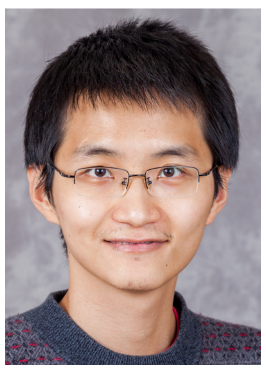
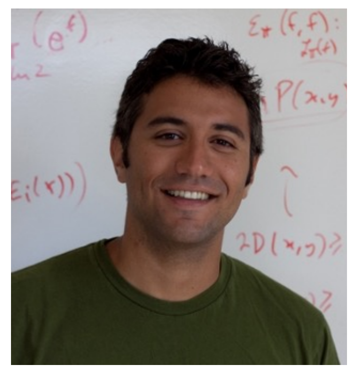
Of the People: Voting with Representative Candidates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Voting and Social Choice

- n candidates: A, B, C.
- Population of voters: Each ranks all candidates $A > B > C$.
- **Voting rule** selects a winner based on voters' preferences.

Ordinal Preferences: Example

- Input:

  A > B > C
  A > B > C
  A > B > C
  B > A > C
  B > A > C

  C > A > B
  C > A > B
  C > A > B
  C > A > B

- Output
  - Plurality: C (each voter casts a vote for top candidate)
  - Copeland: A (pairwise comparison: A > B, A > C, B > C)

Ordinal Preferences

- Arrow's impossibility theorem

System	Mono-tonic	Condorcet winner	Majo-rity	Condorcet loser	Majority loser	Mutual majority	Smith	ISDA	LIIA	Independence of clones	Reversal symmetry	Participation, consistency	Later-no-harm	Later-no-help	Polynomial time	Resol-vability
Anti-plurality[14]	Yes	No	No	No	Yes	No	No	No	No	No	No	Yes	No	No	Yes	Yes
Baldwin	No	Yes	Yes	Yes	Yes	Yes	Yes	No	No	No	No	No	No	No	Yes	Yes
Black	Yes	Yes	Yes	Yes	Yes	No	No	No	No	No	Yes	No	No	No	Yes	Yes
Borda	Yes	No	No	Yes	Yes	No	No	No	No	No	Yes	Yes	No	Yes	Yes	Yes
Bucklin	Yes	No	Yes	No	Yes	Yes	No	No	No	No	No	No	No	Yes	Yes	Yes
Contingent voting	No	No	Yes	Yes	Yes	No	No	No	No	No	No	No	Yes	Yes	Yes	Yes
Coombs[14]	No	No	Yes	Yes	Yes	Yes	No	No	No	No	No	No	No	No	Yes	Yes
Copeland	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	No	Yes	No	No	No	Yes	No
Dodgson[14]	No	Yes	Yes	No	No	No	No	No	No	No	No	No	No	No	No	Yes
Instant-runoff voting	No	No	Yes	Yes	Yes	Yes	No	No	No	Yes	No	No	Yes	Yes	Yes	Yes
Kemeny–Young	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	No	No	No	No	Yes
MiniMax	Yes	Yes	Yes	No	No	No	No	No	No	No	No	No	No	No	Yes	Yes
Nanson	No	Yes	Yes	Yes	Yes	Yes	Yes	No	No	No	Yes	No	No	No	Yes	Yes
Plurality	Yes	No	Yes	No	No	No	No	No	No	No	No	Yes	Yes	Yes	Yes	Yes
Ranked pairs	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	No	No	Yes	Yes
Schulze	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	No	No	No	Yes	Yes
Smith/IRV	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	No	No	No	No	Yes	Yes
Sri Lankan contingent voting	No	No	Yes	No	No	No	No	No	No	No	No	No	Yes	Yes	Yes	Yes
Supplementary voting	No	No	Yes	No	No	No	No	No	No	No	No	No	Yes	Yes	Yes	Yes
Tideman's Alternative	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	No	No	No	No	Yes	Yes

Utilitarian Approach

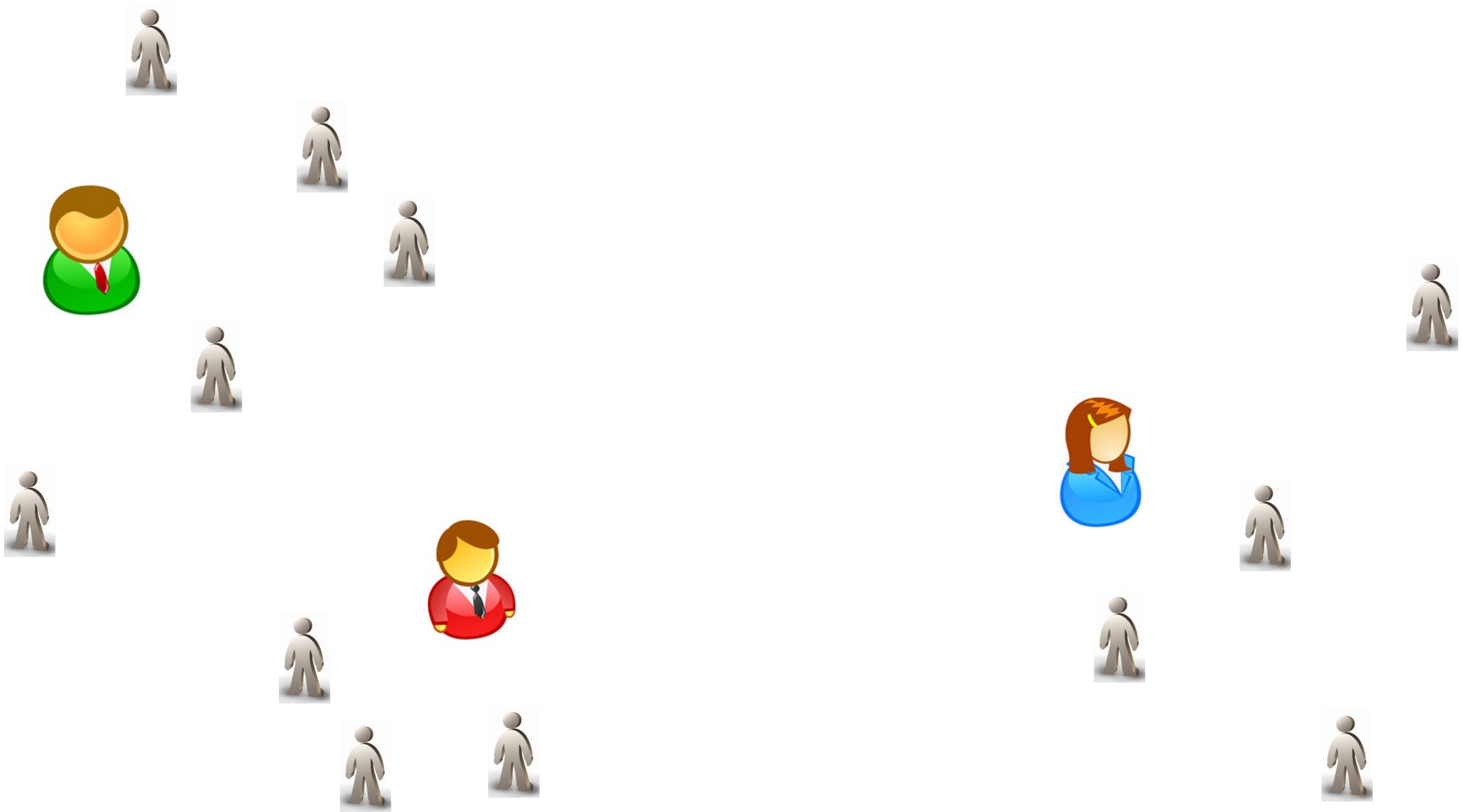
- The preferences come from cardinal utilities, but we can only observe ordinal preferences.

- Want to compare solutions quantitatively.

