Dominik Peters

Proportional Participatory Budgeting

Summer School on Game Theory and Social Choice
2022-06-24

based on joint work with Piotr Skowron and Grzegorz Pierczyński

How to Vote

- You’re a member of an academic society which needs to elect a president. Several candidates are running.
- Could ask each member to rank all candidates, and use one of dozens of voting rules to make a decision (no consensus which rule to use)
  - Plurality, Borda, Instant Runoff, Schulze, Kemeny, …
- Or just use Approval Voting: allow each member to approve an arbitrary number of candidates, elect the one with the most approvals.
  - very well behaved
  - easy to use
  - wins if you ask voting theorists to vote for best voting rule

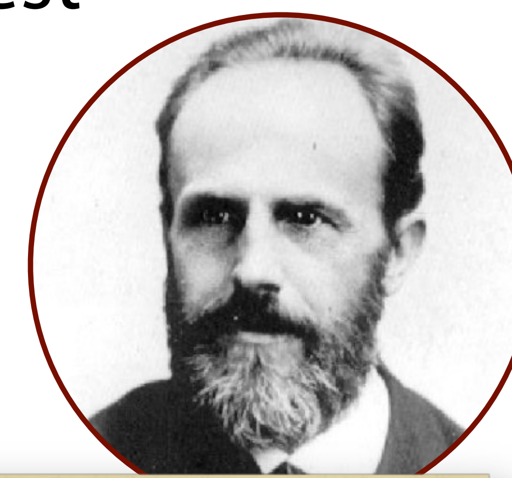
☒ Candidate A
☐ Candidate B
☐ Candidate C
☒ Candidate D
☐ Candidate E

Electing a Council

- Academic society doesn't only have a president. Also has a council with $k = 8$ members. Many people are running.
- Suppose voters submit approvals. How to select the council?
- Easiest way: select the 8 candidates with highest number of approvals.
- Could be bad: suppose field is split into topic A (60%) and topic B (40%). Rule could select 8 candidates from subfield A.

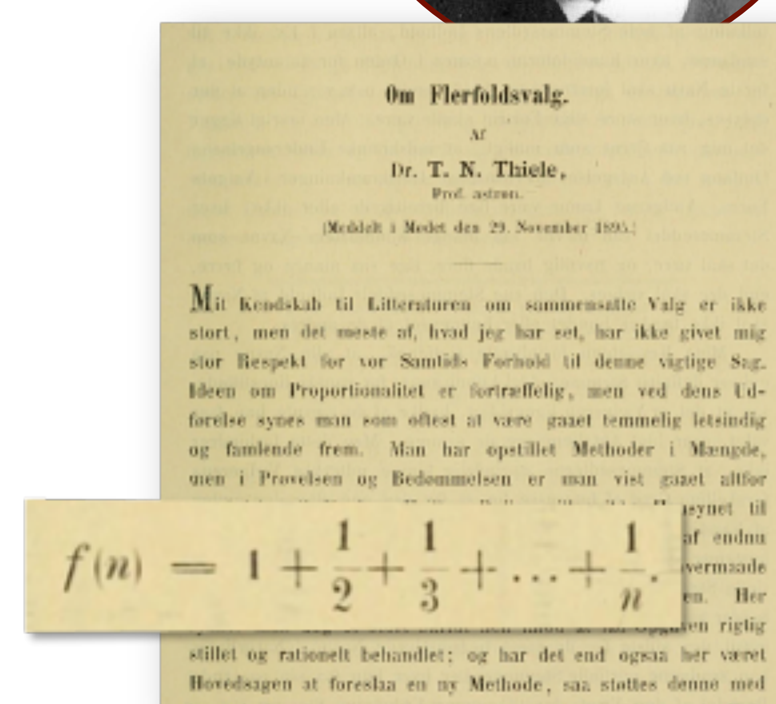
Rules for Committee Elections

- For a committee W of size k, write $u_i(W)$ for the number of committee members that i approves, $|A_i \cap W|$.
- The committee selecting the k candidates with highest approval score is the one maximizing $\sum_i u_i(W)$.
- Idea: to make majorities less overpowering, replace $u_i(W)$ by a concave function
- Thiele proposed this in 1895 for Sweden.



$$\sum_{i \in N} 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{|W \cap A_i|}$$

"Proportional Approval Voting" (PAV)



Om Flerfoldsvalg.

Af

Dr. T. N. Thiele,

Prof. astron.

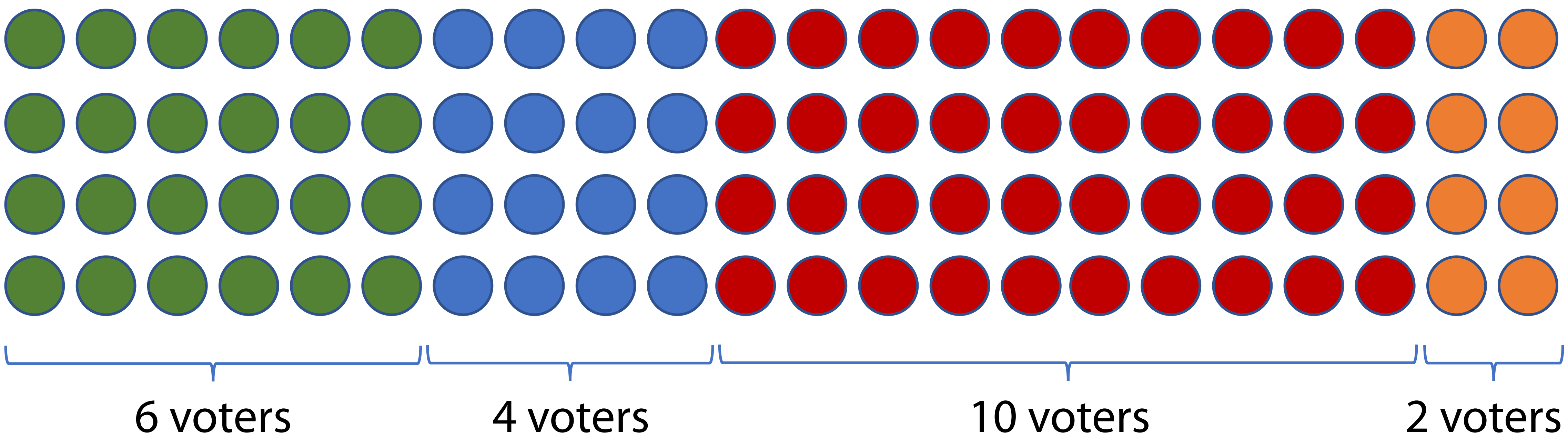
(Meddelt i Mødet den 29. November 1895.)

Mit Kendskab til Litteraturen om sammensatte Valg er ikke stort, men det meste af, hvad jeg har set, har ikke givet mig stor Respekt for vor Samtids Forhold til denne vigtige Sag. Ideen om Proportionalitet er fortræffelig, men ved dens Udførelse synes man som oftest at være gaaet temmelig letsindig og famlende frem. Man har opstillet Methoder i Mængde, men i Prøvelsen og Bedømmelsen er man vist gaaet altfor

$$f(n) = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}.$$

stillet og rationelt behandlet; og har det end ogsaa her været Hovedsagen at foreslaa en ny Methode, saa støttes denne med

PAV works



- Suppose both voters and candidates are partitioned into subfields. Then PAV selects candidates from subfields in proportion to the subfield size. (Thiele showed this in 1895.)

