Energy-Infeasibility Tradeoff in Cognitive Radio Networks: Price-Driven Spectrum Access Algorithms

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***Abstract*—We study the feasibility of the total power minimization problem subject to power budget and Signal-to-Interference-plus-Noise Ratio (SINR) constraints in cognitive radio networks. As both the primary and the secondary users are allowed to transmit simultaneously on a shared spectrum, uncontrolled access of secondary users degrades the performance of primary users and can even lead to system infeasibility. To find the largest feasible set of secondary users (i.e., the system capacity) that can be supported in the network, we formulate a vector-cardinality optimization problem. This nonconvex problem is however hard to solve, and we propose a convex relaxation heuristic based on the sum-of-infeasibilities in optimization theory. Our methodology leads to the notion of admission price for spectrum access that can characterize the tradeoff between the total energy consumption and the system capacity. Price-driven algorithms for joint power and admission control are then proposed that quantify the benefits of energy-infeasibility balance. Numerical results are presented to show that our algorithms are theoretically sound and practically implementable.**

***Index Terms*—Optimization, cognitive radio networks, spectrum access control, power and admission control.**

I. Introduction

ENERGY efficiency in wireless communication is a growing focus as energy consumption by wireless devices increasingly becomes a global environmental concern [1]–[3]. In wireless networks, power control is an important medium access control mechanism used to minimize the total energy consumption [4]. The requirement therein is to ensure that the signal is strong enough for the desired receiver to satisfy the Signal-to-Interference-plus-Noise Ratio (SINR) requirements for reliable reception and yet not so strong that it interferes with the other receivers. A crucial issue in this energy minimization problem is the infeasibility problem, that is to say, it may not be possible to simultaneously meet the SINR constraints of all the users. Since the seminal work by Foschini and Miljanic in [5] on power control algorithms for energy minimization subject to SINR constraints, it has been extended to consider power constraints, e.g., the constrained Distributed Power Control (DPC) algorithm in [6], when there is an individual power constraint for each user. In [7], the authors proposed joint power control and channel access algorithms for robustness against outage. In [8], [9], the authors proposed an energy-robustness tradeoff optimization to balance energy expenditure and robustness in wireless cellular networks.

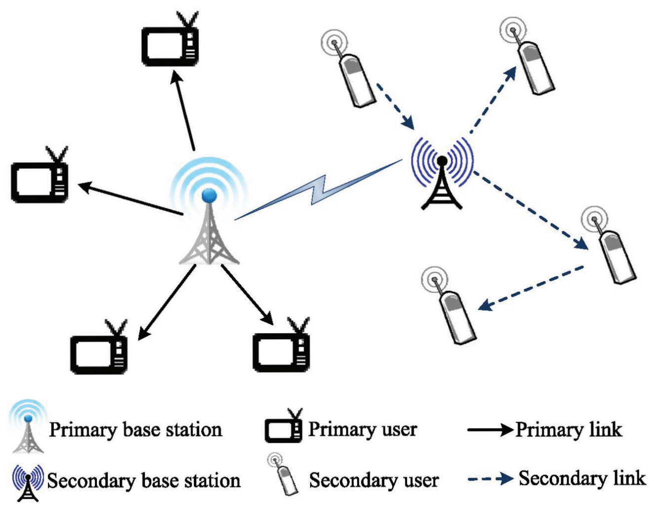

Fig. 1. An illustration of simultaneous transmissions by primary and secondary users in a cognitive radio network, where the secondary users opportunistically exploit the idle frequency bands to transmit. An example of the cognitive radio network is the dynamic access of TV white space spectrum, which consists of unused TV broadcast frequency bands.

In a cognitive radio network, secondary users actively monitor the radio environment for dynamic spectrum access, and can opportunistically transmit simultaneously with the primary users, as illustrated in Fig. 1. When there is infeasibility, existing power control algorithms, e.g., in [5], may not converge or may be unstable in general, e.g., using the algorithm in [6], users may transmit at the maximum possible power and yet still cannot satisfy their SINR constraints leading to undue interference. This infeasibility problem is more severe in a cognitive radio network due to the unplanned deployment of the (unlicensed band) secondary users whose interference can overwhelm the (licensed band) primary users [10]–[14]. Thus, joint power and admission control is necessary to resolve the infeasibility issue in the energy minimization problem [15].

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To simultaneously maximize the number of secondary users that can be supported together with the primary users and to minimize the total energy consumption is generally hard to solve. Mathematically, it is equivalent to computing the maximum feasible set given an infeasible set of linear constraints [16]. In practice, admission control is needed to find the maximum feasible set of users. In [17]–[19], the authors studied the optimal power and admission control for fading channels under stochastic uncertainty. In [20], Mahdavi-Doost et al. proposed an algorithm that removes users based on maximizing the minimum achievable SINR. In [21], Rasti et al. proposed a distributed algorithm to remove secondary users once their instantaneous power exceed certain threshold. The authors in [22]–[24] proposed linear programming relaxation to obtain approximate solution to the system capacity. The authors in [25] proposed a robust distributed uplink power allocation algorithm in a cognitive radio network to maximize the social utility of secondary users that are admitted. The authors in [26] proposed a power control algorithm to maximize the throughput of the secondary users while protecting the primary users. Phunchongharn et al. proposed power control algorithms for transmission under channel uncertainty in [27].

The system capacity is in fact intriguingly related to the amount of energy consumption in the network. Aggressive admission control unduly removes secondary users that leads to the network being under-utilized albeit with a lower total energy consumption. On the other hand, a maximum system efficiency perspective requires supporting as many secondary users as possible albeit with a higher total energy consumption. This *energy-infeasibility tradeoff* determines a desired system operating point that balances the system capacity and the energy consumption. In contrast to the commonly used two-timescale approach (finding a maximum feasible set of secondary users first before minimizing the total energy consumption of all the users in the set) in the literature, we propose a *single timescale approach* to jointly optimize this energy-infeasibility tradeoff using joint power control and admission control algorithms with low implementation complexity. In particular, we first propose an algorithm based on the sum-of-infeasibilities convex relaxation heuristic in optimization theory [28]. Using optimization duality, we refine our algorithm based on the tradeoff analysis between the total energy consumption and the system capacity to compute a (suboptimal) feasible set of users that can be supported subject to a specification constraint on the marginal increase in the total energy consumption. These algorithms are (admission) price-driven in the sense that admission prices are iteratively determined to admit secondary users in the network.

Overall, the contributions in this paper are:

1) the formulation and algorithm design methodology of the energy-infeasibility optimization problem,
2) the joint power and admission control algorithms that are driven by admission prices (can be interpreted as the price that a secondary user pays to be admitted),
3) to quantify the benefits of energy-infeasibility balance in the tradeoff between the number of admitted secondary users and the energy expenditure.

