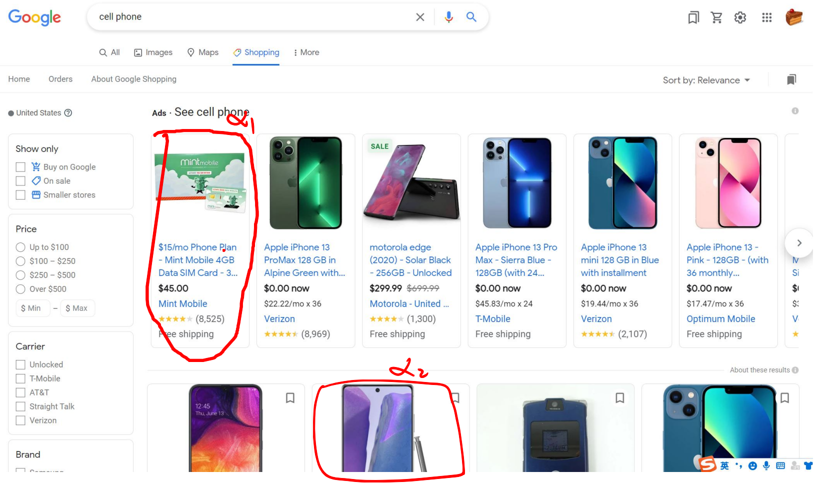
- Design mechanism
- Private information VS. public information
- Objective of designer: truthful + some other objectives
  - social welfare maximization
  - revenue maximization
  - fairness
- Need a game perspective

For second-price mechanism, what is the objective?

- For designer: truthful + maximize social welfare
- For bidders: maximize utility
- Later, we will show this is the "unique" truthful mechanism which maximizes the social welfare.

Sponsored search auction

Consider n companies competing for k ad slots on a search page:



$\alpha_1 > \alpha_2$.

Sponsored search auction

Consider n companies competing for k ad slots on a search page.
Settings:

- k slots for sponsored links, n agents
- For the j-th slot, the click-through-rates is α_j (public information). We assume $\alpha_1 > \alpha_2 > \cdots > \alpha_k$.
- For the i-th agent, the private valuation v_i is the profit for one click.
- If i-th agent wins the j-th slot and pays p_i per search, the utility function is $v_i \cdot \alpha_j - p_i$ per search.

Goals:

- truthful
- maximize social welfare

Single-parameter environment

Setting:

valuation for unit items

- n agents. Each has private information v_i, but will bid b_i.
- bids $\mathbf{b} = (b_1, b_2, \cdots, b_n)$
- mechanism: Allocation rule: $\mathbf{x}(\mathbf{b}) = (x_1(\mathbf{b}), x_2(\mathbf{b}), \cdots, x_n(\mathbf{b}))$
- Payment rule: $\mathbf{p}(\mathbf{b}) = (p_1(\mathbf{b}), p_2(\mathbf{b}), \cdots, p_n(\mathbf{b}))$
- Utility function: $u_i(\mathbf{b}) = v_i \cdot x_i(\mathbf{b}) - p_i(\mathbf{b})$ ←

depends on $x_i(b)$ only, not depend on $x_j(b)$ $(j \neq i)$.

Key points.

1. private information is "single parameter"
2. utility function: has specific form.
3. "feasible" allocation rule

Single-parameter environment

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- bids $\mathbf{b} = (b_1, b_2, \cdots, b_n)$
- Allocation rule: $\mathbf{x}(\mathbf{b}) = (x_1(\mathbf{b}), x_2(\mathbf{b}), \cdots, x_n(\mathbf{b}))$
- Payment rule: $\mathbf{p}(\mathbf{b}) = (p_1(\mathbf{b}), p_2(\mathbf{b}), \cdots, p_n(\mathbf{b}))$
- Utility function: $u_i(\mathbf{b}) = v_i \cdot x_i(\mathbf{b}) - p_i(\mathbf{b})$

For example, consider single item auction, the feasible allocation rules are

- $x_i(b) = 1, x_j(b) = 0 (\forall j \neq i)$: means the i-th bidder wins the item
- $(0, 0, \cdots, 0)$: means no one wins the item

Single-parameter environment

Setting:

- n agents. Each has private information v_i, but will bid b_i.
- bids $\mathbf{b} = (b_1, b_2, \cdots, b_n)$
- Allocation rule: $\mathbf{x}(\mathbf{b}) = (x_1(\mathbf{b}), x_2(\mathbf{b}), \cdots, x_n(\mathbf{b}))$
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For sponsored search problem, the feasible allocation rules are

- For $1 \leq j \leq k$, α_j appears at most once in $(x_1(\mathbf{b}), x_2(\mathbf{b}), \cdots, x_n(\mathbf{b}))$
- For example, when $n = 4, k = 2$: $(\alpha_1, 0, \alpha_2, 0), (\alpha_2, 0, 0, 0)$ are feasible allocation rules.