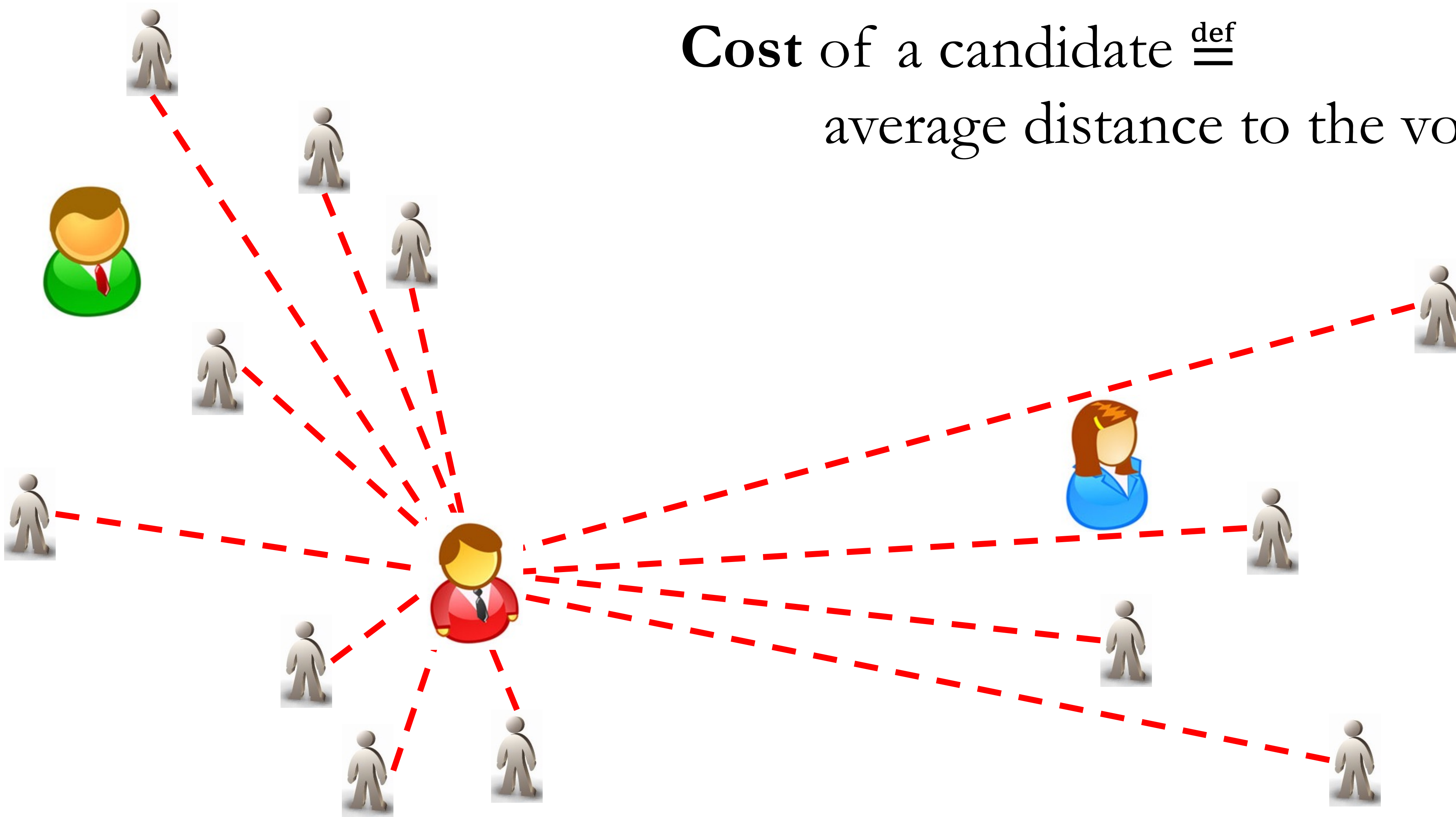
  Recent survey by Anshelevich et al.:

  Distortion in Social Choice Problems: The First 15 Years and Beyond. IJCAI 2021.

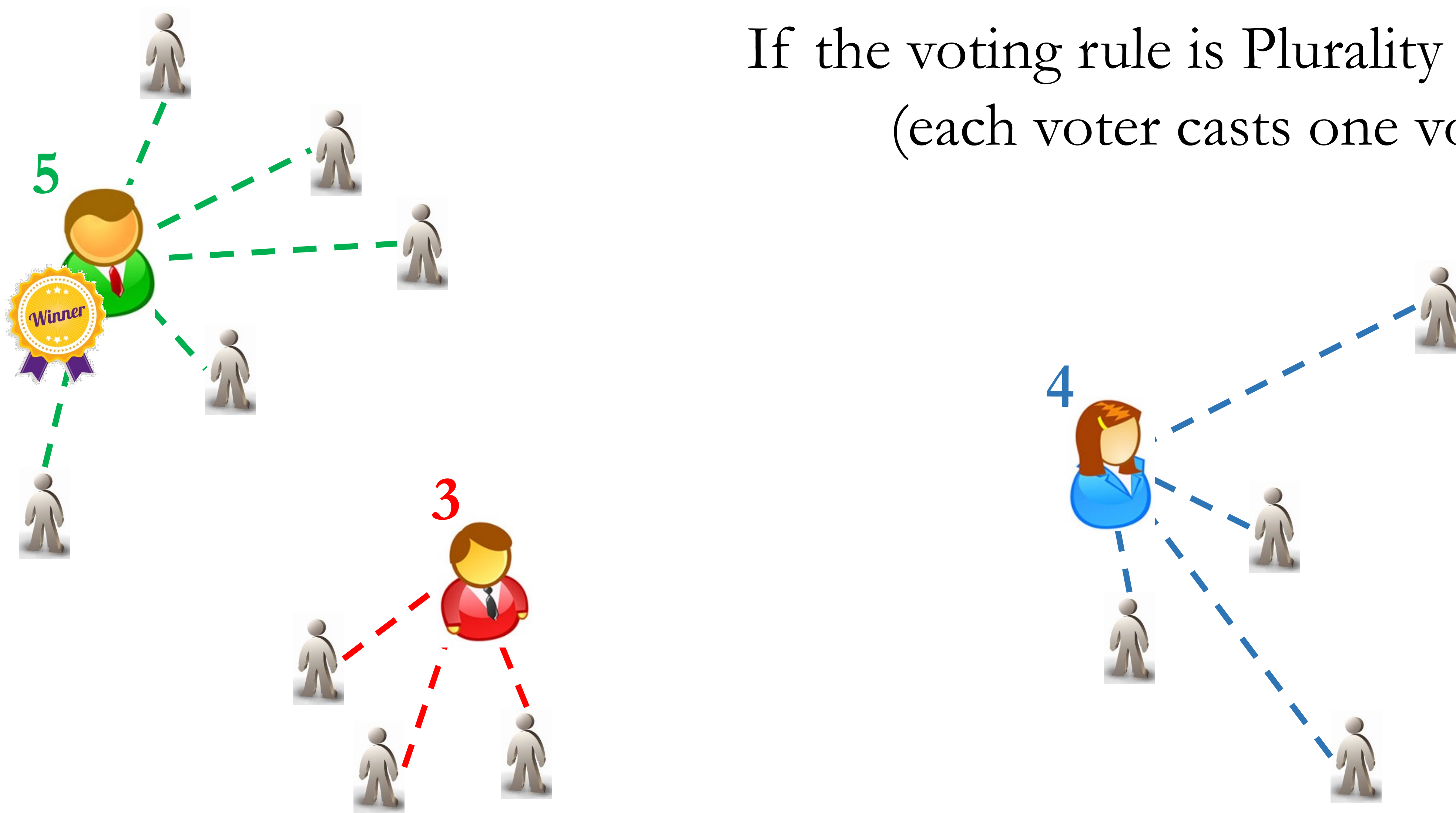
- This talk: metric preferences



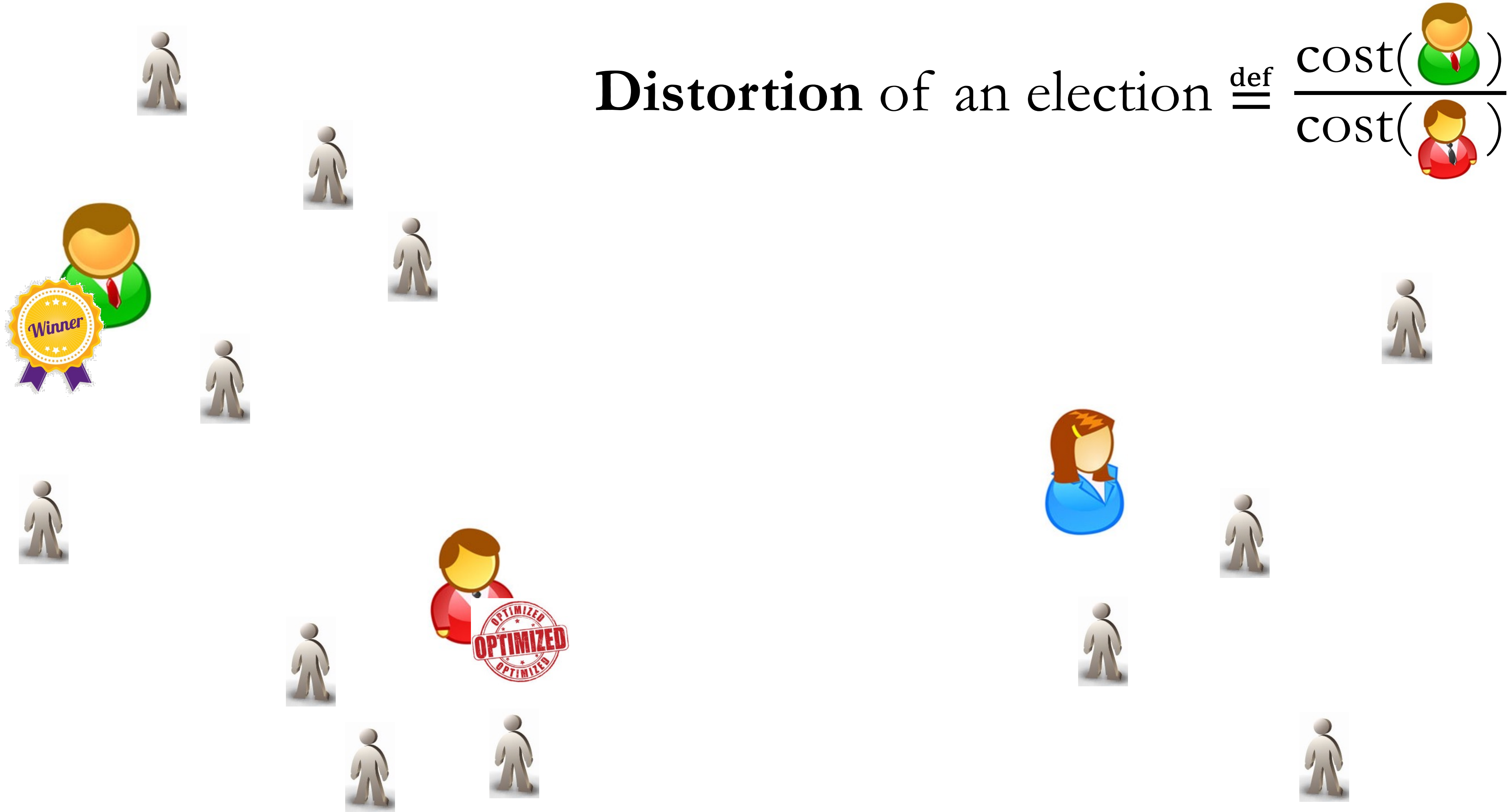
Cost of a candidate $\stackrel{\text{def}}{=}$ average distance to the voters.