The paper is organized as follows: We introduce the system model in Section II. We formulate the energy-infeasibility optimization as a vector-cardinality optimization problem in Section III. We propose a price-driven algorithm based on the sum-of-infeasibilities heuristic to find the system capacity in Section IV. We study the tradeoff between the total energy consumption and the system capacity driven by the admission prices in Section V. We evaluate the performance of our algorithms numerically and compare them to other baseline algorithms in Section VI. Finally, we conclude the paper in Section VII. All the proofs can be found in the appendix.

The following notations are used in this paper: Boldface uppercase letters denote matrices, boldface lowercase letters denote column vectors and italics denote scalars. Let $\rho(\mathbf{A})$ denote the Perron-Frobenius eigenvalue (spectral radius) of a nonnegative matrix $\mathbf{A}$. The super-script $(\cdot)^\top$ denotes the transpose. Let $\|\cdot\|_0$ denote the ℓ_0 norm (the cardinality of a vector). Let $\mathbf{I}$ and $\mathrm{diag}(\mathbf{x})$ denote the identity matrix and the diagonal matrix with the entries of $\mathbf{x}$ on the diagonal, respectively. Let $e^{\mathbf{x}}$ and $\log \mathbf{x}$ denote $(e^{x_1},\ldots,e^{x_n})^\top$ and $(\log x_1,\ldots,\log x_n)^\top$, respectively. We let $[\mathbf{x};\mathbf{y}]$ denote a column vector with entries from $\mathbf{x}$ and $\mathbf{y}$ stacked one after the other, and let $\frac{\mathbf{x}}{\mathbf{y}}$ denote a column vector with entries x_l/y_l for all l.

II. System Model

We consider a cognitive radio network with a finite number of (licensed and higher priority) primary users and (unlicensed and lower priority) secondary users. There are L_m primary users and L_s secondary users (transmitter-receiver pairs) that want to communicate simultaneously over a common frequency-flat fading channel. For notation purpose, we use the super-script m to label the primary users and the super-script s to label the secondary users. Let $\mathbf{p}^m = (p_1^m,\ldots,p_{L_m}^m)^\top$ and $\mathbf{p}^s = (p_1^s,\ldots,p_{L_s}^s)^\top$ be the transmit power vector of the primary and secondary users respectively. The received SINR of the ith primary user and the jth secondary user at the receiver can be given in terms of the transmit power $\mathbf{p} = [\mathbf{p}^m;\mathbf{p}^s]$ as:

$$\mathsf{SINR}_i^m(\mathbf{p}) = \frac{G_{ii}^{mm} p_i^m}{\sum_{\substack{l=1\\ l\neq i}}^{L_m} G_{il}^{mm} p_l^m + \sum_{j=1}^{L_s} G_{ij}^{ms} p_j^s + n_i^m}, \qquad (1)$$

and:

$$\mathsf{SINR}_j^s(\mathbf{p}) = \frac{G_{jj}^{ss} p_j^s}{\sum_{i=1}^{L_m} G_{ji}^{sm} p_i^m + \sum_{\substack{l=1\\ l\neq j}}^{L_s} G_{jl}^{ss} p_l^s + n_j^s}, \qquad (2)$$

respectively, where G_{ij}^{ms} is the channel gain from the jth secondary transmitter to the ith primary receiver, and n_i is the additive white Gaussian noise (AWGN) at the receiver of the ith user.

Let us consider the problem that minimizes the total energy consumption of both the primary and secondary users subject to given power budget and SINR constraints [4], [5]:

$$\begin{array}{ll} \text{minimize} & \sum_{i=1}^{L_m} p_i^m + \sum_{j=1}^{L_s} p_j^s \\ \text{subject to} & \mathsf{SINR}_i^m(\mathbf{p}) \geq \bar{\gamma}_i^m, \quad i = 1, \ldots, L_m, \\ & \mathsf{SINR}_j^s(\mathbf{p}) \geq \bar{\gamma}_j^s, \quad j = 1, \ldots, L_s, \\ & \mathbf{0} \leq \mathbf{p}^m \leq \bar{\mathbf{p}}^m, \\ & \mathbf{0} \leq \mathbf{p}^s \leq \bar{\mathbf{p}}^s, \\ \text{variables :} & \mathbf{p}^m, \mathbf{p}^s, \end{array} \tag{3}$$

where $\bar{\mathbf{p}} = [\bar{\mathbf{p}}^m; \bar{\mathbf{p}}^s]$ is a vector that upper-bounds the transmit powers for all the users, and $\bar{\boldsymbol{\gamma}} = [\bar{\boldsymbol{\gamma}}^m; \bar{\boldsymbol{\gamma}}^s]$ is a given minimum SINR threshold vector that represents the quality-of-service requirement in the cognitive radio network. It is required that the received SINR of all the users are at least larger than $\bar{\boldsymbol{\gamma}}$.

To give a more compact representation to (3), let us define the nonnegative vector:

$$\mathbf{v} = [\mathbf{v}^m; \mathbf{v}^s] = \left(\frac{n_1^m}{G_{11}^{mm}}, \cdots, \frac{n_{L_m}^m}{G_{L_m L_m}^{mm}}, \frac{n_1^s}{G_{11}^{ss}}, \cdots, \frac{n_{L_s}^s}{G_{L_s L_s}^{ss}} \right)^\top, \tag{4}$$

and the nonnegative matrix $\mathbf{F}$:

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}^{mm} & \mathbf{F}^{ms} \\ \mathbf{F}^{sm} & \mathbf{F}^{ss} \end{bmatrix}, \tag{5}$$

where $\mathbf{F}^{ms} \in \mathbb{R}_+^{L_m \times L_s}, \mathbf{F}^{sm} \in \mathbb{R}_+^{L_s \times L_m}, \mathbf{F}^{mm} \in \mathbb{R}_+^{L_m \times L_m}$ and $\mathbf{F}^{ss} \in \mathbb{R}_+^{L_s \times L_s}$ are matrices that have, respectively, entries $F_{ij}^{ms} = G_{ij}^{ms}/G_{ii}^{mm}, F_{ji}^{sm} = G_{ji}^{sm}/G_{jj}^{ss}$, and:

$$F_{li}^{mm} = \begin{cases} 0, & l = i, \\ \frac{G_{li}^{mm}}{G_{ll}^{mm}}, & l \neq i, \end{cases} \tag{6}$$

$$F_{lj}^{ss} = \begin{cases} 0, & l = j, \\ \frac{G_{lj}^{ss}}{G_{ll}^{ss}}, & l \neq j. \end{cases} \tag{7}$$

Moreover, we shall assume that $\mathbf{F}$ is irreducible, i.e., each (primary or secondary) user has at least an interferer. Then, we can rewrite (3) as the following linear program [4]:

$$\begin{array}{ll} \text{minimize} & \mathbf{1}^\top \mathbf{p} \\ \text{subject to} & (\mathbf{I} - \operatorname{diag}(\bar{\boldsymbol{\gamma}})\mathbf{F})\mathbf{p} \geq \operatorname{diag}(\bar{\boldsymbol{\gamma}})\mathbf{v}, \\ & \mathbf{0} \leq \mathbf{p} \leq \bar{\mathbf{p}}, \\ \text{variables :} & \mathbf{p}. \end{array} \tag{8}$$

In general, (8) or equivalently (3) may be feasible or it may not be. This means that it may not be possible to have the received SINR of all the users be larger than $\bar{\boldsymbol{\gamma}}$ in (3).

