Strategyproof Mechanisms for Multiple Facility Location Games

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Based on joint work with **Panagiotis Patsilinakos** (NTUA)
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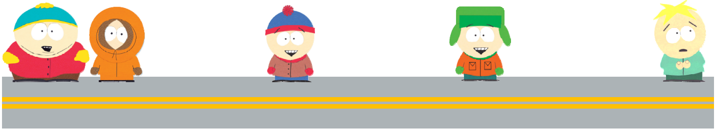
k-Facility Location Games

Public Good Allocation for Strategic Agents with Linear Preferences

- Agents $N = \{1, \ldots, n\}$ on the real **line** .
- Agent i **wants** a facility close to x_i , which is **private information** .



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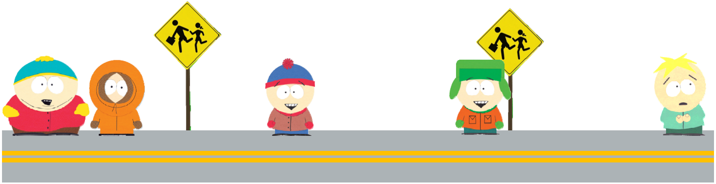
(Randomized) Mechanism

Mechanism F maps reported ideal locations $\boldsymbol{y} = (y_1, \dots, y_n)$ to (probability distribution over) set(s) of k **facilities** .



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- Agent i **wants** a facility close to x_i , which is **private information** .
- Each agent i **reports** y_i that may be **different** from x_i.

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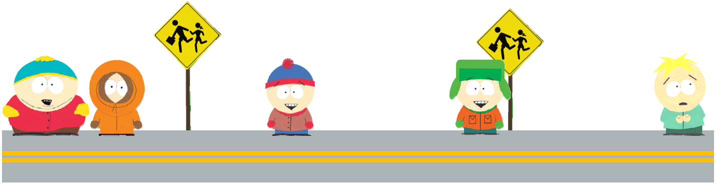
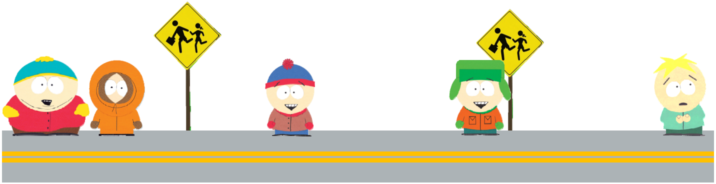


Preferences and Truthfulness

Connection Cost

(Expected) distance of agent i's **true location** to the **nearest** facility:

$$\text{cost}[x_i, F(\mathbf{y})] = \text{dist}(x_i, F(\mathbf{y})) = \min_{c \in F(\mathbf{y})} |x_i - c|$$



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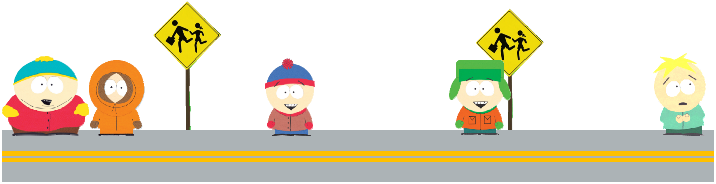
Truthfulness

For any location profile $\boldsymbol{x}$, agent i, and location y:

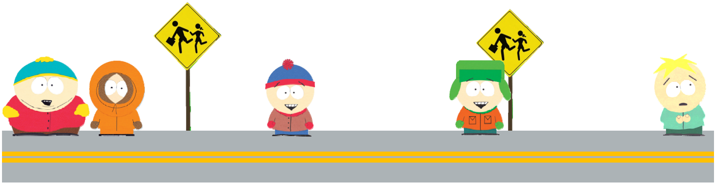
$$\text{cost}[x_i, F(\boldsymbol{x})] \leq \text{cost}[x_i, F(y, \boldsymbol{x}_{-i})]$$



Variants and Social Efficiency

Candidate Facility Locations:

- **Unrestricted** : Any point (esp. agent locations) can be facility.
- **Restricted** : Facilities selected from m candidate locations $\mathcal{C}$
  Motivation from multi-winner elections: Chamberlin-Courant.