If the voting rule is Plurality
(each voter casts one vote)



Distortion of an election $\stackrel{\text{def}}{=}$



Metric Preferences and Distortion

We can study the worst-case distortion of voting rules (maximum distortion over all instances, upper/lower bounds).

We can compare voting rules quantitatively (going beyond which rules satisfy which axioms).

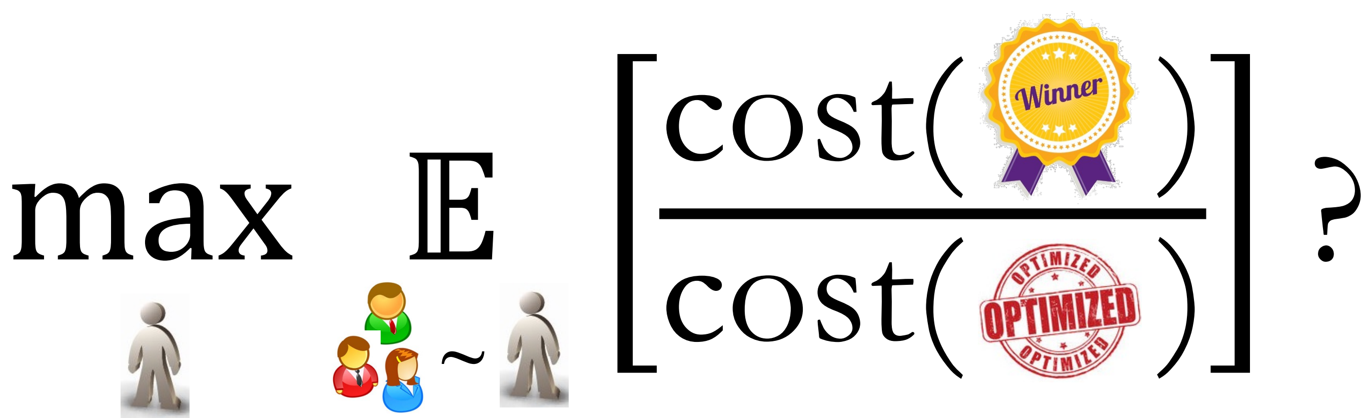
Metric Preferences and Distortion

We can study the worst-case distortion of voting rules (maximum distortion over all instances, upper/lower bounds).

What about the average-case distortion?

Our Contribution: Of the People

Candidates are drawn i.i.d. from the voter population.



Our Contribution: Of the People

Candidates are drawn i.i.d. from the voter population.

Is democracy more effective when candidates are representative?

How good are different voting rules when the candidates are representative?

Our Results

- Two candidates
- Multiple candidates

Our Results

- Two candidates + majority rule
  - Representative candidates $\Longrightarrow$ smaller distortion
  - The amount of improvement depends on the complexity of the underlying metric space.
- Multiple candidates

Our Results

- Two candidates

- Multiple candidates
  - A clean and tight characterization of positional voting rules that have constant distortion.
  - Allows us to distinguish voting rules that appear the same in the worst case.

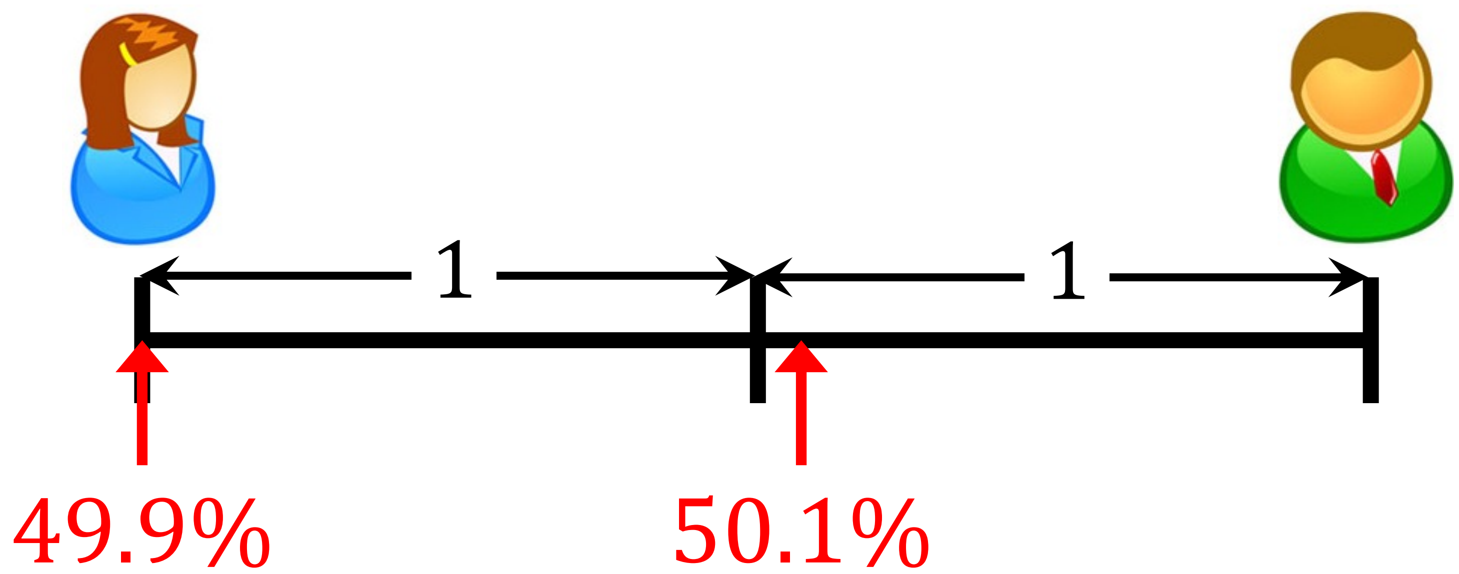
Part I: Two Candidates

Representative candidates $\Longrightarrow$ smaller distortion?

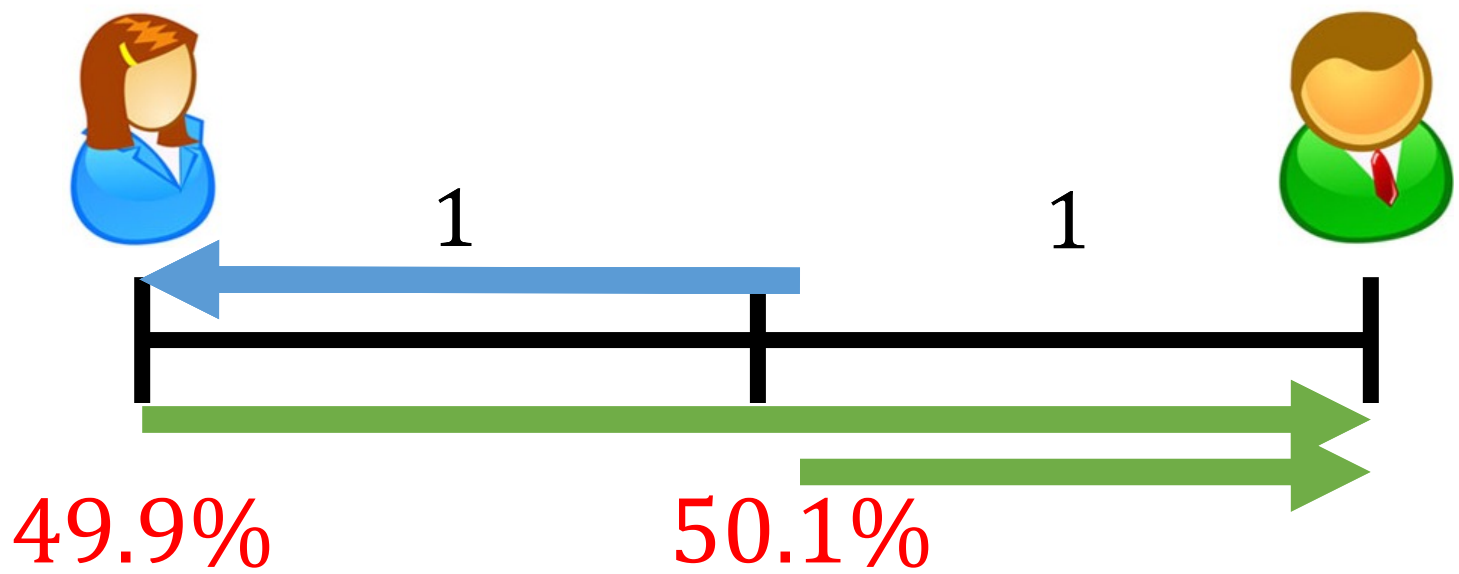
We will focus on:

- Two candidates.
- A majority-rule election.

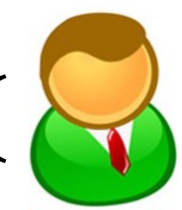
Example of Distortion 3 [Anshelevich et al.]