Now, suppose that (8) is feasible. To solve (8), the following algorithm has been proposed in [6]:

$$p_l(t+1) = \min \left\{ \frac{\bar{\gamma}_l}{\mathsf{SINR}_l(\mathbf{p}(t))} p_l(t), \bar{p}_l \right\}, l = 1, \ldots, L_m + L_s, \tag{9}$$

where $\mathsf{SINR}(\mathbf{p}) = [\mathsf{SINR}^m(\mathbf{p}); \mathsf{SINR}^s(\mathbf{p})]$. This is known as the constrained DPC algorithm, and it converges to the optimal solution of (3) whenever (3) is feasible. Intuitively, the lth user increases its power if $\mathsf{SINR}_l(\mathbf{p})$ is below $\bar{\gamma}_l$ or decreases it otherwise. However, when (3) is infeasible, (9) converges to a point in which only a subset of the users can satisfy their SINR requirements. In a cognitive radio network, it is important to study the impact of secondary users on the primary users. From a system efficiency perspective, it is necessary to find the system capacity, i.e., the maximum number of users that can be supported. Also, we shall make the assumption that the system having only primary users is already feasible without any secondary user, and hence the point of interest is to find out how many additional secondary users (out of the L_s) can be supported in the network.

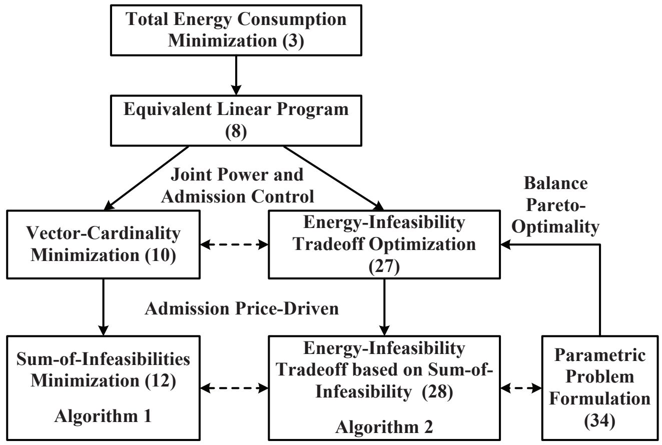

Fig. 2. Overview of the connection between the optimization problems in the paper and the design of price-driven algorithms for joint power and secondary user admission control. The dotted lines indicate that the corresponding blocks share similar parallel features in the analysis or algorithm implementation.

Definition 1: The maximum feasible set of secondary users for (3) is the largest number of secondary users that can be supported subject to the constraints in (3) being all feasible.

In the following, we first study a feasibility optimization problem closely related to (3), which is a vector-cardinality minimization problem in Section III. By leveraging the sum-of-infeasibilities convex relaxation heuristic, we propose a price-driven algorithm to approximate the maximum feasible set of secondary users in Section IV. Furthermore, to balance the infeasibility and total energy consumption in the network, we study a Pareto optimality tradeoff problem in Section V using sensitivity analysis. Fig. 2 gives an overview of the key optimization problems in this paper.

III. Energy-Infeasibility Optimization

In this section, we first formulate the energy-infeasibility optimization as a vector-cardinality problem and then propose a solution methodology using the sum-of-infeasibilities convex relaxation heuristic.

A. Energy-Infeasibility Optimization Problem

In general, finding the largest set of users whose SINR thresholds can all be satisfied in (3) whenever it is infeasible is a NP-hard combinatorial problem [21]. When the number of secondary users is large, it is computationally difficult to find the feasible set with the maximum cardinality. In the following, we formulate an optimization problem related to (3) by adding auxiliary variables q_j^s to the right-hand side of the

SINR constraint for the jth secondary user:

$$\begin{array}{ll} \text{minimize} & \|\mathbf{q}^s\|_0 \\ \text{subject to} & \frac{\bar{\gamma}_i^m}{\mathsf{SINR}_i^m(\mathbf{p})} \le 1, \quad i=1,\ldots,L_m, \\ & \frac{\bar{\gamma}_j^s}{\mathsf{SINR}_j^s(\mathbf{p})} \le 1+q_j^s, \quad j=1,\ldots,L_s, \\ & \mathbf{0} \le \mathbf{p} \le \bar{\mathbf{p}}, \\ \text{variables}: & \mathbf{p}, \mathbf{q}^s, \end{array} \tag{10}$$

where $\mathbf{q}^s$ can be interpreted as an indicator of infeasibility that also has a physical meaning of SINR margins being added to the SINR thresholds, and the objective function $\|\mathbf{q}^s\|_0$ is the ℓ_0 norm that measures the cardinality of $\mathbf{q}^s$. For brevity, we call $\mathbf{q}^s$ the SINR margin variable and let $\mathbf{q} = [\mathbf{q}^m; \mathbf{q}^s]$ where $\mathbf{q}^m$ is a zero vector with L_m zeros.

Lemma 1: If $\mathbf{q}$ is a feasible solution of (10), we have:

$$\rho\left(\text{diag}\left(\frac{\bar{\boldsymbol{\gamma}}}{\mathbf{1}+\mathbf{q}}\right)\left(\mathbf{F}+\frac{1}{\bar{p}_l}\mathbf{v}\mathbf{e}_l^\top\right)\right) \le 1, l=1,\ldots,L_m+L_s. \tag{11}$$

From Lemma 1, (3) is feasible if and only if the optimal value of (10) is zero. We have $q_j^s > 0$ if the SINR threshold of the jth secondary user cannot be achieved. Intuitively, a feasible set of users for (3) can be obtained by removing all the secondary users satisfying $q_l^s > 0$ at the optimality of (10). However, (10) is still a computationally hard problem due to the nonsmooth and nonconvex ℓ_0 norm function.

B. Sum-of-Infeasibilities Based Convex Relaxation Heuristic

We consider the following optimization problem by replacing the ℓ_0 norm objective function of (10) with the sum of $\mathbf{q}^s$, i.e., using the sum-of-infeasibilities[1] heuristic, given by:

$$\begin{array}{ll} \text{minimize} & \mathbf{1}^\top\mathbf{q}^s \\ \text{subject to} & \frac{\bar{\gamma}_i^m}{\mathsf{SINR}_i^m(\mathbf{p})} \le 1, \quad i=1,\ldots,L_m, \\ & \frac{\bar{\gamma}_j^s}{\mathsf{SINR}_j^s(\mathbf{p})} \le 1+q_j^s, \quad j=1,\ldots,L_s, \\ & \mathbf{0} \le \mathbf{p} \le \bar{\mathbf{p}}, \\ & \mathbf{q}^s \ge \mathbf{0}, \\ \text{variables}: & \mathbf{p}, \mathbf{q}^s. \end{array} \tag{12}$$

Let us denote the optimal solution $\mathbf{p}$ in (12) by $\mathbf{p}^\star$. The optimal solution of $\mathbf{q}^s$ in (12) can be expressed as $\mathbf{q}^\star = \left(0,\ldots,0,q_{L_m+1}^\star,\ldots,q_{L_m+L_s}^\star\right)^\top$.