- Above, if $k = 11$, PAV selects 3 green, 2 blue, 5 red, 1 orange committee members.

- Harmonic numbers is the only function $f(|A_i \cap W|)$ that guarantees this (in the sense of following d'Hondt rounding).

  Aziz, H., Brill, M., Conitzer, V., Elkind, E., Freeman, R. and Walsh, T., 2017. Justified representation in approval-based committee voting. *Social Choice and Welfare*.

Extended Justified Representation (EJR)

- Usually, candidates and voters aren't partitioned. Approval sets overlap. We still want to be "proportional". What does this mean formally?

- Consider a group S of voters with $|S| \geq \ell\ n/k$.
  -> They are large enough to decide ℓ seats.

- Suppose there is a set T of candidates with $|T| = \ell$ such that every voter in S approves all of T ("cohesive group").

- For a committee W to satisfy EJR, it cannot be that every member of S prefers T to W
  -> thus at least one voter in S must approve at least ℓ members of W.

PAV satisfies EJR

- Theorem: PAV satisfies EJR.
  Aziz, H., Brill, M., Conitzer, V., Elkind, E., Freeman, R. and Walsh, T., 2017. Justified representation in approval-based committee voting. *Social Choice and Welfare*.

- Swapping argument.

  - Assume not. Then there is a candidate c in T that is not elected, and all voters in S have utility at most ℓ - 1.

  - This means adding c to the PAV committee would increase its score by at least $\frac{1}{\ell} \cdot |S| = \frac{n}{k}$.

  - Can check that, on average, removing a candidate from the PAV committee decreases its score by strictly less than n/k.

  - So by removing the worst candidate and adding c we get a better committee, contradiction.

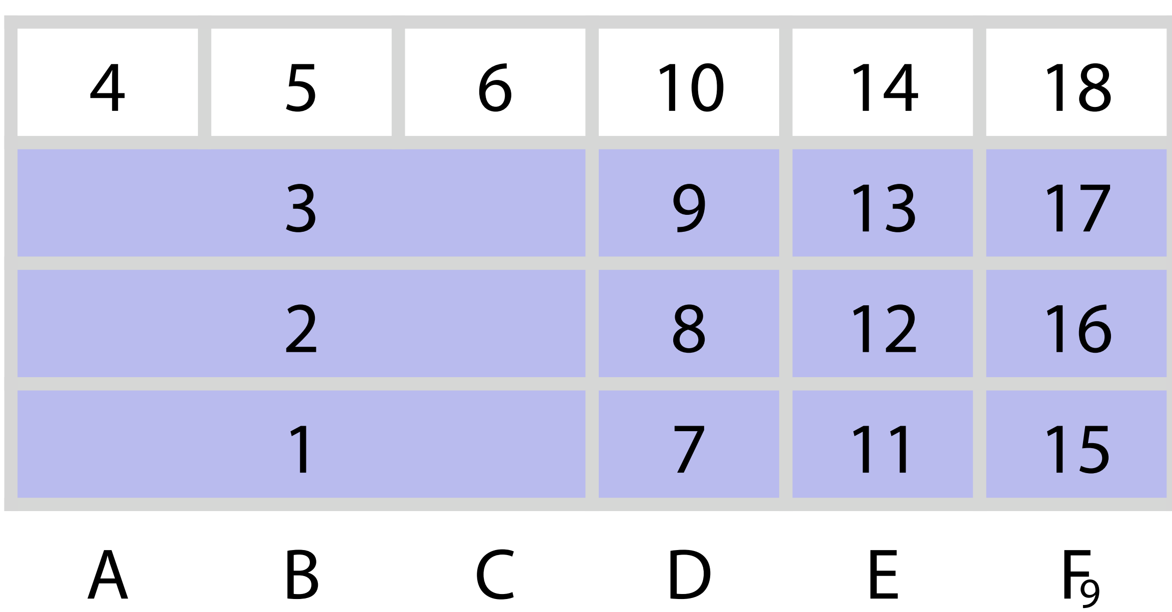
The Core

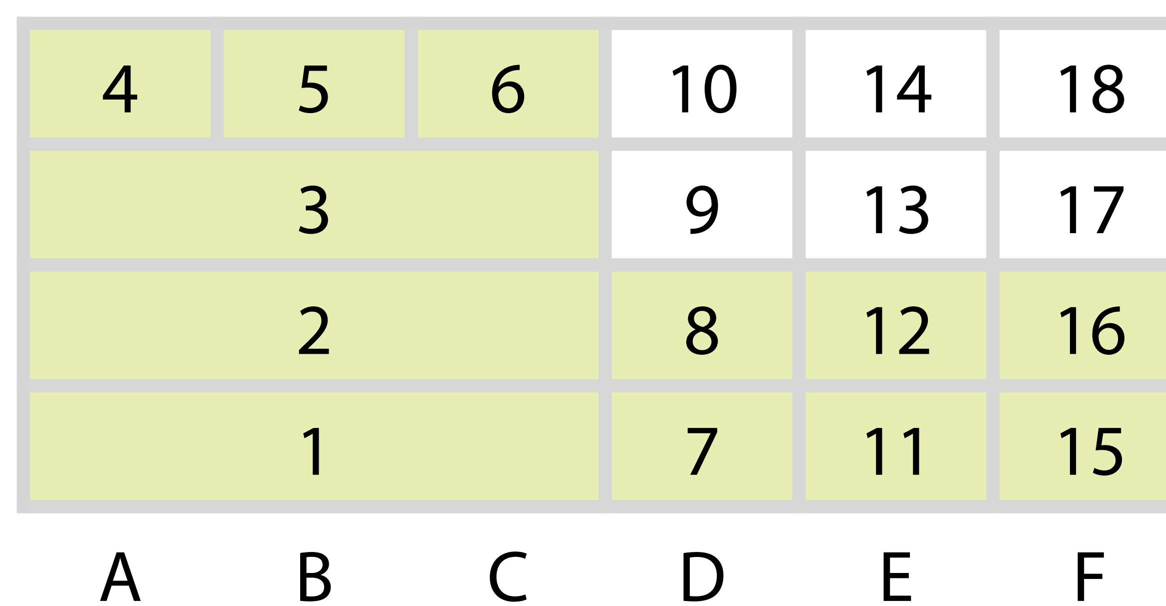
- Can we give a stronger representation guarantee?
- Consider a group *S* of voters with $|S| \geq \ell\ n/k$.
  -> They are large enough to decide ℓ seats.
- Suppose there is a set *T* of candidates with $|T| \leqslant \ell$ but we do not require that everyone in *S* approves everyone in *T*. (~~cohesive~~)
- For a committee *W* to satisfy the core, it cannot be that every member of *S* prefers *T* to *W*.