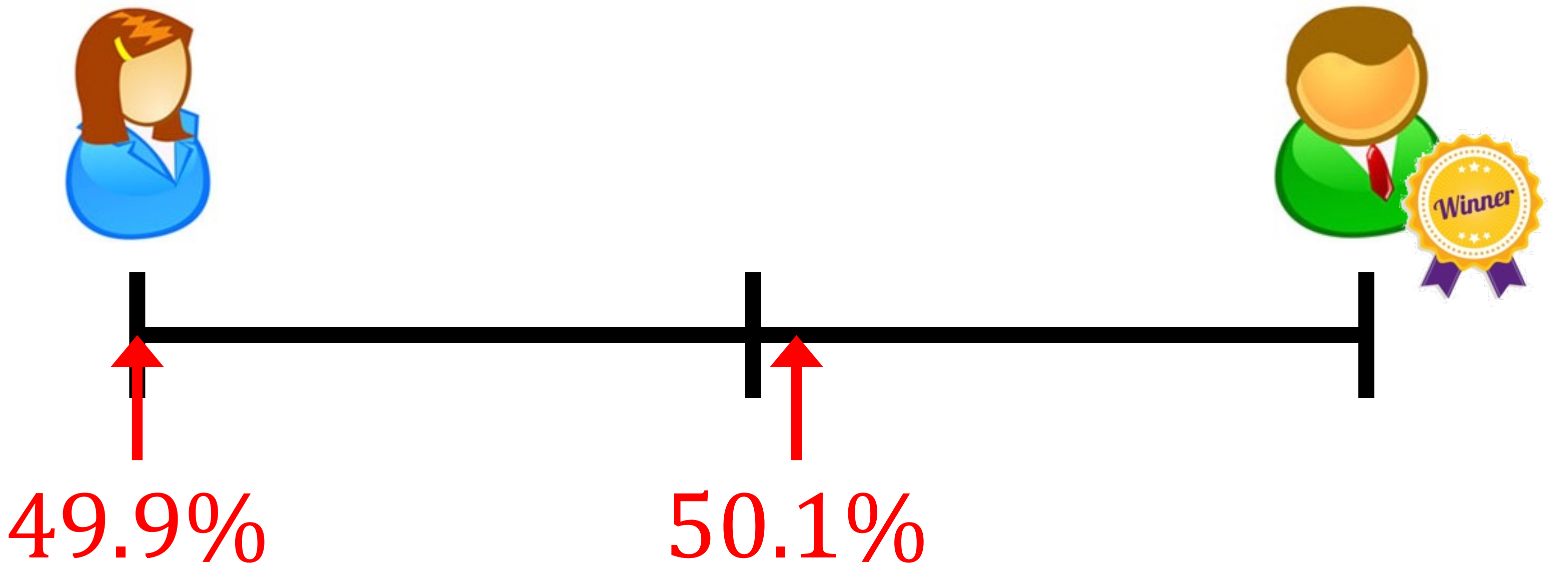
Example of Distortion 3 [Anshelevich et al.]



Cost() ≈ 0.5

Cost() ≈ 1.5

Example of Distortion 3 [Anshelevich et al.]

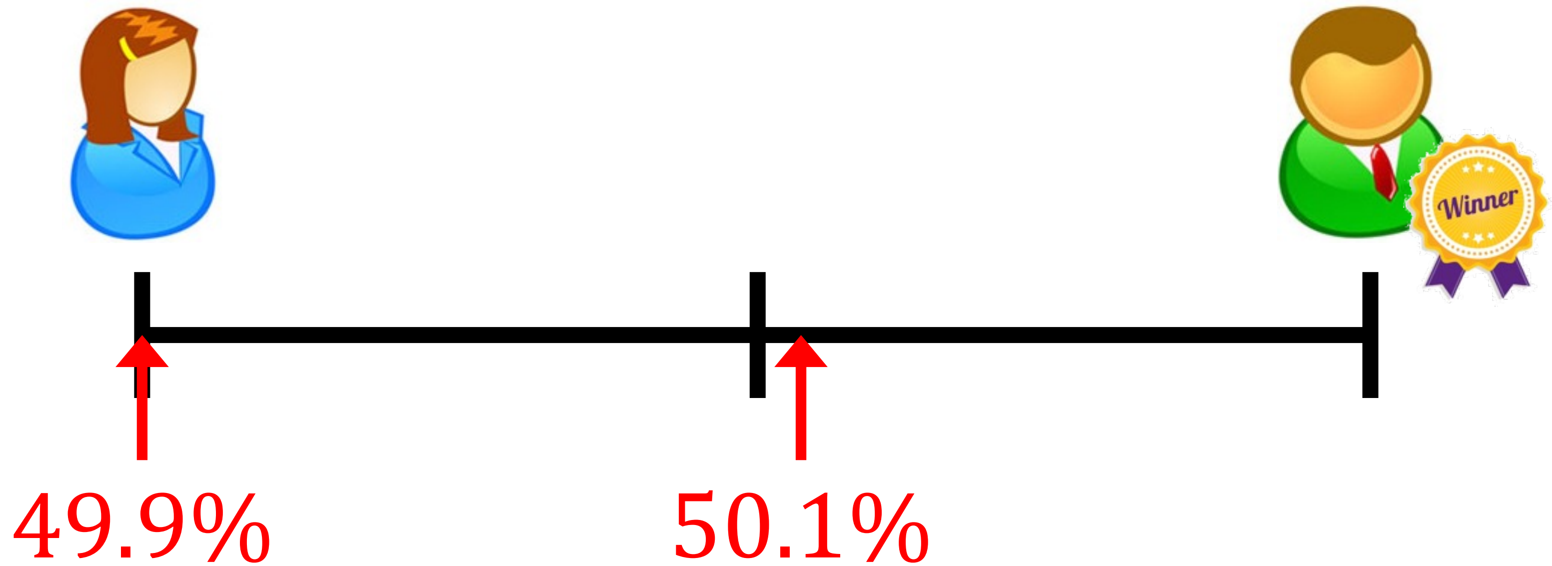


 wins the election,

50.1% against 49.9%.

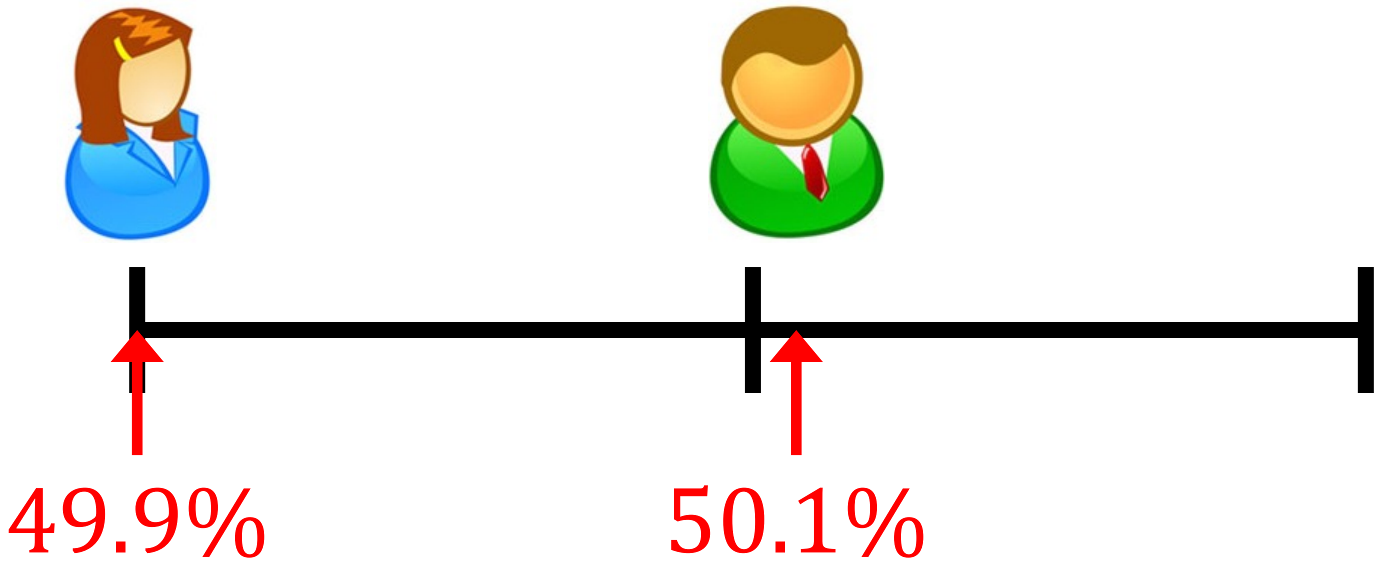
Distortion $\stackrel{\text{def}}{=}$ cost() / cost() ≈ 3

Example of Distortion 3 [Anshelevich et al.]



Reason for high distortion:  is not **of the people**.

Of the People



Had we drawn two candidates from the population, the winner would always be the socially optimal choice.

Our Results

Given candidates drawn from $\boldsymbol{p}$, we study the expected distortion.

- $\boldsymbol{p}$ is arbitrary [non-representative].
- $\boldsymbol{p}$ is uniform over the voters [representative].