Remark 1: Since the nonnegative SINR margin variable $\mathbf{q}^s$ satisfies $1-q_j^s \le \frac{1}{1+q_j^s}$ for all j, the objective function of (12) satisfies:

$$\sum_{j=1}^{L_s} q_j^s \ge \sum_{j=1}^{L_s}\left(1-\frac{\mathsf{SINR}_j^s(\mathbf{p})}{\bar{\gamma}_j^s}\right). \tag{13}$$

The inequality in (13) is tight if (3) is feasible. Otherwise, minimizing the left-hand side of (13) has the effect of minimizing the differences between the SINR thresholds and the achieved SINRs of all the secondary users. This viewpoint thus motivates the sum-of-infeasibilities heuristic as a viable way to approximating the maximum feasible set of secondary users.

[1]The sum-of-infeasibilities method is routinely used in the first phase of many convex programming algorithms, e.g., interior-point method, to find a feasible point. It often violates only a small number of inequalities, and this interesting phenomenon is under active research in sparse recovery, e.g., basis pursuit and ℓ_1 norm regularization (cf. Chapter 11.4 in [28]).

Although (12) is still nonconvex, we can transform it to a convex problem by using a logarithmic transformation on the transmit power, i.e., $\tilde{\mathbf{p}} = \log\mathbf{p}$. Then, we obtain the following equivalent convex optimization problem:

$$\begin{array}{ll} \text{minimize} & \mathbf{1}^\top\mathbf{q} \\ \text{subject to} & \log\frac{\bar{\gamma}_i^m}{\mathsf{SINR}_i^m(e^{\tilde{\mathbf{p}}})} \le 0, \quad i=1,\ldots,L_m, \\ & \log\frac{\bar{\gamma}_j^s}{\mathsf{SINR}_j^s(e^{\tilde{\mathbf{p}}})} \le \log(1+q_j^s), \quad j=1,\ldots,L_s, \\ & e^{\tilde{\mathbf{p}}} \le \bar{\mathbf{p}}, \\ & \mathbf{q}^s \ge \mathbf{0}, \\ \text{variables}: & \tilde{\mathbf{p}}, \mathbf{q}^s. \end{array} \tag{14}$$

Note that the optimal solution $\tilde{\mathbf{p}}^\star$ in (14) is related to $\mathbf{p}^\star$ in (12) by $\tilde{\mathbf{p}}^\star = \log\mathbf{p}^\star$. Next, we present results on the optimality of (12) that will be used to design a price-driven algorithm to solve (10) in Section IV.

Theorem 1: The optimal solution $\mathbf{p}^\star$, $\mathbf{q}^\star$ and the dual solution $(\boldsymbol{\nu}^\star, \boldsymbol{\lambda}^\star)$ of (14) satisfy:

$$\mathbf{p}^\star = \text{diag}\left(\frac{\bar{\boldsymbol{\gamma}}}{\mathbf{1}+\mathbf{q}^\star}\right)(\mathbf{F}\mathbf{p}^\star+\mathbf{v}), \tag{15}$$

$$\nu_l^\star = p_l^\star\left(\sum_{i\neq l}\frac{G_{il}\nu_i^\star}{\sum_{j\neq i}G_{ij}p_j^\star+n_i}+\lambda_l^\star\right), l=1,\ldots,L_m+L_s, \tag{16}$$

$$\lambda_l^\star(p_l^\star-\bar{p}_l)=0, \quad l=1,\ldots,L_m+L_s, \tag{17}$$

and:

$$q_j^\star = \max\{\nu_j^\star-1,0\}, \quad j=L_m+1,\ldots,L_m+L_s, \tag{18}$$

where $\nu_l \in \mathbb{R}_+$ is the dual variable associated with the lth SINR constraint and $\lambda_l \in \mathbb{R}_+$ is the dual variable associated with the lth power constraint.

Interestingly, ν_l can be interpreted as the *admission price* of the lth secondary user (once admitted into the system, the lth secondary user pays this price to maintain his or her SINR requirement in co-existence with the other users in the network). In particular, from (18), the secondary users with the largest SINR margin pays the highest price at the optimality of (14). In Section V, we further elaborate the role of $\boldsymbol{\nu}$, through sensitivity analysis in optimization theory, as prices that characterize the energy-infeasibility tradeoff. Furthermore, by introducing an auxiliary variable $x_l^\star = \nu_l^\star/p_l^\star$ for each l, we can rewrite (16) as:

$$\mathbf{x}^\star = \mathbf{F}^\top\text{diag}\left(\frac{\bar{\boldsymbol{\gamma}}}{\mathbf{1}+\mathbf{q}^\star}\right)\mathbf{x}^\star+\boldsymbol{\lambda}^\star. \tag{19}$$

Remark 2: Theorem 1 is deduced by applying the Karush-Kuhn-Tucker (KKT) optimality conditions (cf. Chapter 5.5 in [28]) to (14). From the KKT complementarity slackness condition, the dual variable $\lambda_l^\star$ is equal to zero whenever $p_l^\star < \bar{p}_l$ at the optimality of (14). If the optimal value of (10) is greater than zero, the dual variables satisfy $\boldsymbol{\nu}^\star > \mathbf{0}$ and $\boldsymbol{\lambda}^\star \neq \mathbf{0}$. In general, $\mathbf{x}$ can be regarded as an auxiliary variable to assist in the computation of the optimal primal and dual solution of (14).

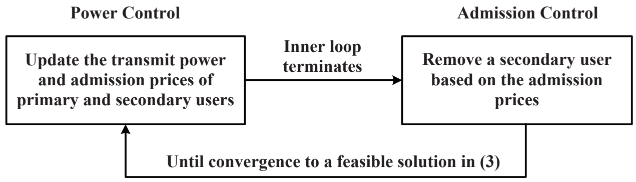

Fig. 3. Block diagram of Algorithm 1.

IV. Price-Driven Spectrum Access Algorithm Design

In this section, we propose a price-driven algorithm for joint power and admission control by leveraging the admission price and the fixed-point equations established in Theorem 1 to solve the energy-infeasibility optimization problem.

A. Sum-of-Infeasibilities Based Joint Power and Admission Control Algorithm

We propose a joint power and admission control algorithm that determines the spectrum access of secondary users iteratively through admission control to identify a subset of secondary users that is feasible in (3). The key idea is to compute the transmit power (primal solution of (14)) and the admission prices (dual solution of (14)) iteratively, and then remove secondary users based on the admission prices in a greedy fashion. Fig. 3 shows a block diagram that illustrates this Algorithm 1 as given in the following.