Single-parameter environment

Setting:

- n agents. Each has private information v_i, but will bid b_i.
- bids $\mathbf{b} = (b_1, b_2, \cdots, b_n)$
- Allocation rule: $\mathbf{x}(\mathbf{b}) = (x_1(\mathbf{b}), x_2(\mathbf{b}), \cdots, x_n(\mathbf{b}))$
- Payment rule: $\mathbf{p}(\mathbf{b}) = (p_1(\mathbf{b}), p_2(\mathbf{b}), \cdots, p_n(\mathbf{b}))$
- Utility function: $u_i(\mathbf{b}) = v_i \cdot x_i(\mathbf{b}) - p_i(\mathbf{b})$

Thus, we have

$b_{-i} = b_1, b_2, \cdots b_{i-1}, b_{i+1}, \cdots b_n$.

true private value

- Truthful: for any i, b_{-i}, b_i, we have $u_i(v_i, b_{-i}) \geq u_i(b_i, b_{-i})$
- Social welfare maximization: $\max \sum_i v_i \cdot x_i(\mathbf{b})$

Social welfare

$\because \sum_i p_i(b) + \sum_{i=1}^{n} u_i(b) = \sum_{i=1}^{n} v_i \cdot x_i(b) - \sum_{i=1}^{n} p_i(b)$.

↑ utility of designer of mechanism.

$k=1$

$x(b) = (\alpha_1, 0 \cdots 0)$

$p(b) = (b_2 \cdot \alpha_1, 0, \cdots 0)$

second price auction

truthful.

Mechanism:

- suppose $b_1 \geq b_2 \geq \cdots \geq b_n$
- $\mathbf{x}(\mathbf{b}) = (\alpha_1, \cdots, \alpha_k, 0, \cdots, 0)$
- $\mathbf{p}(\mathbf{b}) = (b_2 \cdot \alpha_1, b_3 \cdot \alpha_2, \cdots, b_{k+1} \cdot \alpha_k, 0, \cdots, 0)$

Good or not?

Generalized second-price auction mechanism

Mechanism:

- suppose $b_1 \geq b_2 \geq \cdots \geq b_n$
- $\mathbf{x}(\mathbf{b}) = (\alpha_1, \cdots, \alpha_k, 0, \cdots, 0)$
- $\mathbf{p}(\mathbf{b}) = (b_2 \cdot \alpha_1, b_3 \cdot \alpha_2, \cdots, b_{k+1} \cdot \alpha_k, 0, \cdots, 0)$

Good or not?

- Not truthful!
- Example: $n = 3, k = 2, \alpha_1 = 1, \alpha_2 = 0.4$, private value $(7, 6, 1)$. Consider agent 1:

$b_2 \cdot \alpha_1 = 6 \cdot 1$ $\quad$ $7 - 6 = 1$

Bid	Allocation	Payment	Utility
$(7 \text{ (honest)}, 6, 1)$	$(1, 0.4, 0)$	(6,0.4,0)	(1, 2, 0)
$(5 \text{ (cheat)}, 6, 1)$	$(0.4, 1, 0)$	(0.4,5,0)	(2.4, 1, 0)

$b_3 \cdot \alpha_2 = 1 \cdot 0.4$ $\quad$ $7 \cdot 0.4 - 0.4$

Generalized second-price auction mechanism

Mechanism:

- suppose $b_1 \geq b_2 \geq \cdots \geq b_n$
- $\mathbf{x}(\mathbf{b}) = (\alpha_1, \cdots, \alpha_k, 0, \cdots, 0)$
- $\mathbf{p}(\mathbf{b}) = (b_2 \cdot \alpha_1, b_3 \cdot \alpha_2, \cdots, b_{k+1} \cdot \alpha_k, 0, \cdots, 0)$

Good or not?