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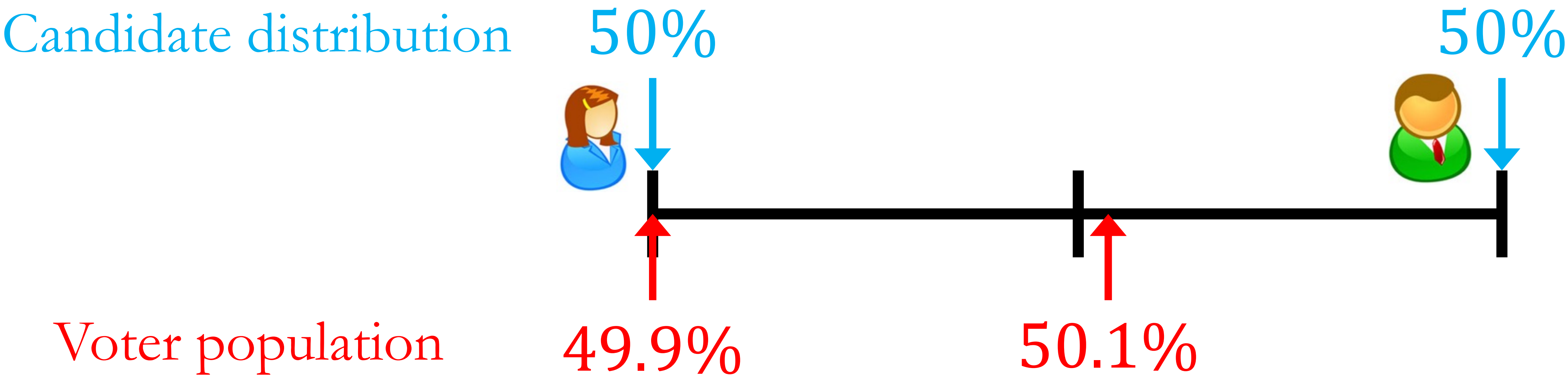
	Representative	Non-Representative
Line Metric		2
General Metric		2

Non-Representative (≥ 2)



With probability $1/2$, we get a distortion of ≈ 3.

So expected distortion $\approx (1/2) \cdot 3 + (1/2) \cdot 1 = 2$.

Our Results

Given candidates drawn from $\boldsymbol{p}$, we study the expected distortion.

- $\boldsymbol{p}$ is arbitrary [non-representative].
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	Representative	Non-Representative
Line Metric	$4 - 2\sqrt{2} \approx 1.1716$	2
General Metric		2

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	Representative	Non-Representative
Line Metric	$4 - 2\sqrt{2} \approx 1.1716$	2
General Metric	$[1.5, 2 - \frac{1}{652})$	2

Our Results

Takeaway Message

Voting is more effective with representative candidates.

Exact improvement depends on the complexity of the metric space.

	Representative	**Non-Representative**
Line Metric	$4 - 2\sqrt{2} \approx 1.1716$	2
General Metric	$[1.5, 2 - \frac{1}{652})$	2

Our Results

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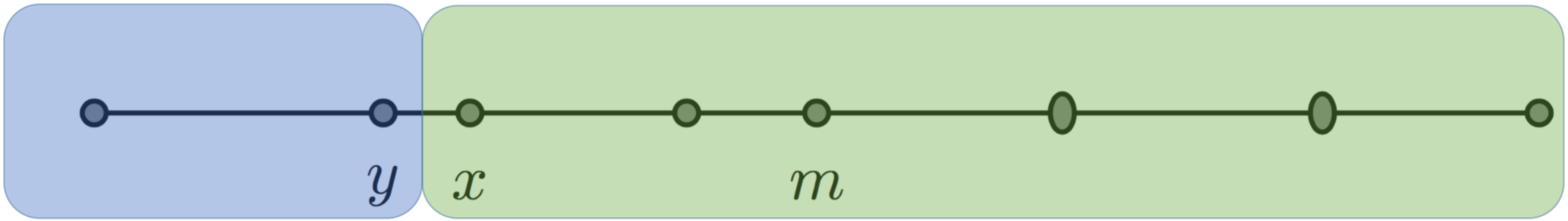
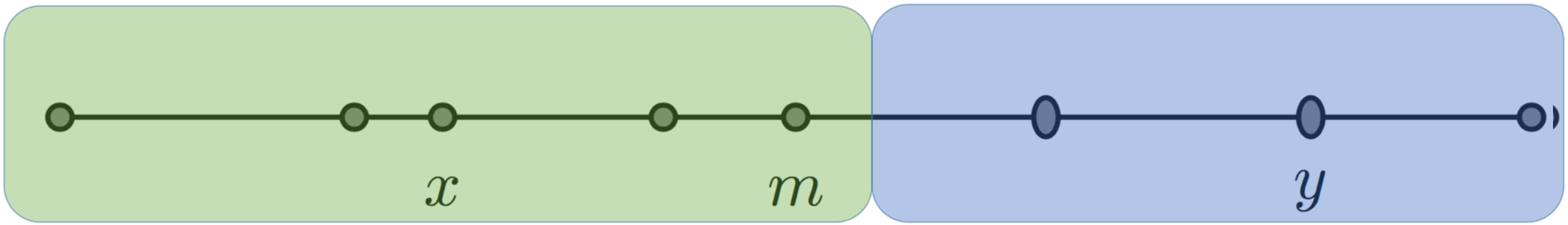
Voting on the Line: Structural Results



The median voter m.

1) The candidate closer to the median m wins the election.



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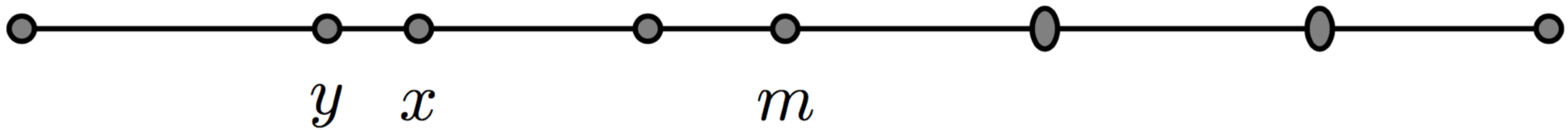
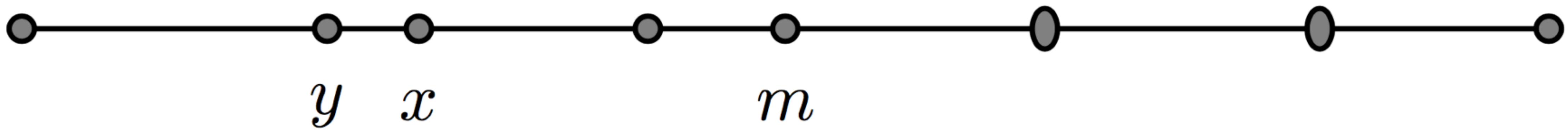


1) The candidate closer to the median m wins the election.

2) For two candidates x, y on the same side of the median m, the one closer to m has smaller social cost.

Intuition: More than half of the population need to first get to x before they can get to y.

Voting on the Line: Structural Results



1) The candidate closer to the median m wins the election.
2) For two candidates x, y on the same side of the median m, the one closer to m has smaller social cost.

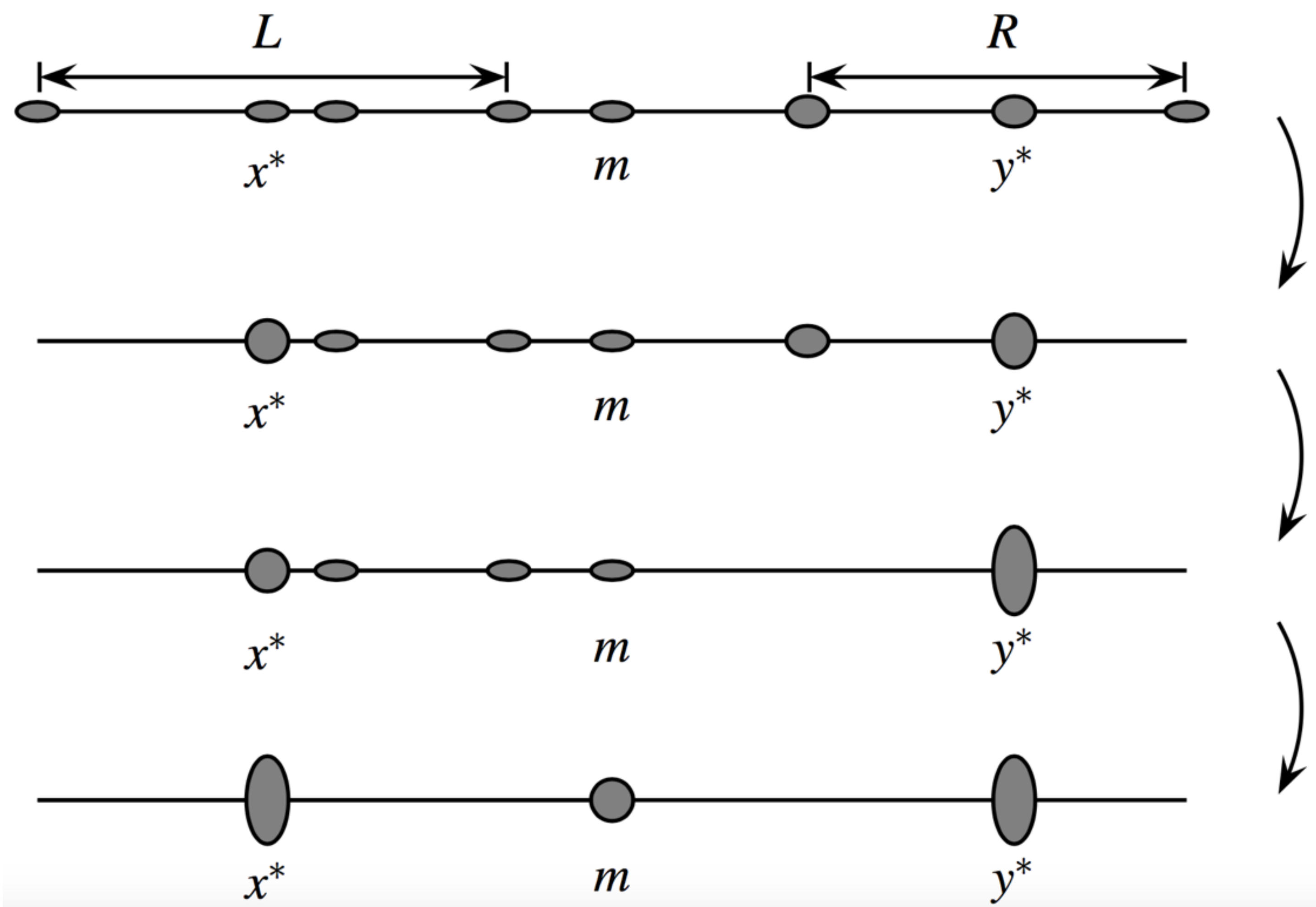
(1) + (2) $\implies$ If both candidates are on the same side of m, then majority voting elects the socially better candidate.