Algorithm 1:

Sum-of-Infeasibilities Based Joint Power and Admission Control

1) **Initialization:**
   - Initialize the set of secondary users $\mathcal{A}(0) = \{L_m + 1, \ldots, L_m + L_s\}$.
2) **Update by each primary user** $i \in \{1, ..., L_m\}$ **and each secondary user** $j \in \mathcal{A}(k)$**:**
   - Update the transmitter power $p_i(k+1)$ at the $(k+1)$th step for all the primary users $i = 1, \ldots, L_m$:

$$p_i(k+1) = \min\left\{\frac{\bar{\gamma}_i}{\mathsf{SINR}_i(\mathbf{p}(k))}p_i(k), \bar{p}_i\right\}. \quad (20)$$

   - Update the transmitter power $p_j(k+1)$ at the $(k+1)$th step for all the secondary users $j \in \mathcal{A}(k)$:

$$p_j(k+1) = \min\left\{\frac{\bar{\gamma}_j p_j(k)}{\max\{\nu_j(k), 1\}\mathsf{SINR}_j(\mathbf{p}(k))}, \bar{p}_j\right\}. \quad (21)$$

3) **Update by each user** $l \in \{1, ..., L_m\} \bigcup \mathcal{A}(k)$**:**
   If $p_l(k+1) < \bar{p}_l$
   - Update the auxiliary variable $x_l(k+1)$:

$$x_l(k+1) = \sum_{i=1}^{L_m} F_{il}\bar{\gamma}_i x_i(k) + \sum_{j \in \mathcal{A}(k)} \frac{F_{jl}\bar{\gamma}_j x_j(k)}{\max\{\nu_j(k), 1\}}. \quad (22)$$

   - Update the admission price $\nu_l(k+1)$:

$$\nu_l(k+1) = x_l(k+1)p_l(k+1). \quad (23)$$

   else
   - Update the admission price $\nu_l(k+1)$:

$$\nu_l(k+1) = \frac{\bar{\gamma}_l}{\mathsf{SINR}_l(\mathbf{p}(k+1))}. \quad (24)$$

   - Update the auxiliary variable $\mathbf{x}(k+1)$:

$$x_l(k+1) = \nu_l(k+1)/p_l(k+1). \quad (25)$$

   end
4) **Inner loop stopping condition:**
   - If $\|\mathbf{p}(k+1) - \mathbf{p}(k)\|_2 < \epsilon$ or the iterations of (20)-(25) exceed a predefined threshold T, go to Step 5.
   - Otherwise, go to Step 2.
5) **Secondary user admission control:**
   - Let $q_j(k+1) = \max\{\nu_j(k+1) - 1, 0\}$ for all the secondary users $j \in \mathcal{A}(k)$. If $\mathbf{1}^\top \mathbf{q}(k+1) > 0$, then remove a secondary user z satisfying:

$$z = \arg\max_{j \in \mathcal{A}(k)} \nu_j(k+1). \quad (26)$$

   - Update the set $\mathcal{A}(k+1) \leftarrow \mathcal{A}(k) - z$ and go to Step 2.

Theorem 2: Let us define a locally asymptotically stable solution in the Lyapunov sense to be one such that all solutions starting near the stable solution remain near it and tend towards it as $k \to \infty$ [29]. Algorithm 1 converges to a locally asymptotically stable solution that is feasible in (3).

Remark 3: The computation of (22) and (26) can be made distributed by message passing. We may have more than one secondary user satisfying (26). In this case, we remove secondary users by breaking ties uniformly at random. The limit point of $\lim_{k\to\infty} \mathbf{q}(k)$ and its condition that $\lim_{k\to\infty} \mathbf{1}^\top \mathbf{q}(k) = 0$ implies that $\lim_{k\to\infty} \mathbf{p}(k)$ is a feasible solution to (3).

Remark 4: Theorem 2 only characterizes the local convergence behavior of Algorithm 1, and its global convergence is an open problem. Our numerical evaluation in Section VI however demonstrates that Algorithm 1 has good empirical convergence behavior even when the iterates are far from the fixed-point solution.

Remark 5: The dual variable ν_i of the ith primary user also carries the practical meaning of admission price. In this paper, we only consider the spectrum access control of secondary users using these admission prices. However, in general, the admission prices for both the primary and the secondary users can be used to control their joint spectrum access.

From the condition that $\mathsf{SINR}_j(\mathbf{p}^\star) = \frac{\bar{\gamma}_j}{1+q_j^\star}$, $q_j^\star = 0$ implies that the jth user can achieve its SINR threshold. Otherwise, $q_j^\star > 0$ implies that the jth user cannot reach its SINR threshold and it can possibly be removed. Now, if we remove all the users that satisfy $q_j^\star > 0$ for all j, then (3) with a reduced number of constraints is guaranteed to be feasible. However, some users may be unnecessarily removed since we have used the optimality conditions in (12) instead of that in (10). An educated guess to reduce the sum of infeasibilities is to remove the secondary user corresponding to $\arg\max_{j \in \mathcal{A}(k)} \nu_j(k+1)$ at the kth iteration. This is implemented in Step 5. This secondary user removal criterion is motivated by (18) in Theorem 1, namely that the secondary user with the largest SINR margin variable pays the highest price. This secondary user is removed to reduce the interference to other (primary and secondary) users in subsequent iterations. Upon convergence, the total energy consumption is minimized on the set of the primary users and a subset of secondary users, whose SINR constraints are all satisfied. In general, other secondary user removal criterions based on the admission price can also be considered.

B. Discussion of Inner Loop Threshold T

The convergence of Algorithm 1 depends on the predefined threshold T at Step 4 (to stop the inner loop iteration of Step 2 and 3). If T is too large, Algorithm 1 may converge rather slowly. As T becomes smaller, Algorithm 1 converges faster but may prematurely remove more secondary users than necessary based on admission prices (yet to converge in the inner loop). Therefore, the choice of T affects the aggressiveness of admission control and the convergence behavior. To understand this better, we use the outage probability, which is defined as the ratio of the final number of removed users to the initial total number of secondary users, as a parameter to evaluate the performance of Algorithm 1 by choosing different T. The total number of iterations of Algorithm 1 for convergence is affected by T which in turn affects the number of secondary users eventually removed. From a practical perspective, the system should become feasible as soon as possible. To facilitate this, we describe a heuristic to adapt T. First, we empirically get an (a priori) outage probability r_o in the cognitive radio network. Suppose we desire an expected convergence time of Algorithm 1 that is given by $\bar{T}$. Then, we propose to set the threshold $T = \bar{T}/(L_s \times r_o)$ for admission control.