- very simple and intuitive, widely used
- still has many nice properties and in some sense stable
- Reference
    - B. Edelman, M. Ostrovsky and M. Schwarz (2007), "Internet Advertising and the Generalized Second-Price Auction: Selling Billions of Dollars Worth of Keywords". American Economic Review 97(1): 242-259

Sponsored search auction

Can we design a truthful mechanism which maximizes the social welfare?

Can we design a truthful mechanism which maximizes the social welfare?

Mechanism:

$\max_{x} \sum_{i \geq 1}^{n} v_i \cdot x_i(b)$

- Suppose $b_1 \geq b_2 \geq \cdots \geq b_n$
- Allocation rule which maximizes the social welfare: $\mathbf{x}(\mathbf{b}) = (\alpha_1, \cdots, \alpha_k, 0, \cdots, 0)$
  - The same as the generalized second-price auction
- Is there a suitable "payment rule" so that $(\mathbf{x}, \mathbf{p})$ is truthful?

Sponsored search auction

Can we design a truthful mechanism which maximizes the social welfare?

Mechanism:

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- Is there a suitable "payment rule" so that $(\mathbf{x}, \mathbf{p})$ is truthful?

Answer: Myerson's lemma

Definition (Implementable allocation rule)

For any single-parameter problem, an allocation rule $\mathbf{x}$ is implementable if there exists a payment rule $\mathbf{p}$ such that $(\mathbf{x}, \mathbf{p})$ is truthful.

Definition (Monotone allocation rule)

For any single-parameter problem, an allocation rule $\mathbf{x}(\mathbf{b})$ is monotone if for any $i, b_{-i}, b_i > b_i'$, we have $x_i(b_i, b_{-i}) \geq x_i(b_i', b_{-i})$.

$x(b) = (x_1(b), \cdots\ x_n(b))$

$x_i(b) = x_i(b_1, \cdots, b_n) = x_i(b_i, b_{-i})$ is monotone w.r.t. b_i

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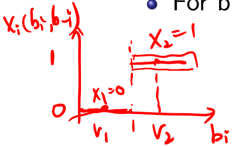
Theorem (Myerson's lemma)

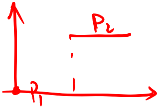
For any single-parameter problem, an allocation rule $\mathbf{x}$ *is monotone if and only if it is implementable.*

Single item auction

- Step 1: Consider the allocation rule which maximizes the social welfare
  - Allocate the item to the highest bid
- Step 2: Show that the allocation rule is monotone
  - For bidder i, let $b_{\max} = \max_{j \neq i} b_j$, then we have



$$x_i(\mathbf{b}) = \begin{cases} 1 & \text{if } b_i \geq b_{\max} \\ 0 & \text{otherwise} \end{cases}$$



Single item auction

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$$x_i(\mathbf{b}) = \begin{cases} 1 & \text{if } b_i \geq b_{\max} \\ 0 & \text{otherwise} \end{cases}$$

- Step 3: Compute the payment rule according to Myerson's lemma
  - For bidder i

$$p_i(\mathbf{b}) = \begin{cases} b_{\max} & \text{if } b_i \geq b_{\max} \\ 0 & \text{otherwise} \end{cases}$$

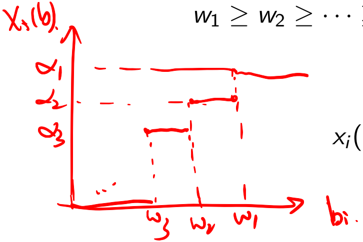
= second highest bid

- This is actually the second price auction

Application of Myerson's Lemma

Sponsored Search Auction

- Step 1: Fix the allocation rule
  - Suppose $b_1 \geq b_2 \geq \cdots \geq b_n$
  - Allocation rule: $\mathbf{x}(\mathbf{b}) = (\alpha_1, \cdots, \alpha_k, 0, \cdots, 0)$
- Step 2: Show the allocation rule is monotone
  - For bidder i, suppose bids of the other $n-1$ bidders are $w_1 \geq w_2 \geq \cdots \geq w_{n-1}$



$$x_i(\mathbf{b}) = \begin{cases} \alpha_1 & \text{if } b_i \geq w_1 \\ \alpha_2 & \text{if } w_2 \leq b_i < w_1 \\ \cdots & \\ \alpha_k & \text{if } w_k \leq b_i < w_{k-1} \\ 0 & \text{if } b_i < w_k \end{cases}$$

- Step 3: Compute the payment rule according to Myerson's lemma

Myerson's lemma

Very nice result! monotone allocation $\rightarrow$ truthful for free!
Roger Myerson:

- 2007 Nobel Memorial Prize in Economic Sciences - with Leonid Hurwicz and Eric Maskin
- Contribution: having laid the foundations of mechanism design theory

Reference:

- R. Myerson (1981). Optimal auction design. Mathematics of Operations Research, 6(1):58-73

Limitation:

- Works only for single parameter environment

What about general case?