<table>
<tr><td>4</td><td>5</td><td>6</td><td>10</td><td>14</td><td>18</td></tr>
<tr><td colspan="3">3</td><td>9</td><td>13</td><td>17</td></tr>
<tr><td colspan="3">2</td><td>8</td><td>12</td><td>16</td></tr>
<tr><td colspan="3">1</td><td>7</td><td>11</td><td>15</td></tr>
<tr><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>F</td></tr>
</table>

The Core

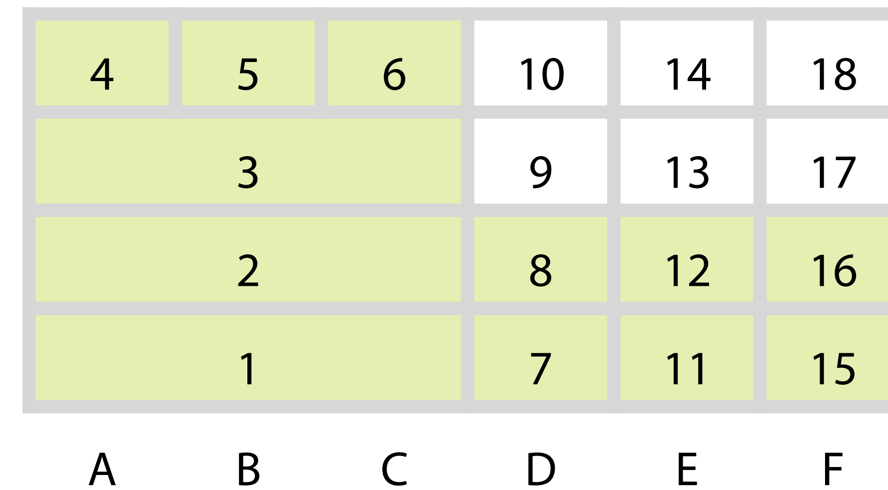
- Can we give a stronger representation guarantee?
- Consider a group S of voters with $|S| \geq \ell\, n/k$.
  -> They are large enough to decide ℓ seats.
- Suppose there is a set T of candidates with $|T| \leqslant \ell$ but we do not require that everyone in S approves everyone in T. (~~cohesive~~)
- For a committee W to satisfy the core, it cannot be that every member of S prefers T to W.



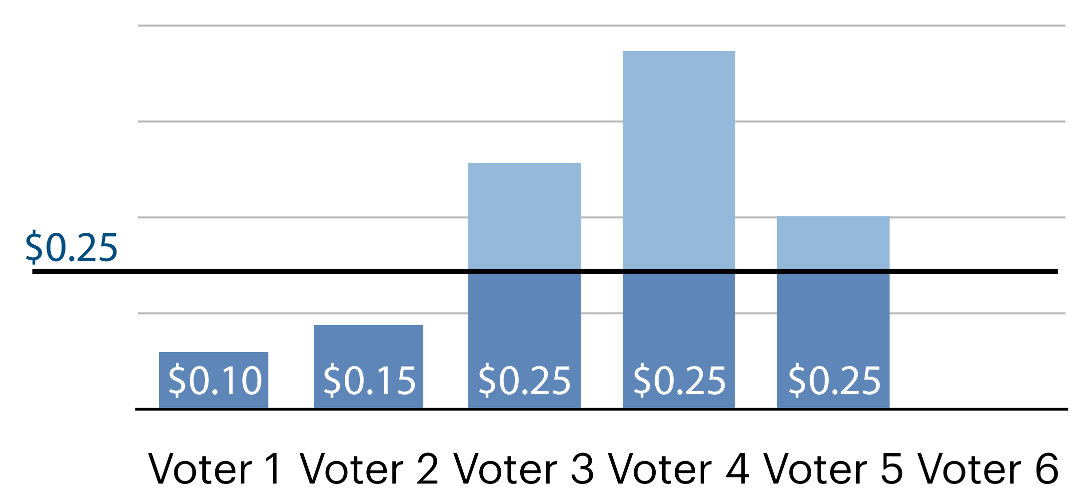


Proportional Rules

- Phragmén proposed a rule selecting the green committee in 1894.



- Sequential, poly time, but fails EJR.
- We proposed a new rule that selects the green committee and is EJR, sequential, poly time: the Method of Equal Shares.
  Peters, Dominik, and Piotr Skowron. "Proportionality and the limits of welfarism." *Proceedings of the 21st ACM Conference on Economics and Computation*. 2020.
- Split \$k equally among the voters. It costs \$1 to elect a candidate. We repeatedly choose a candidate whose approvers have at least \$1 left. We spread the \$1 as evenly as possible, and if several candidates are available we choose the one with the most even spread.



- (Still fails core.)

Participatory Budgeting

- In Participatory Budgeting, the city government allows residents to vote over how the budget is spent.

— Ballot Paper —

Total available budget: € 3 000 000.

Approve up to 4 projects.

☒ Extension of the Public Library
Cost: € 200 000

☐ Photovoltaic Panels on City Buildings
Cost: € 150 000

☒ Bicycle Racks on Main Street
Cost: € 20 000

☐ Sports Equipment in the Park
Cost: € 15 000

☐ Renovate Fountain in Market Square
Cost: € 65 000

☐ Additional Public Toilets
Cost: € 340 000

☐ Digital White Boards in Classrooms
Cost: € 250 000

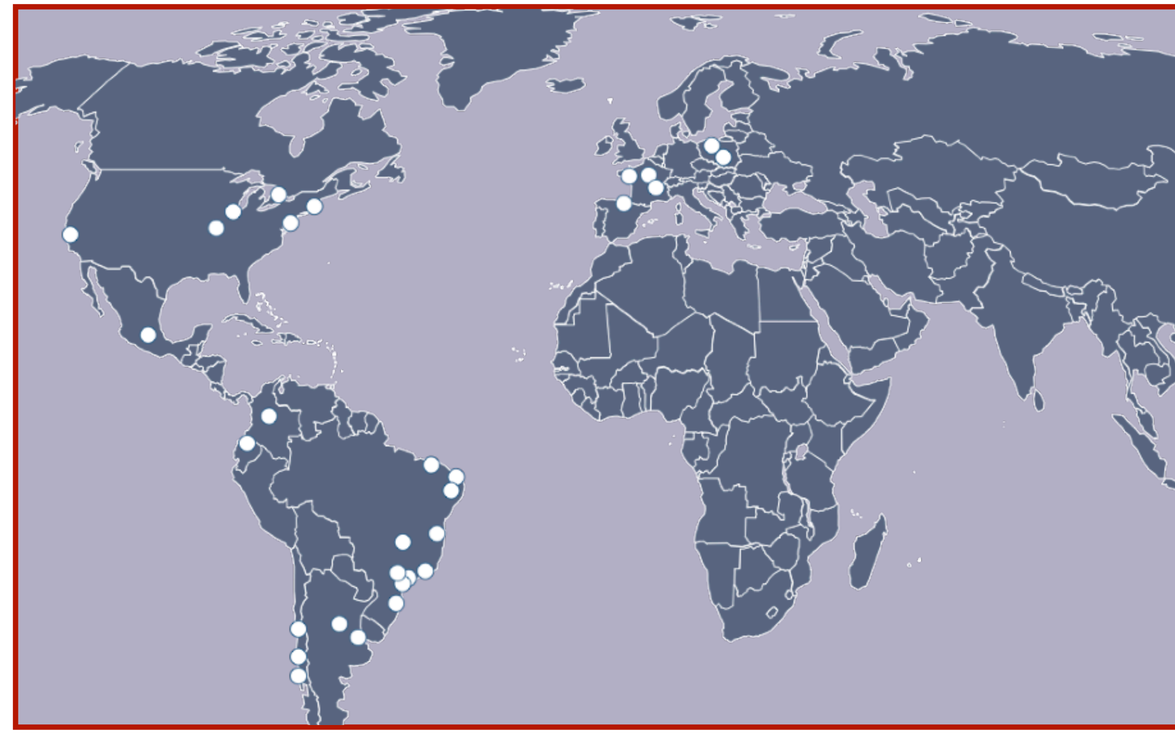
☐ Improve Accessibility of Town Hall
Cost: € 600 000