V. Energy-Infeasibility Tradeoff

In general, the optimal solution of (10) is not unique, i.e., the secondary users that make up the maximum feasible set can be different. Different choice of the maximum feasible set of secondary users can influence the objective value of (3). Using optimization duality, we study this Pareto optimality tradeoff using the following optimization problem to strike a balance between infeasibility and the total energy consumption:

$$\begin{array}{ll} \text{minimize} & \omega \mathbf{1}^\top \mathbf{p} + (1-\omega)\|\mathbf{q}^s\|_0 \\ \text{subject to} & \frac{\bar{\gamma}_i^m}{\mathsf{SINR}_i^m(\mathbf{p})} \le 1, \quad i = 1,\ldots,L_m, \\ & \frac{\bar{\gamma}_j^s}{\mathsf{SINR}_j^s(\mathbf{p})} \le 1 + q_j^s, \quad j = 1,\ldots,L_s, \\ & \mathbf{0} \le \mathbf{p} \le \bar{\mathbf{p}}, \\ \text{variables :} & \mathbf{p}, \mathbf{q}^s. \end{array} \tag{27}$$

Note that (27) is a parametric optimization problem that uses the parameter ω to weigh the objective function. In particular, (27) reduces to (3) if $\omega = 1$, and reduces to (10) if $\omega = 0$. A smaller ω emphasizes admission control to maximize the system capacity over minimizing the total energy consumption.

To overcome the nonconvexity in (27), we again employ the sum-of-infeasibilities heuristic in Section III and consider the optimization problem:

$$\begin{array}{ll} \text{minimize} & \omega \mathbf{1}^\top \mathbf{p} + (1-\omega)\mathbf{1}^\top \mathbf{q}^s \\ \text{subject to} & \frac{\bar{\gamma}_i^m}{\mathsf{SINR}_i^m(\mathbf{p})} \le 1, \quad i = 1,\ldots,L_m, \\ & \frac{\bar{\gamma}_j^s}{\mathsf{SINR}_j^s(\mathbf{p})} \le 1 + q_j^s, \quad j = 1,\ldots,L_s, \\ & \mathbf{0} \le \mathbf{p} \le \bar{\mathbf{p}}, \\ & \mathbf{q}^s \ge \mathbf{0}, \\ \text{variables :} & \mathbf{p}, \mathbf{q}^s. \end{array} \tag{28}$$

Note that (28) is different from (12) only in the objective function, and in fact the optimality conditions of (28) share similar features as that of (12). By exploiting this fact, the approach used to design Algorithm 1 can be used to compute the optimal solution of (28) as given in the following algorithm.

Algorithm 2:

Balancing the Energy-Infeasibility Tradeoff

Run Algorithm 1 with (21), (22) and (24), respectively, replaced by the following computations:

- Update the transmit power $p_j(k+1)$ for all the secondary users $j \in \mathcal{A}(k)$:

$$p_j(k+1) = \min\left\{ \frac{\bar{\gamma}_j p_j(k)}{\max\left\{\frac{\nu_j(k)}{1-\omega}, 1\right\} \mathsf{SINR}_j(\mathbf{p}(k))}, \bar{p}_j \right\}. \tag{29}$$

- Update the auxiliary variable $x_l(k+1)$:

$$x_l(k+1) = \sum_{i=1}^{L_m} F_{il}\bar{\gamma}_i x_i(k) + \sum_{j\in\mathcal{A}(k)} \frac{F_{jl}\bar{\gamma}_j x_j(k)}{\max\left\{\frac{\nu_j(k)}{1-\omega}, 1\right\}} + \omega. \tag{30}$$

- Update the admission price $\nu_l(k+1)$:

$$\nu_l(k+1) = \frac{(1-\omega)\bar{\gamma}_l}{\mathsf{SINR}_l(\mathbf{p}(k+1))}. \tag{31}$$

Through Lagrange duality, the admission prices used in the design of Algorithm 1 are based on a connection between infeasibility and the dual solution. Duality has a more profound role in analyzing the energy-infeasibility tradeoff. In particular, through the relationship between the dual solution and sensitivity analysis in optimization theory, we study how removing secondary users affects the total energy consumption in the cognitive radio network. First, for the lth SINR constraint in (28), we introduce a perturbed SINR margin $u_l = 1 + q_l$. We have $0 < u_l < 1$ or $u_l > 1$ if we tighten or loosen the lth SINR constraint, respectively. Next, we denote $f^\star(\mathbf{u})$ as the optimal value of (28), given by:

$$f^\star(\mathbf{u}) = \inf\left\{ \sum_{l=1}^{L_m+L_s} p_l \;\middle|\; \frac{\bar{\gamma}}{\mathsf{SINR}(\mathbf{p})} \le \mathbf{u}, \mathbf{0} \le \mathbf{p} \le \bar{\mathbf{p}} \right\}. \tag{32}$$

If $f^\star(\mathbf{u})$ does not exist for some $\mathbf{u}$, then we let $f^\star(\mathbf{u}) = \infty$. Using the logarithmic transformation $\tilde{p}_l = \log p_l$, $\tilde{u}_l = \log u_l$, and taking logarithm on both sides of the SINR constraints in (32), we denote $f^\star(\mathbf{u})$ in terms of $\tilde{\mathbf{u}}$ as $\tilde{f}^\star(\tilde{\mathbf{u}})$. Then, $\tilde{f}^\star(\tilde{\mathbf{u}})$ is determined by solving the following convex optimization problem:

$$\begin{array}{ll} \text{minimize} & \sum_{l=1}^{L_m+L_s} e^{\tilde{p}_l} \\ \text{subject to} & \log \frac{\bar{\gamma}_l}{\mathsf{SINR}_l(e^{\tilde{\mathbf{p}}})} \leq \tilde{u}_l, \quad l=1,\ldots,L_m+L_s, \\ & e^{\tilde{\mathbf{p}}} \leq \bar{\mathbf{p}}, \\ \text{variables :} & \tilde{\mathbf{p}}. \end{array} \tag{33}$$

From sensitivity analysis in optimization theory [28], we have $\partial \tilde{f}^\star(\tilde{\mathbf{u}})/\partial \tilde{u}_l = -\nu_l^\star$ for all l. If $\nu_l^\star$ is large, it means that if the lth SINR constraint is loosened or tightened a bit, the effect on the optimal value will be large. However, if $\nu_l^\star$ is small, it means that the lth SINR constraint can be loosened or tightened a bit without much effect on the optimal value. Therefore, if we relax the lth SINR constraint with the largest $\nu_l^\star$, then the optimal value is expected to decrease by a relatively large amount accordingly. By connecting (33) to (28) and noting that $\tilde{u}_l = \log(1+q_l)$, this offers an alternative viewpoint that using the largest q_j to remove the jth secondary user is equivalent to removing the secondary user that satisfies $\arg\max_{j\in\mathcal{A}} \nu_j^\star$ where $\mathcal{A}$ is the set of secondary users.