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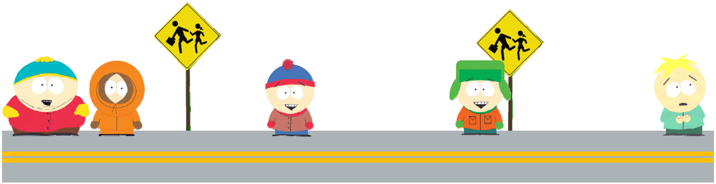
Social Objective

$F(\boldsymbol{x})$ should optimize (or approximate) a given **objective function** .

- **Social Cost** : minimize $\sum_{i=1}^{n} \text{cost}[x_i, F(\boldsymbol{x})]$
- **Social Welfare** : maximize $\sum_{i=1}^{n} \left(L - \text{cost}[x_i, F(\boldsymbol{x})]\right)$
- **Maximum Cost** : minimize $\max\{\text{cost}[x_i, F(\boldsymbol{x})]\}$



1-Facility Location on the Line

Median Mechanism

- **Median** of $(x_1, \ldots, x_n)$: **truthful** and **optimal** , when unrestricted.

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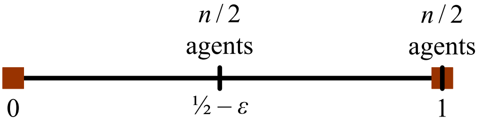
  - OPT social cost $\approx n/4$. OPT social welfare $\approx 3n/4$.
  - Median social cost $\approx 3n/4$. Median social welfare $\approx n/4$.



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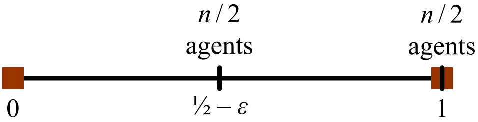
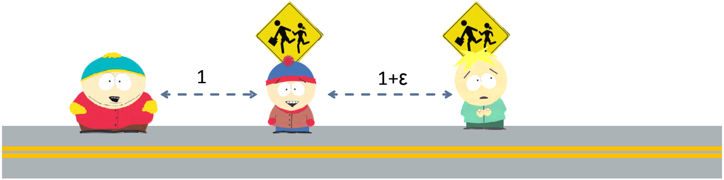
- Anonymity and truthfulness iff **generalized** median [Moulin 80]



2-Facility Location on the Line

Optimal Sensitive to Deviations!

The optimal solution for social cost (and welfare) is **not truthful** !



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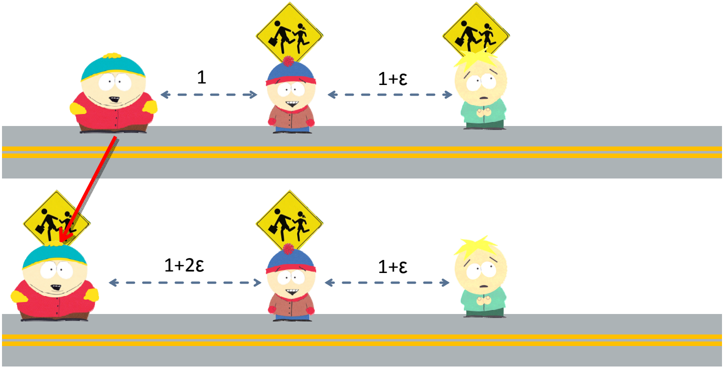

A Tale about Truthfulness in k-Facility Location

Three (+ One) Roads to Truthfulness (with Reasonable Efficiency)

- **Order Statistics**: (generalized) median, two-extremes, percentile mechanisms.
- **Align Incentives** with Optimal (for maximum cost): (randomized) equal-cost mechanism.
- **Restriction to Stable** instances: optimal, almost rightmost, random.