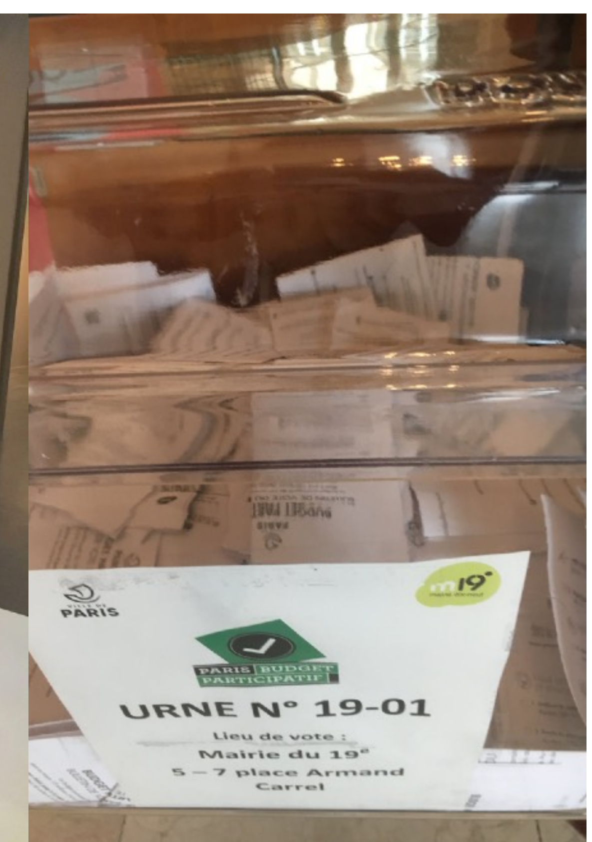
☒ Beautiful Night Lighting of Town Hall
Cost: € 40 000

☐ Resurface Broad Street
Cost: € 205 000

Participatory Budgeting

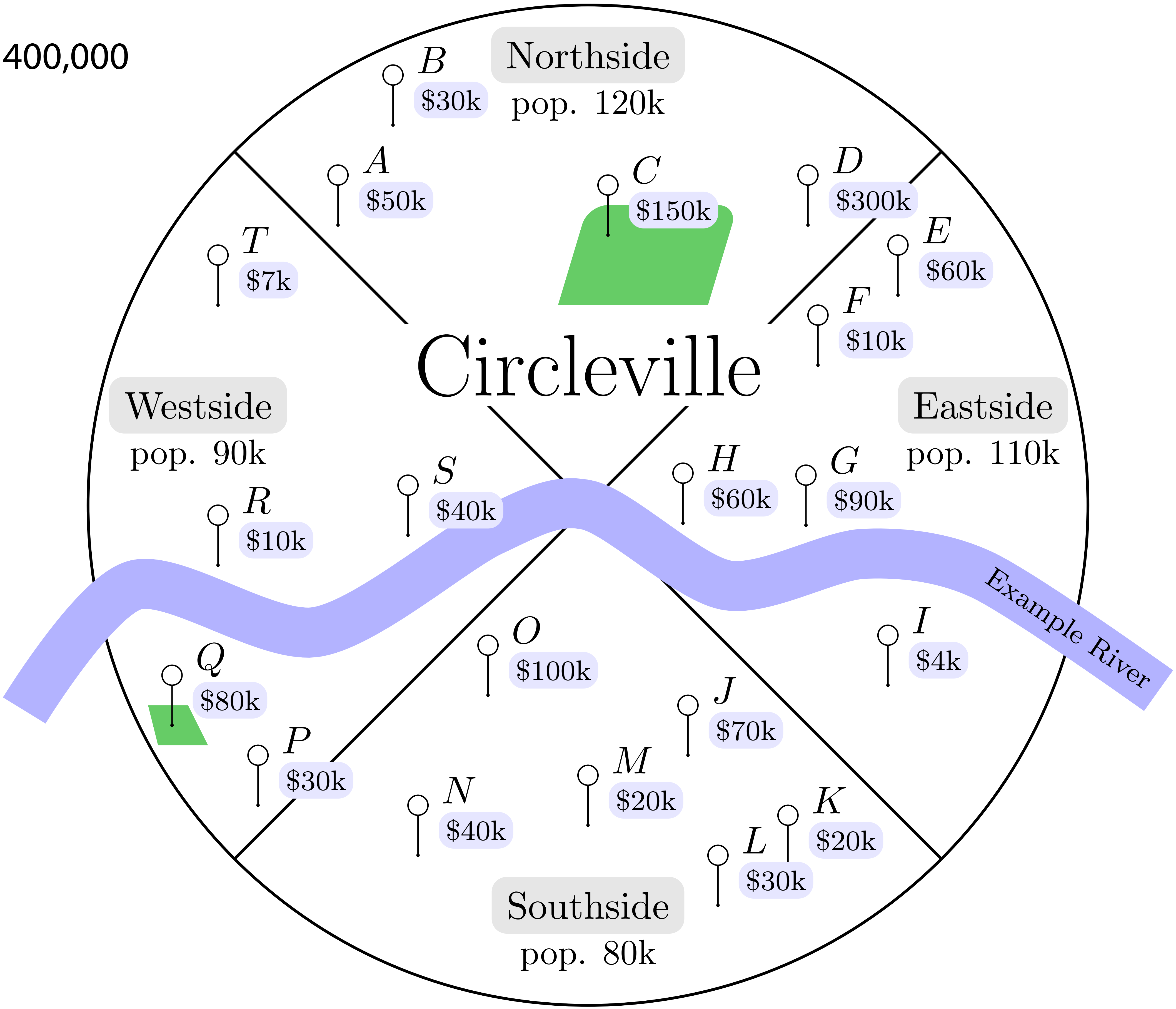
- Participatory Budgeting now happens in 100s of cities
- During 2016-2021, largest in Paris (€ 100 million per year)
- Better theory could improve practice around the world
- Same or similar models for other important applications:
  - ➡ Research grant funding
  - ➡ Scheduling