Besides the sum-of-infeasibilities heuristic, there are also other methodologies, e.g., see [30], that can be used to approximately solve the vector-cardinality problem. We present another parametric problem formulation that leverages Lagrange duality to study the tradeoff between minimizing the total energy consumption and maximizing the system capacity given by:

$$\begin{array}{ll} \text{minimize} & \mathbf{1}^\top \mathbf{p} + \phi(\mathbf{q}^s) \\ \text{subject to} & \frac{\bar{\gamma}_i^m}{\mathsf{SINR}_i^m(\mathbf{p})} \leq 1, \quad i=1,\ldots,L_m, \\ & \frac{\bar{\gamma}_j^s}{\mathsf{SINR}_j^s(\mathbf{p})} \leq 1+q_j^s, \quad j=1,\ldots,L_s, \\ & \mathbf{0} \leq \mathbf{p} \leq \bar{\mathbf{p}}, \\ & \mathbf{q}^s \geq \mathbf{0}, \\ \text{variables :} & \mathbf{p}, \mathbf{q}^s, \end{array} \tag{34}$$

where $\phi(\mathbf{q}^s)$ is a twice differentiable function that is used to capture the energy-infeasibility tradeoff related to the SINR margin variable $\mathbf{q}^s$. For example, by letting $\phi(\mathbf{q}^s) = \frac{1-\omega}{\omega}\mathbf{1}^\top \mathbf{q}^s$, (34) reduces to (28). The key idea to quantify the benefits of the energy-infeasibility balance is this: A more general function $\phi(\mathbf{q}^s)$ can be designed by *reverse engineering* to study the tradeoff between the total energy consumption and a maximal feasible set of secondary users through suitably controlling the curvature of the function $\phi(\mathbf{q}^s)$.

Theorem 3: The optimal solution $(\mathbf{p}^\star, \mathbf{q}^\star)$ and the dual solution $(\boldsymbol{\nu}^\star, \boldsymbol{\lambda}^\star)$ in (34) satisfy:

$$\mathbf{p}^\star = \operatorname{diag}\left(\frac{\bar{\gamma}}{\mathbf{1}+\mathbf{q}^\star}\right)(\mathbf{F}\mathbf{p}^\star + \mathbf{v}), \tag{35}$$

$$\mathbf{x}^\star = \mathbf{F}^\top \operatorname{diag}\left(\frac{\bar{\gamma}}{\mathbf{1}+\mathbf{q}^\star}\right)\mathbf{x}^\star + \boldsymbol{\lambda}^\star + \mathbf{1}, \tag{36}$$

$$\lambda_l^\star(p_l^\star - \bar{p}_l) = 0, \quad l=1,\ldots,L_m+L_s, \tag{37}$$

and:

$$q_j^\star\left(\frac{\partial\phi(\mathbf{q}^\star)}{\partial q_j} - \frac{\nu_j^\star}{1+q_j^\star}\right) = 0, \quad j = L_m+1,\ldots,L_m+L_s. \tag{38}$$

Since (34) shares similar optimality conditions as that of (28), the proof of Theorem 3 is similar to that of Theorem 1. Also, a price-driven algorithm can be obtained by replacing the iteration of $\max\{\nu_j(k)/(1-\omega), 1\}$ in Algorithm 2 with $1+q_j(k)$, where $q_j(k)$ is a solution of the following equation for a given $\nu_j(k)$ for $j\in\mathcal{A}(k)$:

$$q_j(k)\left(\frac{\partial\phi(\mathbf{q}(k))}{\partial q_j} - \frac{\nu_j(k)}{1+q_j(k)}\right) = 0. \tag{39}$$

Using (39), we now deduce a particular $\phi(\mathbf{q}^s)$ that can be used to control the maximal feasible set of secondary users such that the total energy consumption (with the dynamic admission of secondary users) does not exceed a given limit. Suppose the network can tolerate at most an increase of δ in the total energy consumption, then we can choose an appropriate approximation to $\|\mathbf{q}^s\|_0$ using sensitivity analysis in optimization theory. Using the Taylor series expansion, the perturbation $\tilde{u}_j$ leads to a change in the total energy consumption $\tilde{f}^\star(\tilde{\mathbf{u}})$ deviating from $\tilde{f}^\star(\mathbf{0})$ by:

$$\tilde{f}^\star(\tilde{u}_j\mathbf{e}_j) - \tilde{f}^\star(\mathbf{0}) = -\tilde{u}_j\nu_j^\star + o(\tilde{u}_j), \tag{40}$$

where $\mathbf{e}_j$ is the jth unit coordinate vector. Connecting to (34), the total change in the total energy consumption due to the perturbation of $\mathbf{q}^\star$ is given by:

$$\sum_{j=L_m+1}^{L_m+L_s} \left.\frac{\partial\phi(\mathbf{q})}{\partial q_j}\right|_{\mathbf{q}=\mathbf{q}^\star} (1+q_j^\star)\log(1+q_j^\star). \tag{41}$$

Using the given constant bound δ and (41), we have:

$$\frac{\partial\phi(\mathbf{q})}{\partial q_j} = \frac{\xi_j\delta}{(1+q_j)\log(1+q_j)}, j = L_m+1,\ldots,L_m+L_s, \tag{42}$$

where $\xi_j \geq 0$ and $\sum_{j=L_m+1}^{L_m+L_s}\xi_j = 1$. Then integrating (42) yields:

$$\phi(\mathbf{q}^s) = \delta\sum_{j=L_m+1}^{L_m+L_s}\xi_j\log\log(1+q_j). \tag{43}$$

Hence, the $\phi(\mathbf{q}^s)$ given by (43) can be used in (34) to restrain the total energy consumption from exceeding δ. Note that $\log(1+q_j) \approx q_j+\epsilon$ when q_j is relatively small where ϵ is a small regularization constant. In particular, with $\xi_j = 1/L_s$ for all j, $\phi(\mathbf{q}^s)$ in (43) approximates as the function $\delta\sum_{j=L_m+1}^{L_m+L_s}\log(q_j+\epsilon)$ which incidentally can be viewed as a (nonconvex) smooth surrogate for $\|\mathbf{q}^s\|_0$ [30].

VI. Numerical Examples

In this section, we evaluate the performance of our algorithms.