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- **Restriction to Stable** instances: optimal, almost rightmost, random.
- **Winner Imposing** verification: if declared location gets facility, agent must be served by that [F. Tzamos, WINE 10]

k-Facility Location – Social Welfare

Percentile Mechanisms [Sui Boutilier Sandholm, IJCAI 13]

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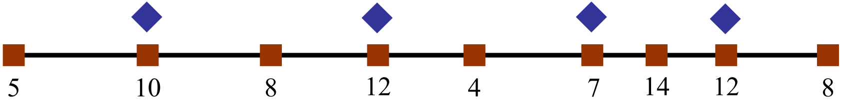
$(\alpha_1, \ldots, \alpha_k)$-**percentile** mechanism $(0 \leq \alpha_1 < \alpha_2 < \cdots < \alpha_k \leq 1)$:

- vote(ℓ) $=$#agents preferring $\ell \in \mathcal{C}$ to other candidates in $\mathcal{C}$.
- j-th facility at leftmost $\ell \in \mathcal{C}$ with $\geq \alpha_j$ **fraction** of vote on ℓ **and its left** .
  - Median is 0.5-percentile. Two-Extremes is $(0, 1)$-percentile.



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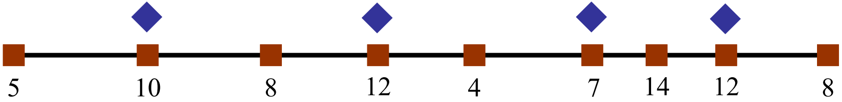
  - Median is 0.5-percentile. Two-Extremes is $(0, 1)$-percentile.
- Percentile mechanisms are **anonymous** and **truthful** (only one?).
- For any $k \geq 2$, $(1/(2k), 3/(2k), \ldots, (2k-1)/(2k))$-percentile mechanism is $(1 + O(1/k))$**-approximate** for social **welfare** [F. Gourvés Monnot, WINE 16].



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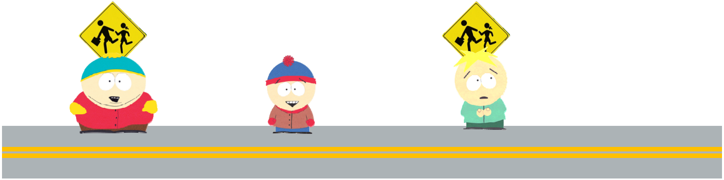
k-Facility Location – Social Cost

Truthful Location of 2 Facilities

Two-Extremes is **truthful** and $(n-2)$**-approximate** (best possible).
[Procaccia Tenneholtz, EC 09], [F. Tzamos, ICALP 13]



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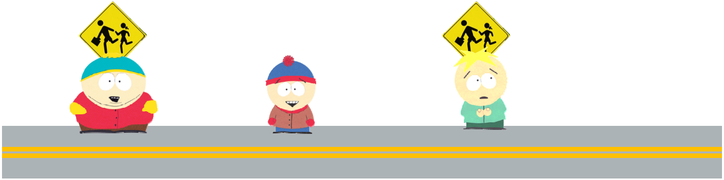
Truthful Location of $k \geq 3$ Facilities

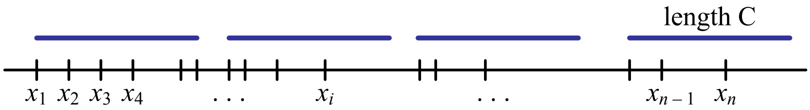
- **Deterministic** anonymous mechanisms have **unbounded** (in terms of n and k) approximation ratio [F. Tzamos, ICALP 13]
- Best known **randomized** mechanism is n**-approximate** for social cost [F. Tzamos, EC 13]



Randomized Mechanisms [F. Tzamos, EC 13]

Equal-Cost Mechanism for k-Facility Location

- **Optimal maximum** cost of declared instance $= C/2$.
- **Cover** all agents with k **disjoint intervals** of length C.