Voting on the Line (≈ 1.17)

Given any instance with support size larger than 3,
we can reduce its support to 3 using a series of operations,
without decreasing the distortion.

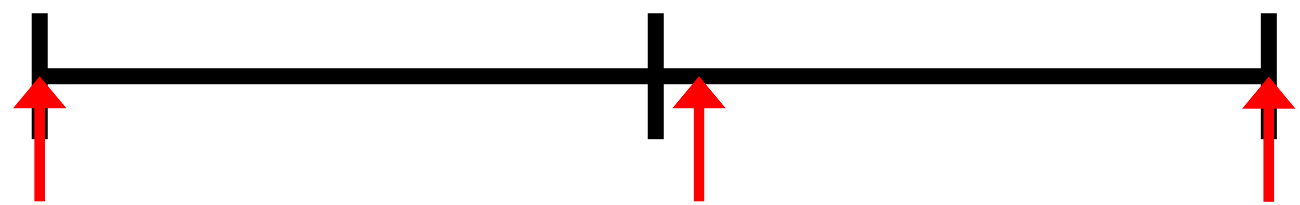
When shifting the probabilities, we use a global argument to show that the operation increases the distortion **on average**.

Voting on the Line (≈ 1.17)



Voting on the Line (≈ 1.17)

For support 3 distributions, we can optimize the locations and probabilities of these 3 points.



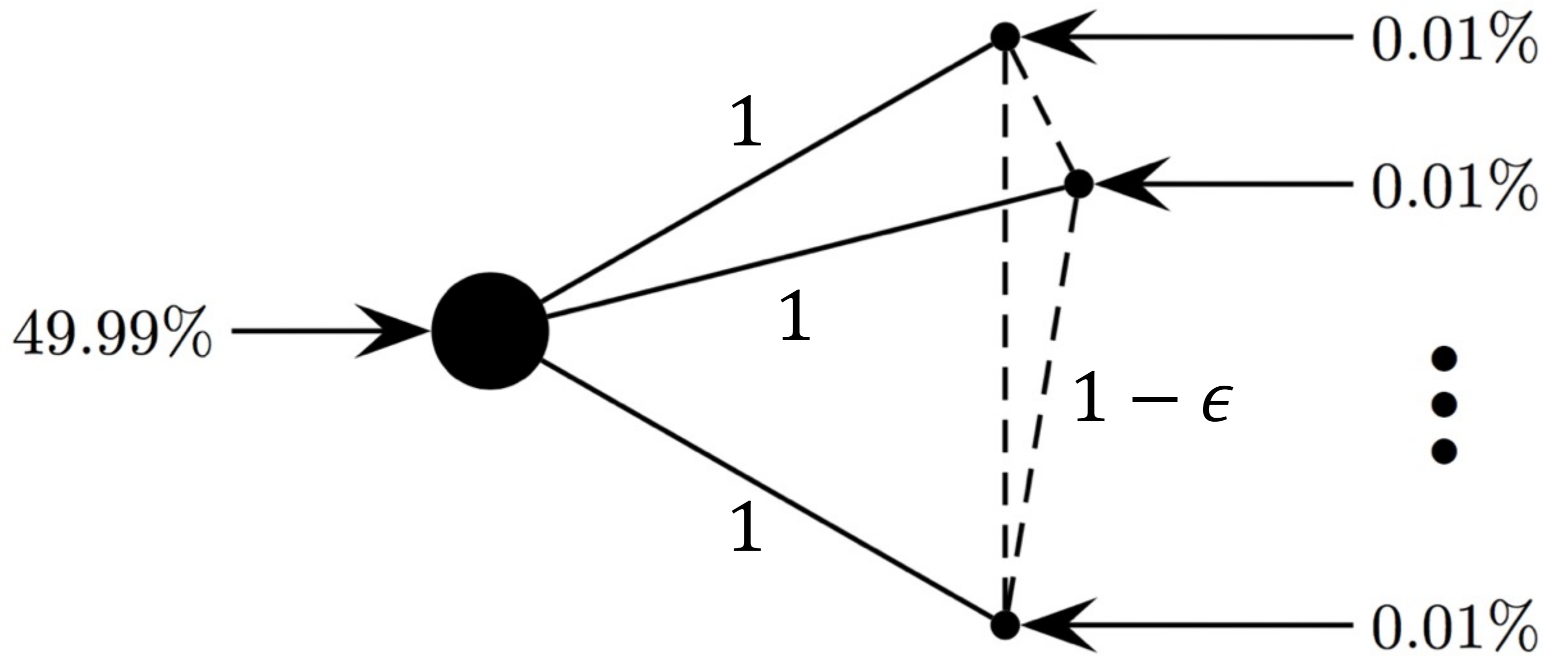
$p_1 \approx 49.9\%$ $p_2 \approx 29.3\%$ $p_3 \approx 20.7\%$

$$\text{Distortion} = 2p_1p_3 \cdot \frac{cost_3}{cost_1} + (1 - 2p_1p_3) \cdot 1 \approx 1.17.$$

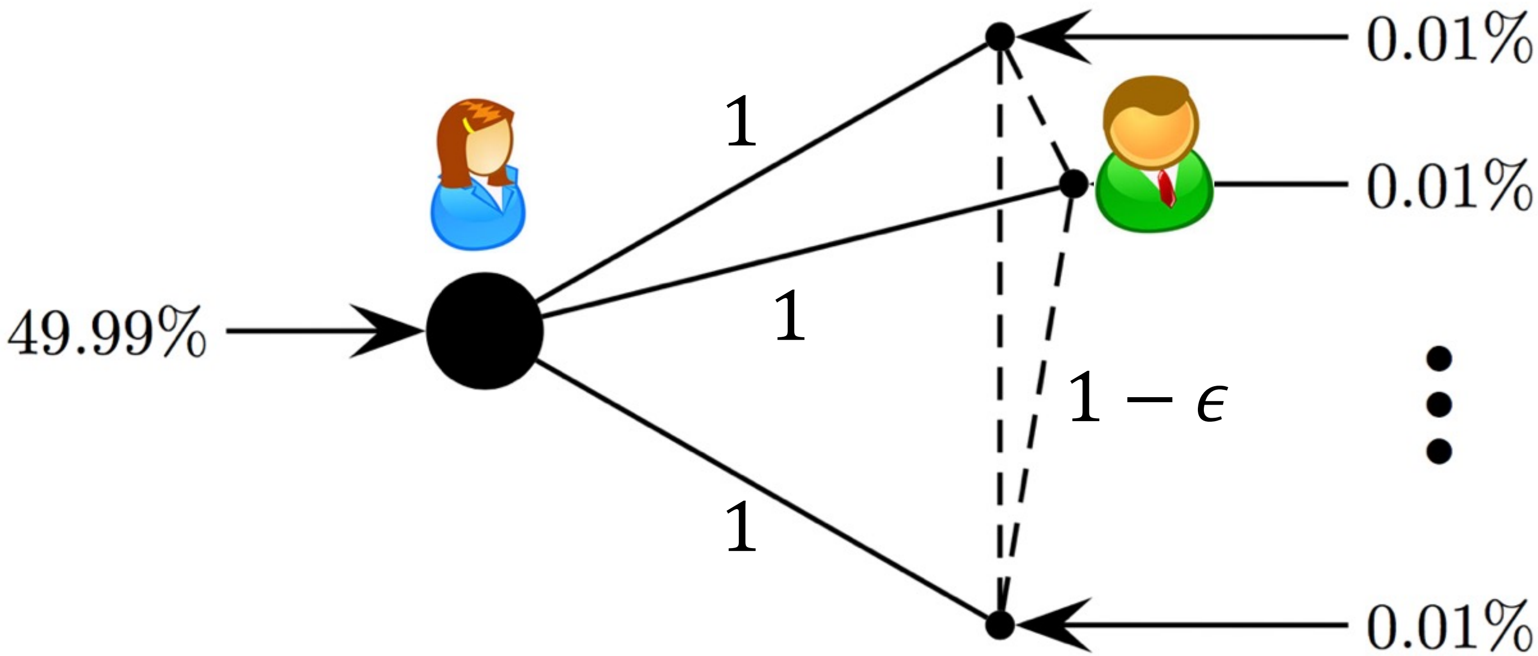
Our Results

	Representative	Non-Representative
Line Metric	1.1716	2
General Metric	$[1.5, 2 - \frac{1}{652})$	2

General Metric Space (≥ 1.5)