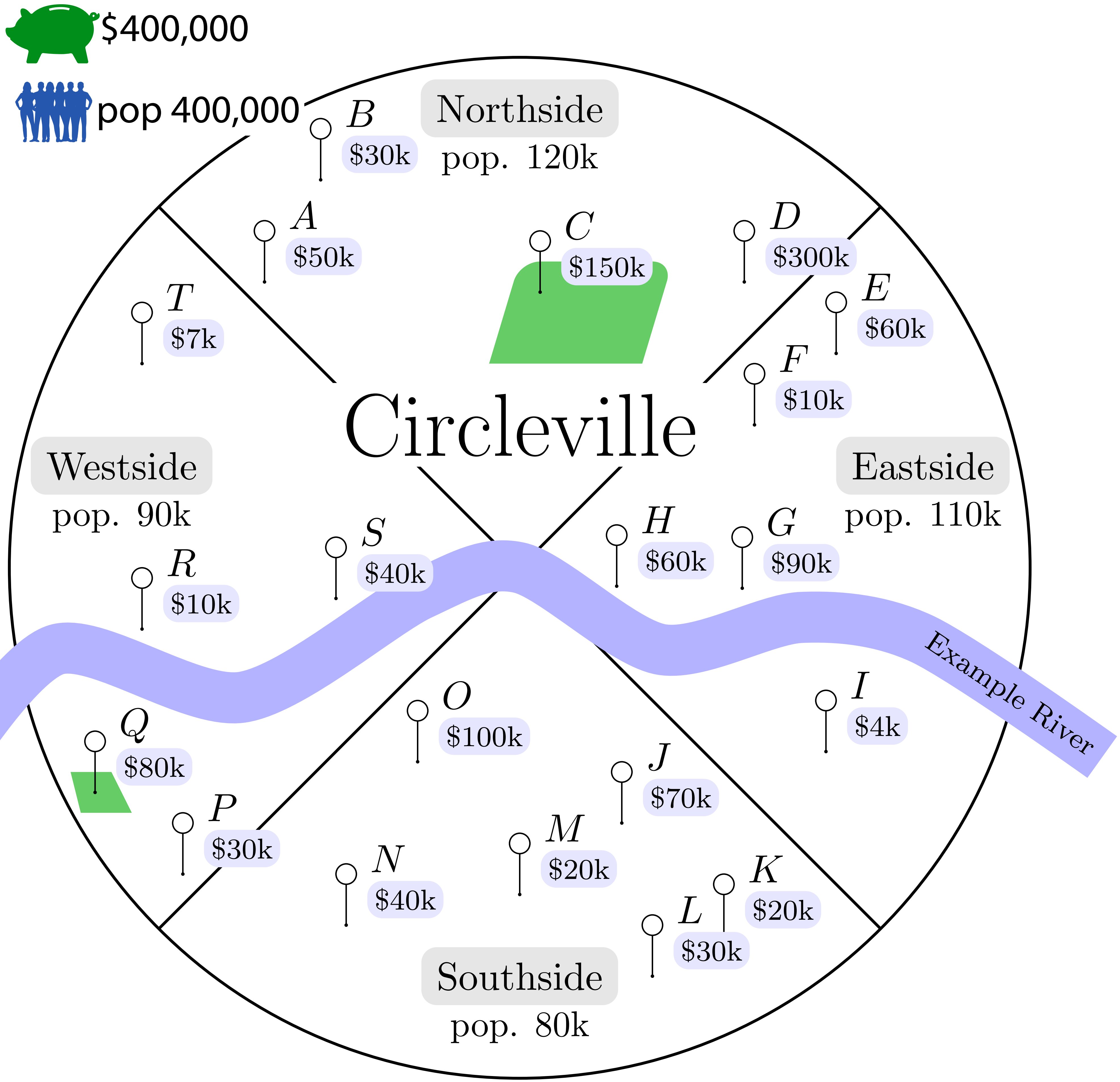


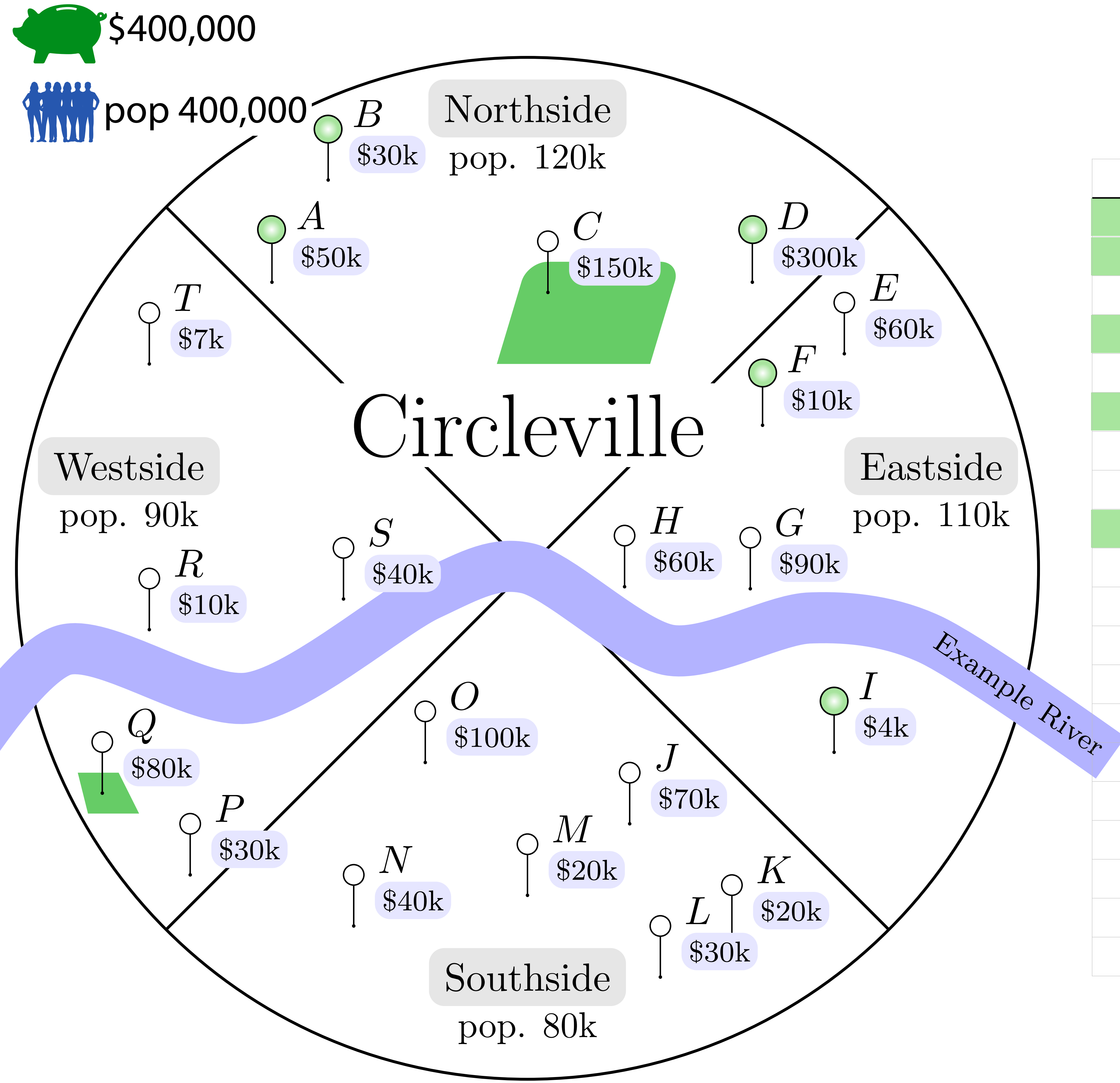


$400,000

pop 400,000

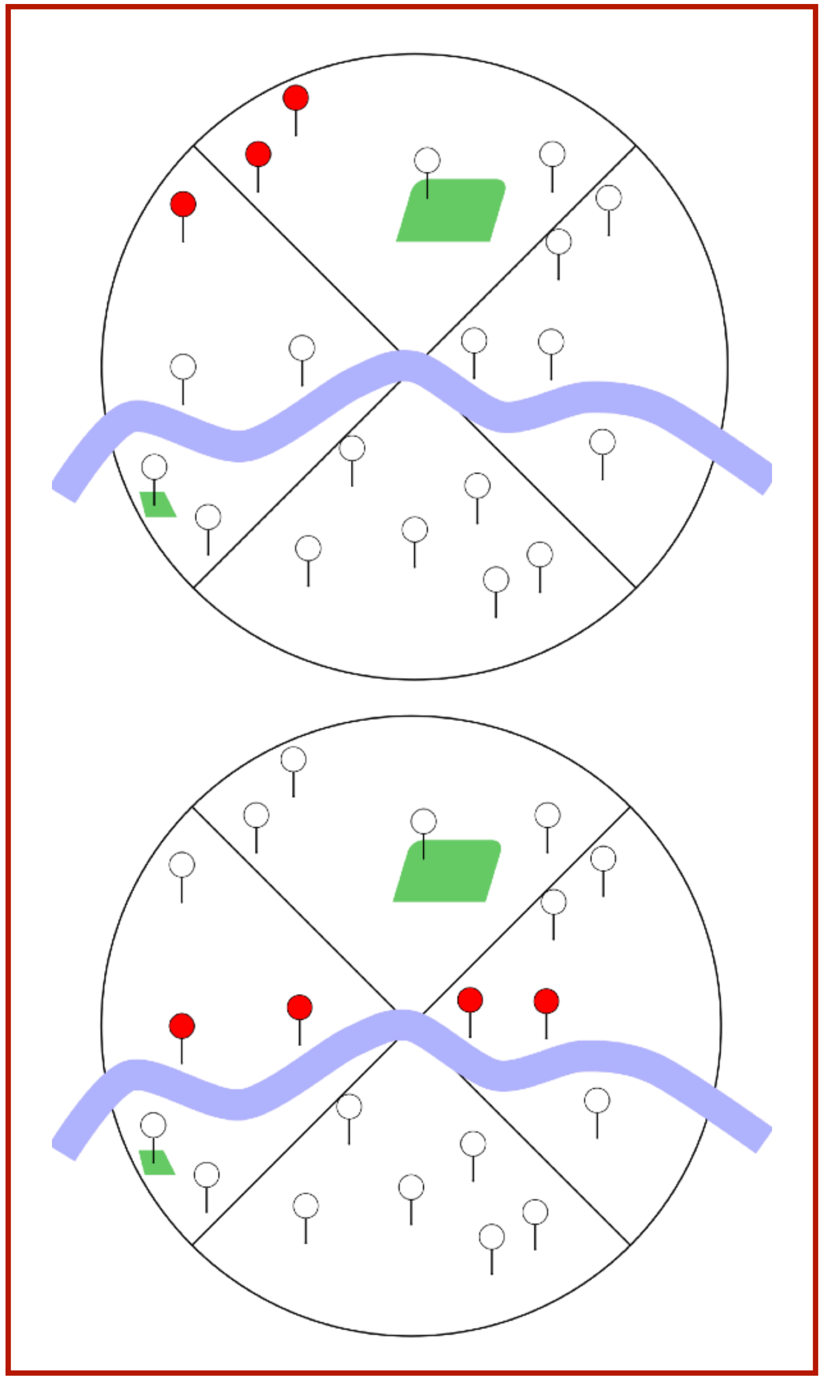



$400,000
pop 400,000
Northside
pop. 120k
Westside
pop. 90k
Eastside
pop. 110k
Southside
pop. 80k
Circleville
Example River
B
$30k
A
$50k
C
$150k
D
$300k
E
$60k
F
$10k
T
$7k
S
$40k
R
$10k
H
$60k
G
$90k
I
$4k
O
$100k
Q
$80k
P
$30k
J
$70k
M
$20k
N
$40k
K
$20k
L
$30k



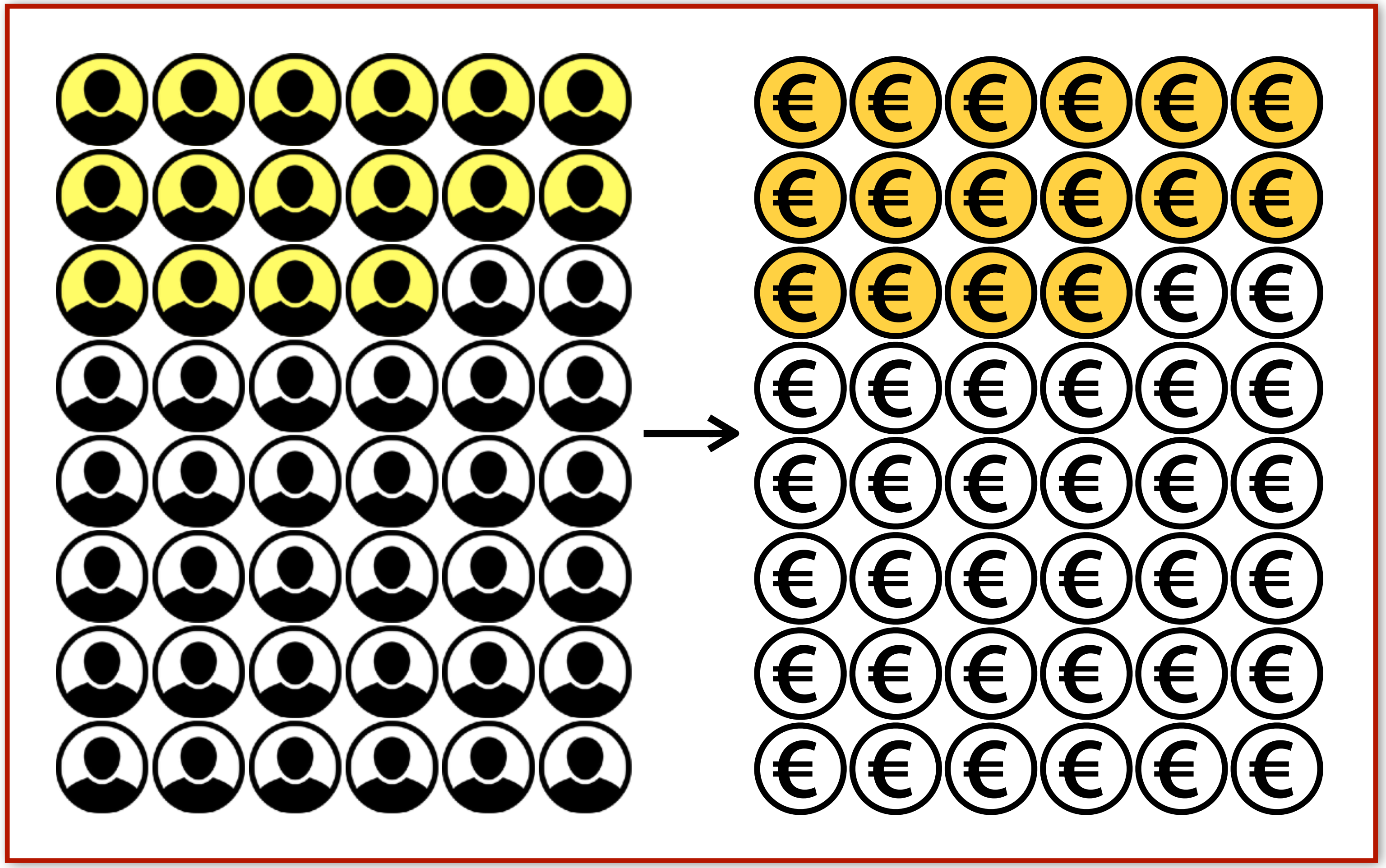
	Votes	Cost	
A	120k	$50k	✓
B	120k	$30k	✓
C	120k	$150k	
D	120k	$300k	✓
E	110k	$60k	
F	110k	$10k	✓
G	110k	$90k	
H	110k	$60k	
I	110k	$4k	✓
J	80k	$70k	
K	80k	$20k	
L	80k	$30k	
M	80k	$20k	
N	80k	$40k	
O	80k	$100k	
P	90k	$30k	
Q	90k	$80k	
R	90k	$10k	
S	90k	$40k	
T	90k	$7k	

Proportionality



Proportional Representation requires that a group of 30% of voters with similar interests should be represented by spending of about 30% of the budget.