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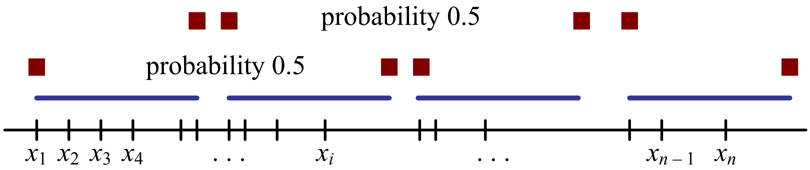

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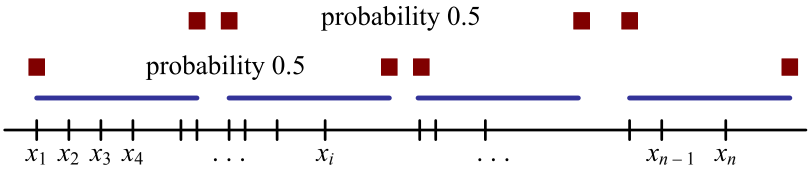
Agents' Cost and Approximation Ratio

- Agent i has expected **cost** $= (C - x_i)/2 + x_i/2 = C/2$.



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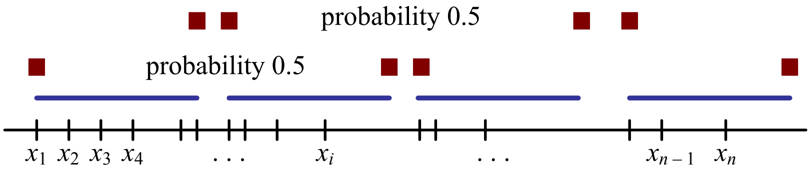
Agents' Cost and Approximation Ratio

- Agent i has expected **cost** $= (C - x_i)/2 + x_i/2 = C/2$.
- Approx. ratio: 2 for the **maximum** cost, n for the **social** cost.



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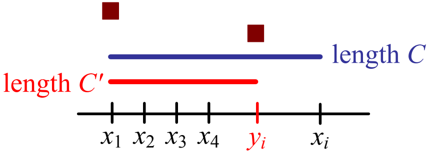

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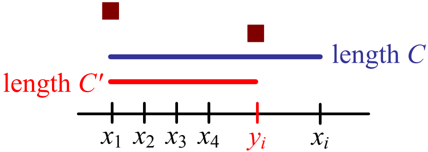
- Agents do not have incentive to **lie** and **increase** optimal maximum cost, i.e. $C/2$.
- Let agent i declare y_i and **decrease** optimal maximum cost to $C'/2 < C/2$.
- Then, i's expected **cost** $= \frac{1}{2}C + \frac{1}{2}(C - C') > C/2$



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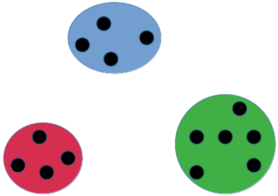
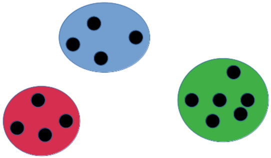
- **Deterministic** anonymous mechanisms have **unbounded** approximation ratio [F. Tzamos, ICALP 13]
- Best known **randomized** mechanism is n**-approximate** [F. Tzamos, EC 13]
- Bounded approximation requires facility in **each optimal** cluster. But optimal clustering is **sensitive** to agent deviations.
- Focus on instances with **stable** optimal clustering.

Perturbation Stability for k-Facility Location

Perturbation Stability in Clustering [Bilu Linial, ITCS 10]

- **γ-stability** : scaling down any distances by factor $\leq \gamma$ (while maintaining metric property) **does not affect optimal** solution.





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- For $\gamma \geq 2$, (metric) k-Facility Location solvable in **poly-time** !
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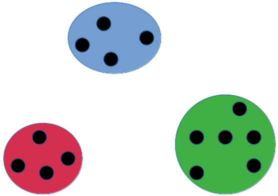
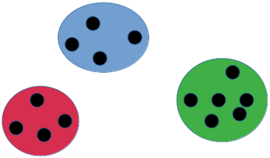
  k-Facility Location remains **hard** for $\gamma \leq 2 - \varepsilon$.
- Real-world instances are supposed to be **stable**: *"Clustering is hard when it doesn't matter"* [Roughgarden 17]





Truthful k-Facility Location in Stable Instances

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Assume that **"true"** instances are indeed **stable** .