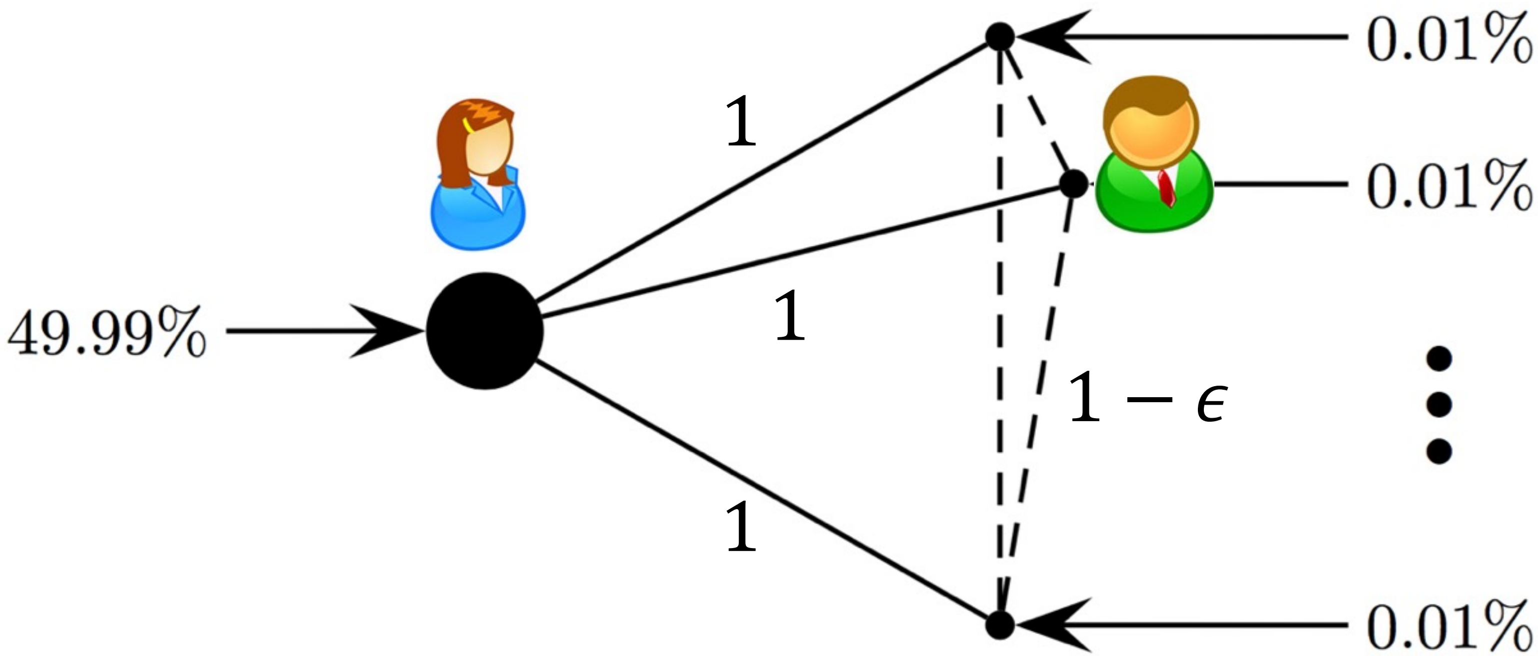
General Metric Space (≥ 1.5)



Cost() ≈ 0.5

Cost() ≈ 1

General Metric Space (≥ 1.5)

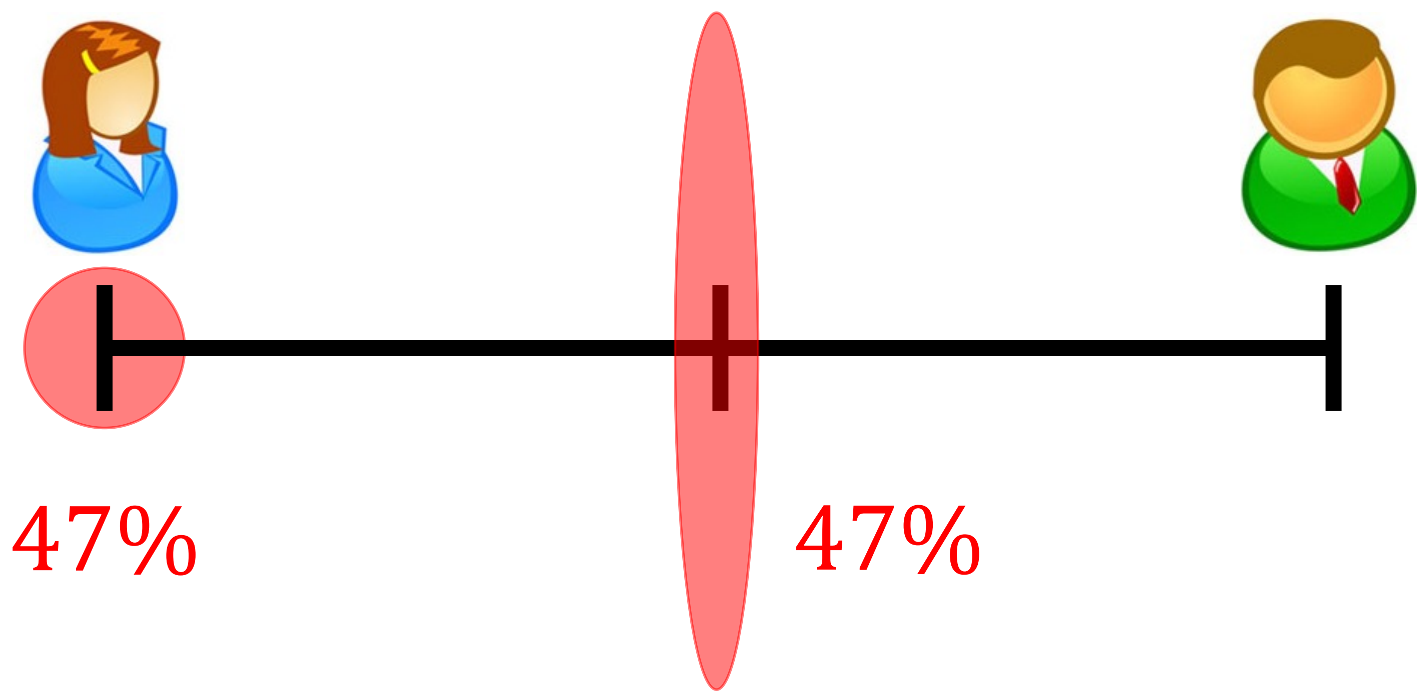


With probability $1/2$, the distortion ≈ 2

$\Longrightarrow$ Expected distortion ≈ 1.5.

General Metric Space ($\leq 2 - \frac{1}{652}$)

- If the expected distortion is sufficiently close to 2, there is a pair of candidates whose distortion is close to 3; we show that then the instance must have special structure.



Part II: Multiple Candidates

We are interested in the distortion of various voting rules when the candidates are representative.

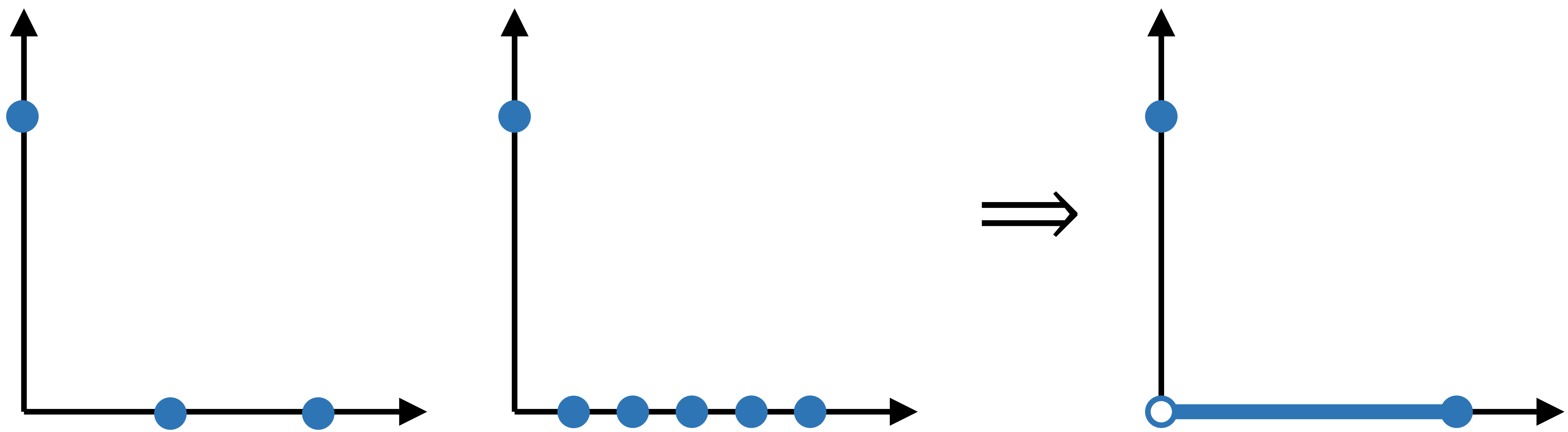
We will focus on:

- n candidates, m voters.
- Positional voting rules.
- Constant distortion (i.e., does not scale with n or m).