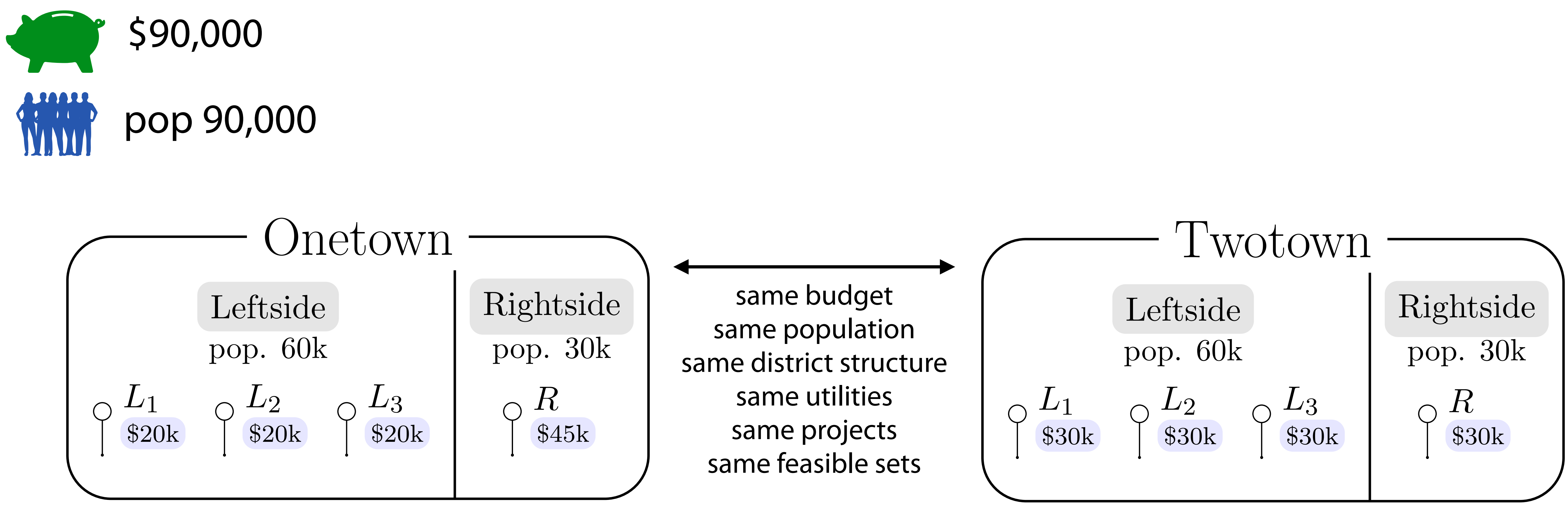
A voter could be part of several interest groups!



Research Question:
Can we design a rule that *on its own* finds all interest groups and represents all of them proportionally?

Reduction to Committee Voting

- Suppose our budget is $k, and each project costs $1 (the "*unit cost assumption*").
- Then PB voting turns into a committee election!
- So maybe we can generalize known rules to work for the knapsack constraint.
- For example, maximize the PAV objective over all feasible knapsacks. —> Fails badly.



not proportional!
Rightside deserves $30k

Maximal Budget-Feasible Knapsacks:

$\{L_1, L_2, L_3\} \longrightarrow$ PAV-score 110,000

$\{L_1, L_2, R\} \longrightarrow$ PAV-score 120,000

not proportional!
Leftside deserves $60k

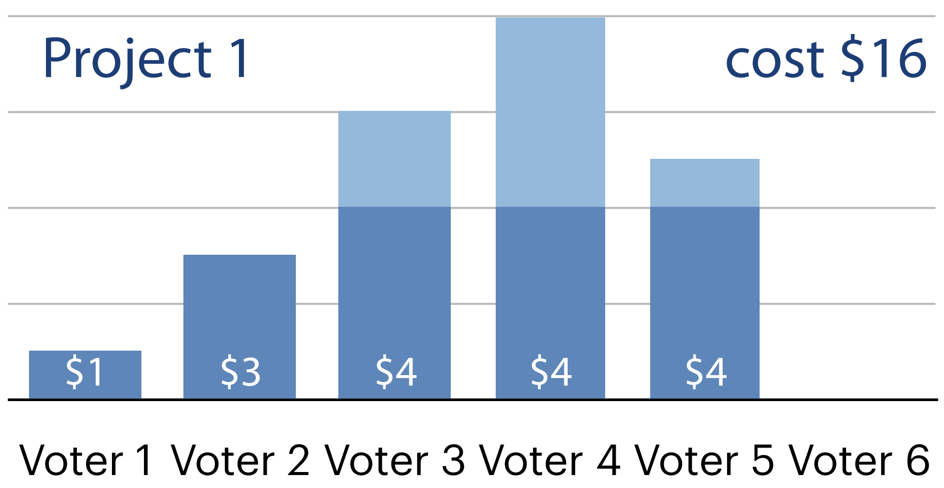
$\{L_1, L_2, L_3\} \longrightarrow$ PAV-score 110,000

$\{L_1, L_2, R\} \longrightarrow$ PAV-score 120,000

Theorem. Every voting rule that only depends on voters' utility functions and the collection of budget-feasible sets must fail proportionality, even on instances with a district structure.

Method of Equal Shares

- Split the city budget evenly among residents.
- Put each resident's share in a virtual bank account.
- Repeatedly, until the budget runs out:
    - identify a project whose supporters have enough money left to afford it
    - divide the cost among supporters *as evenly as possible*, and charge them



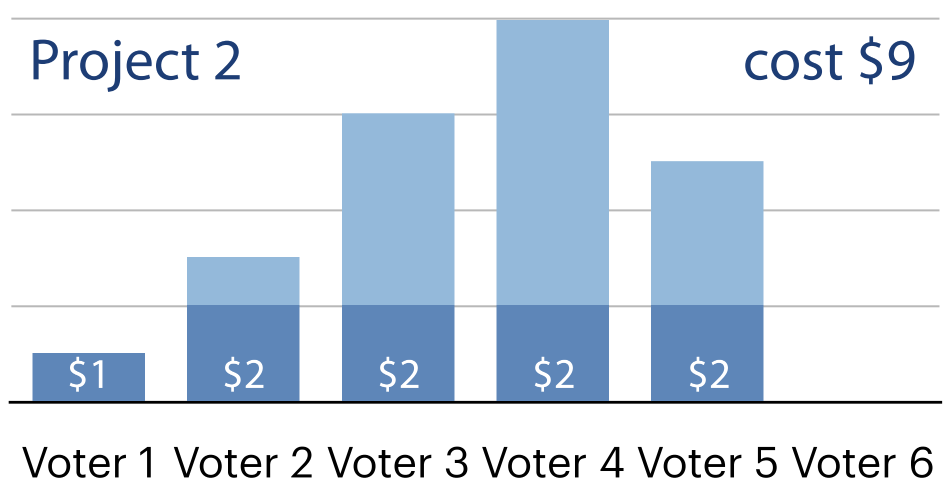
Q: How to choose between affordable projects?