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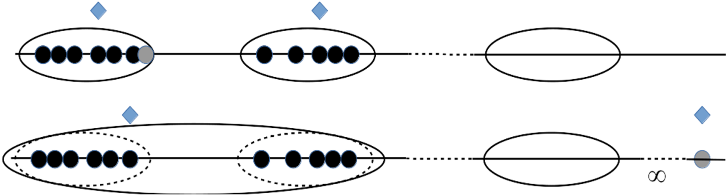

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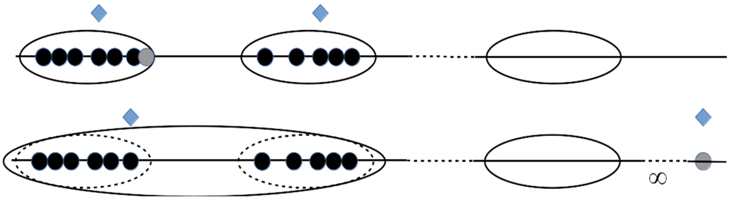
- Optimal solution **not truthful** for any stability $\gamma \geq 1$.
- For $k \geq 3$, deterministic anonymous truthful mechanisms for $(\sqrt{2} - \varepsilon)$**-stable** instances have **unbounded** approximation (based on [F. Tzamos, ICALP 13])



Truthful k-Facility Location in Stable Instances

Remedy and Main Results

- **Optimal** clustering $(C_1, \ldots, C_k)$ due to bounded approximation.
- Stability verification (necessary cond.): allocate facilities only if

$$\max\{\mathrm{diam}(C_i), \mathrm{diam}(C_{i+1})\} < d(C_i, C_{i+1})$$

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- For 5**-stable** instances, facility at **second from the right** in each optimal cluster is **truthful** and $(n-2)/2$**-approximate** .
- For 5**-stable** instances, facility at **random** agent in each optimal cluster is **truthful** and 2**-approximate** .

Optimal Mechanism for Stable k-Facility Location

Optimal Mechanism and Approach to Truthfulness

If optimal clustering $(C_1, \ldots, C_k)$ has **singleton** clusters or $\max\{\text{diam}(C_i), \text{diam}(C_{i+1})\} \geq d(C_i, C_{i+1})$, do **not allocate** facilities!

Otherwise, facilities at $(\text{med}(C_1), \ldots, \text{med}(C_k))$.

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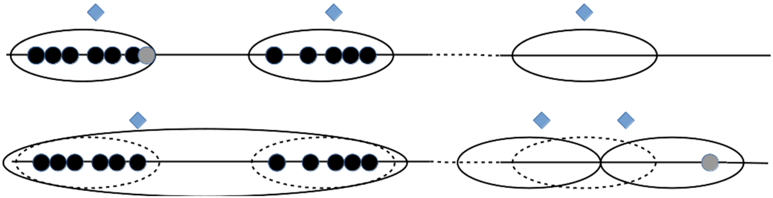
- Key deviation: rightmost agent of C_i deviates to C_j, causing C_j **to split** and C_i **to merge** with C_{i+1} .
- "Simulate" increase in cost of C_j by γ**-perturbation** and decrease in cost of C_i by agent's **cost improvement** .