Positional Scoring Rules

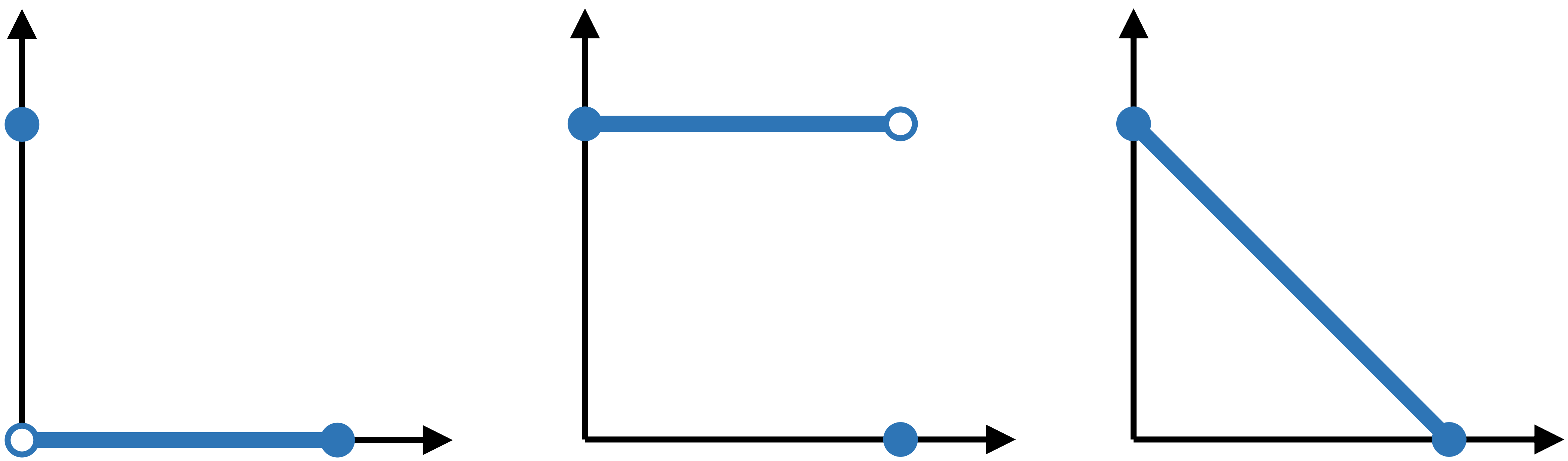
- Plurality: $(1, 0, \dots, 0)$
- Veto: $(1, \dots, 1, 0)$
- Borda: $(n-1, n-2, \dots, 1, 0) \Rightarrow \left(1, \frac{n-2}{n-1}, \dots, \frac{1}{n-1}, 0\right)$
- k-Approval: $(1, \dots, 1, 0, \dots, 0)$
- Dowdall: $(1, 1/2, 1/3, \dots, 1/n)$

Example: Plurality



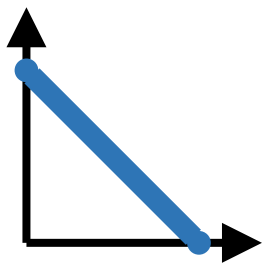
g_3 g_6 $\Longrightarrow$ $g_n \to g$

Limit Scoring Rule

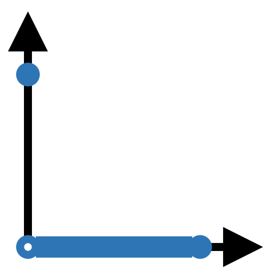


Plurality Veto Borda

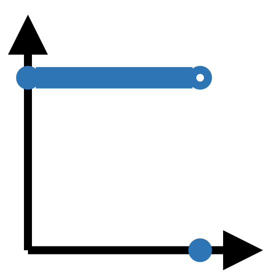
Characterization Result



- g not constant on $(0, 1)$ $O(1)$ distortion



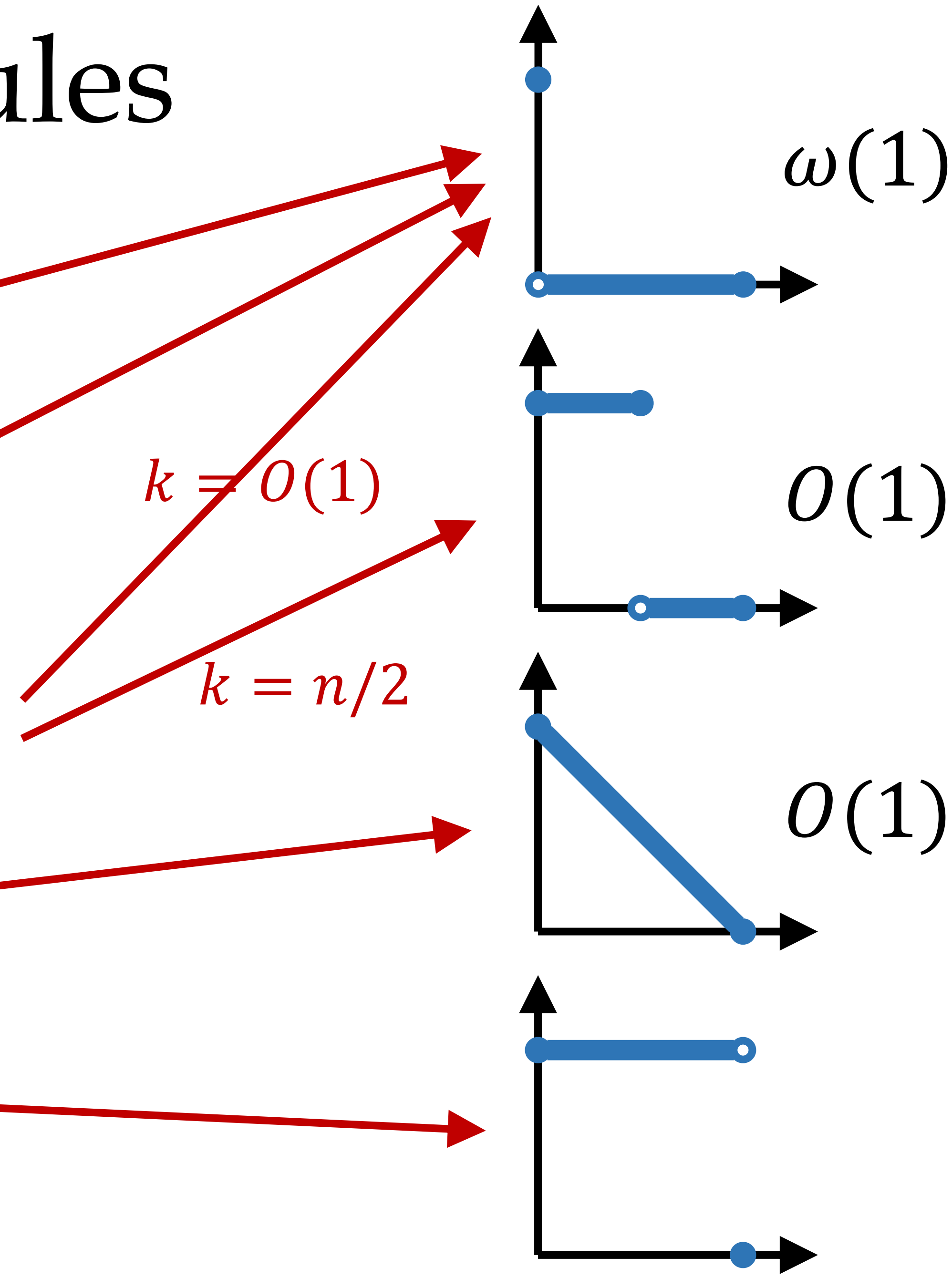
- g constant $\neq 1$ on $(0, 1)$ $\omega(1)$ distortion



- g constant $= 1$ on $(0, 1)$ more subtle condition

Positional Scoring Rules

- Plurality: $(1, 0, \dots, 0)$
- Dowdall: $\left(1, \frac{1}{2}, \frac{1}{3}, \dots, \frac{1}{n}\right)$
- k-Approval: $(1, \dots, 1, 0, \dots, 0)$
- Borda: $\left(1, \frac{n-2}{n-1}, \dots, \frac{1}{n-1}, 0\right)$
- Veto: $(1, \dots, 1, 0)$



Our Results

Voting Rule	$\mathbb{E}$[Distortion]
Plurality	$\Theta(n)$
Dowdall	$\Theta(n)$
k-Approval, $k = O(1)$	$\Theta(n)$

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Plurality	$\Theta(n)$
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k-Approval, $k = O(1)$	$\Theta(n)$
Borda	$\Theta(1)$
k-Approval, $k = n/2$	$\Theta(1)$

Our Results

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Plurality	$\Theta(n)$
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k-Approval, $k = O(1)$	$\Theta(n)$
Borda	$\Theta(1)$
k-Approval, $k = n/2$	$\Theta(1)$
Veto	$\Theta(n)$

Our Results

Voting Rule	$\mathbb{E}$[Distortion] (This paper)	Worst-Case [Anshelevich et al.]
Plurality	$\Theta(n)$	$\Theta(n)$
Dowdall	$\Theta(n)$	
k-Approval, $k = O(1)$	$\Theta(n)$	
Borda	$\Theta(1)$	$\Theta(n)$
k-Approval, $k = n/2$	$\Theta(1)$	
Veto	$\Theta(n)$	∞

Open Questions

- Other notions of average-case voting.
  - How does the distortion degrade as the voter and candidate distributions become more dissimilar?

- Average-case distortion in other problems (e.g., bipartite matching).

Open Questions: Two Candidates

	Representative	Non-Representative
Line Metric	1.1716	2
General Metric	$[1.5, 2 - \frac{1}{652})$	2

Maximum expected distortion in general metric spaces.

Distortion of restricted metric space (e.g., d-dimensional Euclidean space).

Open Questions: Multiple Candidates

- What can we say for voting rules that are not positional?
- How robust are the results to other notions of cost?