A: Take the one where max payment is smallest.

=> cheaper better

=> wealthier supporters better

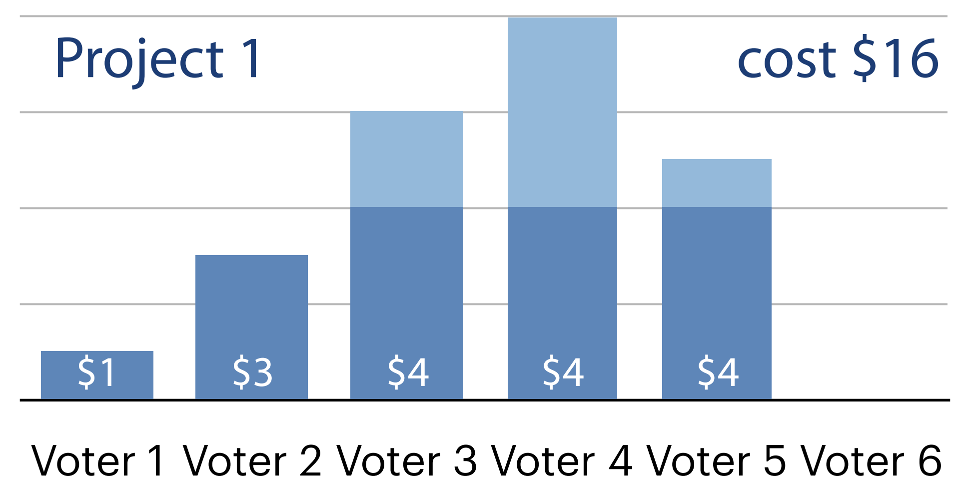
=> more supporters better

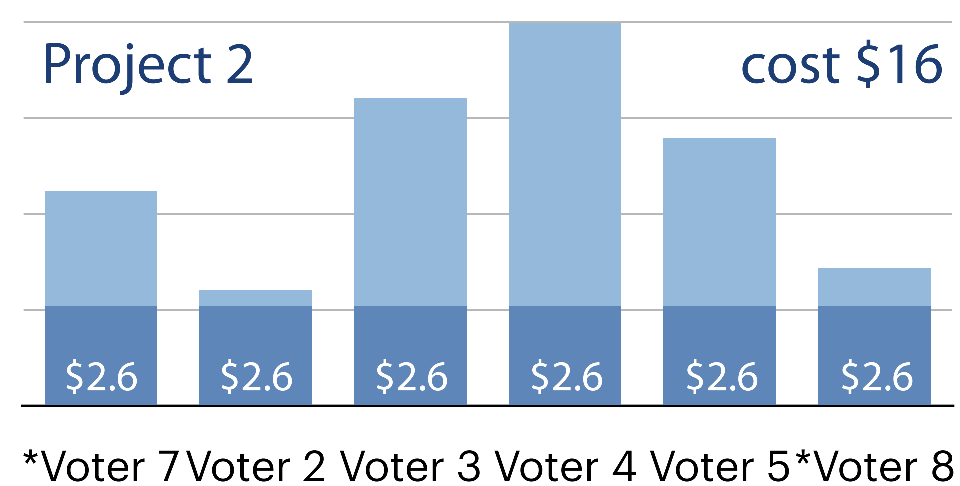


Peters, Dominik, Grzegorz Pierczyński, and Piotr Skowron. "Proportional participatory budgeting with additive utilities." Advances in Neural Information Processing Systems 34 (2021): 12726-12737.

Method of Equal Shares

- Split the city budget evenly among residents.
- Put each resident's share in a virtual bank account.
- Repeatedly, until the budget runs out:
  - identify a project whose supporters have enough money left to afford it
  - divide the cost among supporters *as evenly as possible*, and charge them





Q: How to choose between affordable projects?

A: Take the one where max payment is smallest.

=> cheaper better

=> wealthier supporters better

=> more supporters better

Peters, Dominik, Grzegorz Pierczyński, and Piotr Skowron. "Proportional participatory budgeting with additive utilities." Advances in Neural Information Processing Systems 34 (2021): 12726-12737.

EJR for Budgeting

- Consider a group S of voters with $|S| \geq \alpha B/n$.
- Suppose there is a set $|T| = \ell$ of candidates with cost(T) $\leqslant \alpha$B such that every voter in S approves all of T ("cohesive group").
- For a committee W to satisfy EJR, it cannot be that every member of S prefers T to W
  -> thus at least one voter in S must approve at least ℓ members of W.
- Theorem: The Method of Equal Shares satisfies EJR.

Extending to Additive Utilities

- Equal Shares: can extend using following idea: a voter's payment for a candidate should be proportional to the voter's utility for the candidate.
  - this rule satisfies EJR "up to one project".
  - Consider case $n = 1$. Then EJR means we must solve the knapsack problem. So no strongly polynomial time rule can satisfy EJR.
- We also have a rule satisfying stronger guarantees — but extremely hard to compute!

Example:

2019, Paris, 16th arrondissement

€560k: refurbish sports facility — 775 votes

€3k: materials for classroom project — 670 votes

— *1.15x as popular, 186x the cost!*

We can still use approval voting, but instead of using 0/1 utilities, we can use "0/cost" utilities:

- Approved: utility = cost of project
- Not approved: utility = 0

Discussion

- Idea of proportionality and fairness can be applied to all kinds of decision making situations (scheduling, design, recommendations, rankings)
- Can we implement sophisticated voting rules in public applications? What about computational complexity?
- Fairness over time: participatory budgeting happens every year.

Some Directions

Mutually Exclusive Projects

- Assume there is an empty plot of land, and several ideas what to build there. We can only choose one.
- Easy to adapt Equal Shares. Easy to adapt EJR. But Equal Shares doesn't satisfy EJR.
- Can EJR be satisfied?
- Related to "Public Decision Making", committee elections with variable number of winners.
- Existing very recent work on this question: very special cases, or without proportionality.

Divisible Projects

- Some projects can take an arbitrary amount of funding (e.g. how much should we spend fixing potholes?)
- Can easily incorporate by introducing lots of projects, \$1 each.
- This fixes exhaustiveness issue: entire budget will be used.
- But: we know PAV is best for “copyable” projects (see party-approval).
- Equal Shares behaves weirdly: two parties *A* and *B*. 20% of voters approve both. Of the rest, 60% approve *A* and 40% approve *B*. Then Equal Shares will give 68% to *A* and 32% to *B*.

Projects with Milestones

- Suppose a project comes in three possible sizes:
  \$100k, \$150k, or \$170k.

- Voters can indicate up to which size they approve the project.

- Similarly: divisible project where voters indicate the maximum amount of spending they approve.