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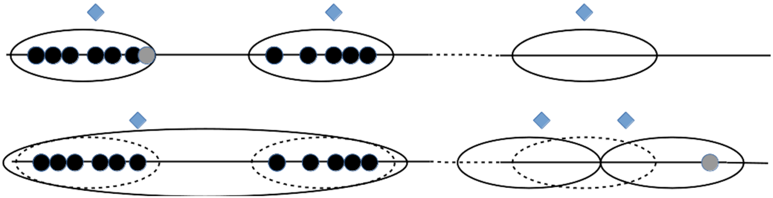
- "Simulate" increase in cost of C_j by γ**-perturbation** and decrease in cost of C_i by agent's **cost improvement** .
- Stability: optimal clustering **not affected** by deviation.



k-Facility Location Resistant to Singleton Deviations

Increase Stability to $\gamma \geq 5$ to Resist Singleton Deviations

If optimal $(C_1, \ldots, C_k)$ has $\max\{\mathrm{diam}(C_i), \mathrm{diam}(C_{i+1})\} \geq d(C_i, C_{i+1})$, do **not allocate** facilities!

Almost Rightmost : Facility at **second to the right** in each optimal C_i.

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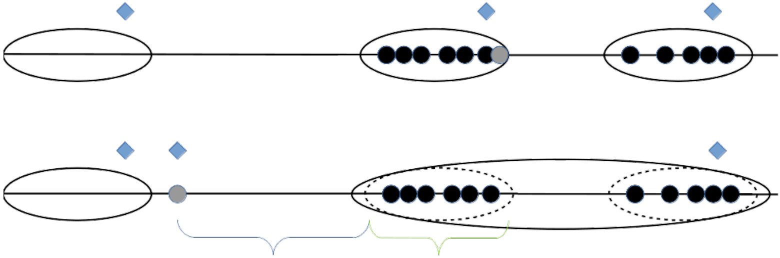

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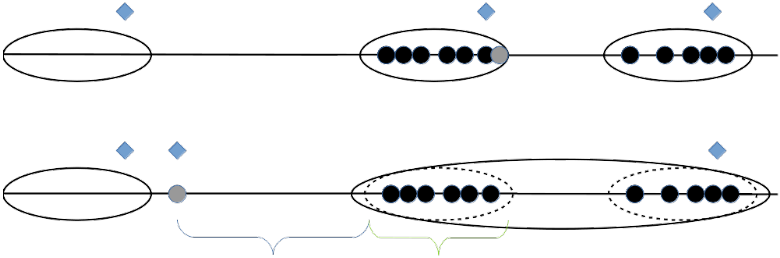

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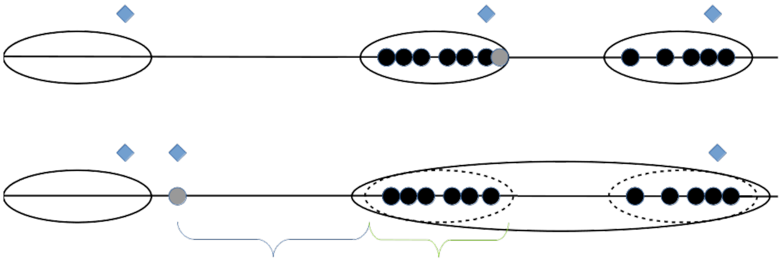
- **5-stable** instances: $x \in C_i$ needs to deviate by $\geq \text{diam}(C_i)$ for **singleton** cluster.
- $x \in C_i$ **cannot** deviate to singleton and **be served** by that facility.



Restriction to Stable Instances Necessary

"Global" Truthfulness and Bounded Approximation Only for Stable

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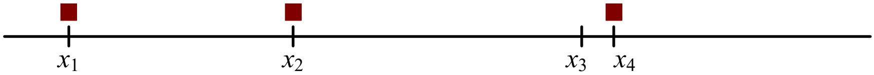
Well-Separated Instances

Instance with $k+1$ agents is **well-separated** if it consists of $k-1$ **isolated** and 2 **nearby** agents.

well-separated instance for $k = 3$



Consistent Allocation for Well-Separated Instances

The Nearby Agents Slide on the Left

- Let $\boldsymbol{x}$ be **well-separated** instance with k-th facility on x_{k+1} .
- For any well-separated instance $\boldsymbol{x}' = (\boldsymbol{x}_{-\{k,k+1\}}, x'_k, x'_{k+1})$ with $x'_{k+1} \leq x_{k+1}$, k-th facility stays with x'_{k+1} .