Setting

- n agents, Ω: a finite set of outcomes
- For i-th agent, private valuation function $v_i : \Omega \to \mathbb{R}_{\geq 0}$, bid $b_i : \Omega \to \mathbb{R}_{\geq 0}$
- $\mathbf{x}(\mathbf{b}) \in \Omega, \mathbf{p}(\mathbf{b}) \in \mathbb{R}^n$
- Utility function for i-th agent: $u_i(\mathbf{b}) = v_i(\mathbf{x}(\mathbf{b})) - p_i(\mathbf{b})$

single parameter case. $v_i \cdot x_i(b)$

Setting

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For single item auction $(n = 3)$:

- Outcome $\Omega = \{(0,0,0), (1,0,0), (0,1,0), (0,0,1)\}$
- Private valuation function for bidder 1: $(0, v_1, 0, 0)$

$$(0, v_1, \varepsilon, 0), \quad \varepsilon > 0.$$

Setting

- n agents, Ω: a finite set of outcomes
- For i-th agent, private valuation function $v_i : \Omega \to \mathbb{R}_{\geq 0}$, bid $b_i : \Omega \to \mathbb{R}_{\geq 0}$
- $\mathbf{x}(\mathbf{b}) \in \Omega, \mathbf{p}(\mathbf{b}) \in \mathbb{R}^n$
- Utility function for i-th agent: $u_i(\mathbf{b}) = v_i(\mathbf{x}(\mathbf{b})) - p_i(\mathbf{b})$

Thus, we have

- Truthful: for any i, b_{-i}, b_i, we have $u_i(v_i, b_{-i}) \geq u_i(b_i, b_{-i})$
- Social welfare maximization: $\max_\omega \sum_i v_i(\omega)$

Theorem (VCG mechanism)

In every general mechanism design environment, there is a truthful mechanism which maximizes the social welfare.

- Allocation rule: $\mathbf{x}(\mathbf{b}) = \arg\max_{\omega \in \Omega} \sum_i b_i(\omega)$

Vickrey-Clarke-Groves mechanism

Theorem (VCG mechanism)

In every general mechanism design environment, there is a truthful mechanism which maximizes the social welfare.

- Allocation rule: $\mathbf{x}(\mathbf{b}) = \arg\max_{\omega \in \Omega} \sum_i b_i(\omega)$
- Payment rule: $p_i(\mathbf{b}) = \max_{\omega \in \Omega}(\sum_{j \neq i} b_j(\omega)) - \sum_{j \neq i} b_j(\omega^*)$ where $\omega^* = \arg\max_{\omega \in \Omega} \sum_i b_i(\omega)$

$= x(b)$.

social welfare maximization without agent i.

$= b_i(\omega^*) - [\sum_j b_j(\omega^*) - \max_{\omega} \sum_{j \neq i} b_j(\omega)]$.

bid of agent i.

rebate: "increase of social welfare" attribute to i's presence

Application: Single Item Auction

- Allocation rule: $\mathbf{x}(\mathbf{b}) = \arg\max_{\omega \in \Omega} \Sigma_i b_i(\omega)$
  - Allocate the item to the bidder with highest bid

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  - Case 1: bidder i's bid is the highest bid
    - $\max_{\omega \in \Omega}(\Sigma_{j \neq i} b_j(\omega))$ = second highest bid
    - $\Sigma_{j \neq i} b_j(\omega^*) = 0$
    - So $p_i(\mathbf{b})$ = second highest bid

Application: Single Item Auction

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  - Allocate the item to the bidder with highest bid
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    - $\sum_{j \neq i} b_j(\omega^*) = 0$
    - So $p_i(\mathbf{b})$ = second highest bid
  - Case 2: bidder i's bid is not the highest bid
    - $\max_{\omega \in \Omega}(\sum_{j \neq i} b_j(\omega))$ = highest bid
    - $\sum_{j \neq i} b_j(\omega^*)$ = highest bid
    - So $p_i(\mathbf{b}) = 0$