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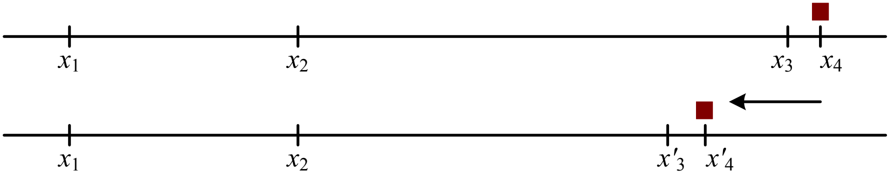

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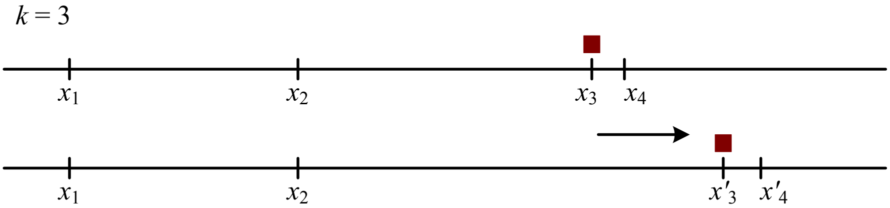

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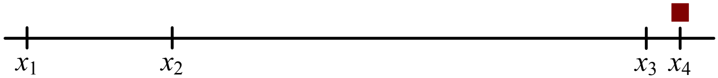
  Set of locations where a **facility** can be **forced by agent** 3 in $\boldsymbol{x}_{-3}$.
- F **truthful** iff all agents get the **best in** their option set .



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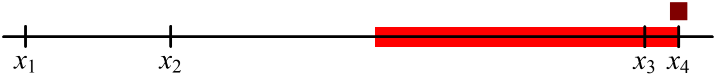

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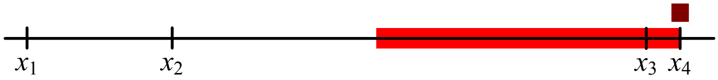

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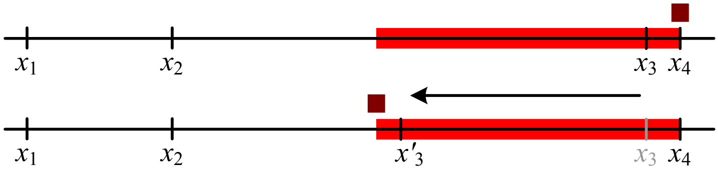

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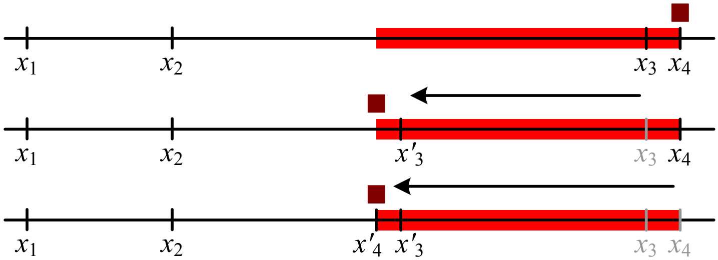

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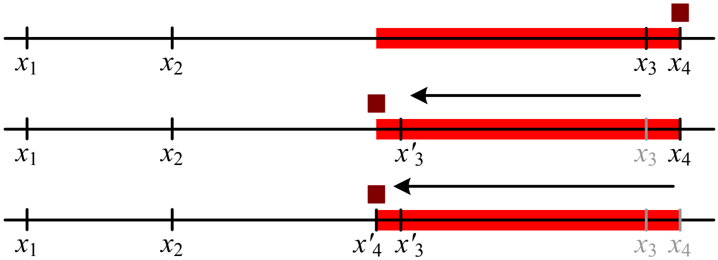

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- Learning-augmented truthful mechanisms for $k \geq 3$ **facilities** . [Xu Lu, 22], [Agrawal Balkanski Gkatzelis Ou Tan, EC 22] for $k \in \{1, 2\}$.

Thank You!