Application: Single Item Auction

- Allocation rule: $\mathbf{x}(\mathbf{b}) = \arg\max_{\omega \in \Omega} \Sigma_i b_i(\omega)$
  - Allocate the item to the bidder with highest bid
- Payment rule: $p_i(\mathbf{b}) = \max_{\omega \in \Omega}(\Sigma_{j \neq i} b_j(\omega)) - \Sigma_{j \neq i} b_j(\omega^*)$ where $\omega^* = \arg\max_{\omega \in \Omega} \Sigma_i b_i(\omega)$
  - Case 1: bidder i's bid is the highest bid
    - $\max_{\omega \in \Omega}(\Sigma_{j \neq i} b_j(\omega)) =$ second highest bid
    - $\Sigma_{j \neq i} b_j(\omega^*) = 0$
    - So $p_i(\mathbf{b}) =$ second highest bid
  - Case 2: bidder i's bid is not the highest bid
    - $\max_{\omega \in \Omega}(\Sigma_{j \neq i} b_j(\omega)) =$ highest bid
    - $\Sigma_{j \neq i} b_j(\omega^*) =$ highest bid
    - So $p_i(\mathbf{b}) = 0$
- This is exactly second-price auction.

Application: Sponsored Search Problem

Suppose $b_1 \geq b_2 \geq \cdots \geq b_n$

- Allocation rule: $\mathbf{x}(\mathbf{b}) = \arg\max_{\omega \in \Omega} \Sigma_i b_i(\omega)$
  - $\mathbf{x}(\mathbf{b}) = (\alpha_1, \alpha_2, \cdots, \alpha_k, 0, \cdots, 0)$

Application: Sponsored Search Problem

Suppose $b_1 \geq b_2 \geq \cdots \geq b_n$

- Allocation rule: $\mathbf{x}(\mathbf{b}) = \arg\max_{\omega \in \Omega} \Sigma_i b_i(\omega)$
  - $\mathbf{x}(\mathbf{b}) = (\alpha_1, \alpha_2, \cdots, \alpha_k, 0, \cdots, 0)$ ← w^*.
- Payment rule: $p_i(\mathbf{b}) = \max_{\omega \in \Omega}(\Sigma_{j \neq i} b_j(\omega)) - \Sigma_{j \neq i} b_j(\omega^*)$ where $\omega^* = \arg\max_{\omega \in \Omega} \Sigma_i b_i(\omega)$. Consider bidder 1:
  - $\max_{\omega \in \Omega}(\Sigma_{j \neq 1} b_j(\omega)) = \alpha_1 b_2 + \alpha_2 b_3 + \cdots + \alpha_k b_{k+1}$ ←
    - the social welfare maximization without bidder 1
  - $\Sigma_{j \neq 1} b_j(\omega^*) = \alpha_2 b_2 + \alpha_3 b_3 + \cdots + \alpha_k b_k$
    - ω^* is the allocation which maximizes the social welfare
  - So $p_1(\mathbf{b}) = \alpha_1 b_2 - \alpha_2(b_2 - b_3) - \cdots - \alpha_k(b_k - b_{k+1})$
    - It is less than $\alpha_1 b_2$ (payment rule in generalized second-price auction)

Vickrey-Clarke-Groves mechanism

Advantage:

- If you want to maximize social welfare, "truthful" is always possible in principle.
- clean mechanism

Disadvantage:

- Cannot deal with other objective functions
- Parameter space may very large $\Omega \to \mathbb{R}$
- It is not always practical to compute the social welfare maximization

Reference

- W. Vickrey (1961). Counterspeculation, auctions, and competitive sealed tenders. Journal of Finance, 16(1):8-37
- E. H. Clarke (1971). Multipart pricing of public goods. Public Choice, 11(1):17-33
- T. Groves (1973). Incentives in teams. Econometrica, 41(4):617-631

Conclusion

- How to design truthful mechanism
- Myerson's Lemma
- VCG Mechanism

	Myerson's lemma	VCG mechanism
Setting	single parameter environment	general environment
Objective	any monotone allocation	max social welfare
Practicality	usually practical	less practical

Thank you! Any questions?