Example 1:

We compare our algorithms with the distributed power control algorithm with temporary removal and feasibility check (DFC) in [21]. Although the model in [21] is the special case for a single cell where the channel gain for each user is the same $G_{lj} = G_{jj}$, we use the same setup for the convenience of comparison. The AWGN at the receiver, i.e., $n = \sigma^2$, is assumed to be 5×10^{-15} W. The channel gain

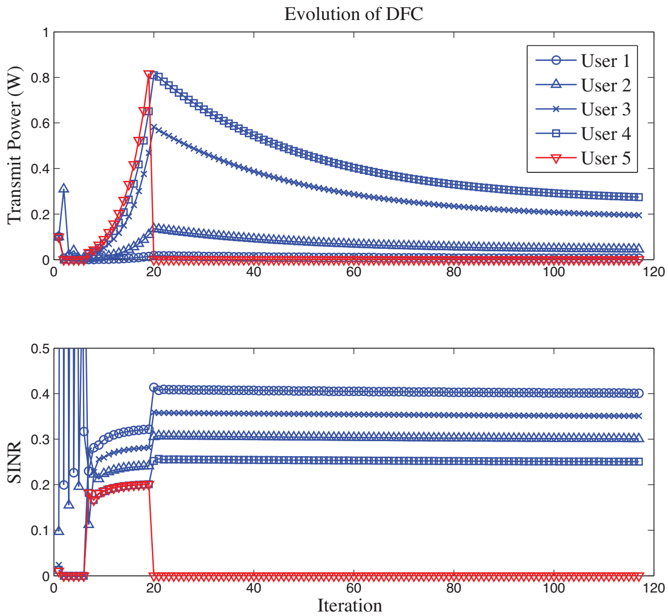

Fig. 4. The evolution of transmit power and SINR for the DFC algorithm with proper initial point. The blue lines are the four supported users. The red line is the removed secondary user.

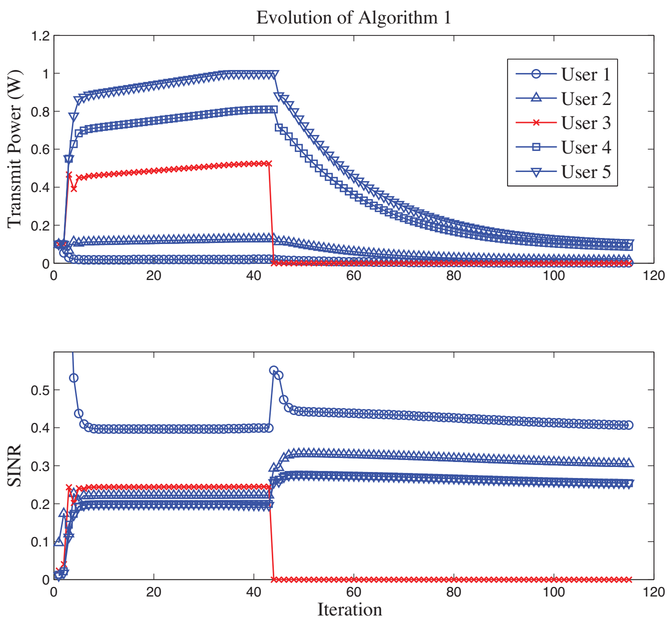

Fig. 5. The evolution of transmit power and SINR for Algorithm 1. The blue lines are the four supported users. The red line is the removed secondary user.

is adopted from the well-known model $G_{jj} = kd_j^{-4}$, where d_j is the distance between the jth transmitter and its receiver, and $k = 0.09$ is the attenuation factor that represents power variations due to path loss. The transmit power upper bounds for all the users are the same, i.e., $\bar{p}_l = 1$ W for all l. There are 5 users indexed by 1 to 5 with a distance vector $d = (300, 530, 740, 860, 910)^\top$m, where each entry is the distance of the corresponding receiver from its transmitter. The SINR threshold vector is $\bar{\gamma} = (0.40, 0.30, 0.35, 0.25, 0.25)^\top$, which is equivalent to $\bar{\gamma} = (-4, -5.2, -4.6, -6, -6)^\top$dB. User 1 is the primary user (cannot be removed) while the rest are the secondary users. These problem parameters make (3) infeasible.

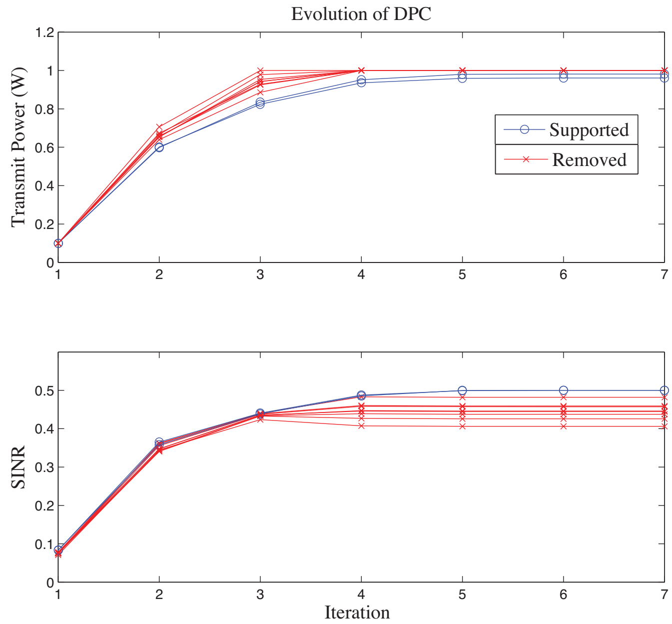

Fig. 6. The evolution of transmit power and SINR for the constrained DPC algorithm. The blue lines are the two supported users. The red lines are the eight removed secondary users.

Fig. 4 shows the same simulation results of the DFC algorithm as [21], which sets $p_5 = 0$ W to switch off User 5 to yield a solution $\mathbf{p} = (0.0061, 0.0483, 0.2063, 0.2904, 0)^\top$W, and the remaining four users satisfy their SINR thresholds. Fig. 5 shows that Algorithm 1 obtains the same maximal feasible set in terms of the cardinality. In particular, it yields $\mathbf{p} = (0.0015, 0.0121, 0, 0.0728, 0.0912)^\top$W, but User 3 is removed instead of User 5. In comparison, our solution gives an additional energy saving of $\frac{0.5511-0.1776}{0.5511} \times 100\% = 67.8\%$. The main reason is that the DFC algorithm greedily (with a local view) removes the user that first hits the upper bound of the transmit power, whereas Algorithm 1 relies on the admission prices of all the users (with a slightly global view) to make the admission control decision.

We also compare our solution with that obtained by the algorithm in [20], which (also with a global view) removes the user to maximize the minimum achievable SINR once the user is removed, and both obtain the same solution.

Example 2:

We compare Algorithm 1 with the constrained DPC algorithm in (9) for networks that have more general channel gains $G_{lj} \neq G_{jj}$ for all $l \neq j$. We consider a network with 2 primary users (cannot be removed) and 8 secondary users. The channel gains are generated randomly to make (3) infeasible. The upper bounds of the power constraints and the SINR thresholds are the same for all l, i.e., $\bar{p}_l = 1$ W and $\bar{\gamma}_l = 0.5$, respectively.

Fig. 6 shows the evolution of the DPC algorithm where the eight secondary users are transmitting at their maximum power and yet cannot achieve the SINR thresholds. Once these secondary users are all removed, the remaining two primary users can achieve their SINR thresholds. Fig. 7 shows the evolution of Algorithm 1 where there are altogether seven users that can achieve their SINR thresholds after the $\{5, 3, 10\}$-th users (which are the secondary users) are iteratively removed.

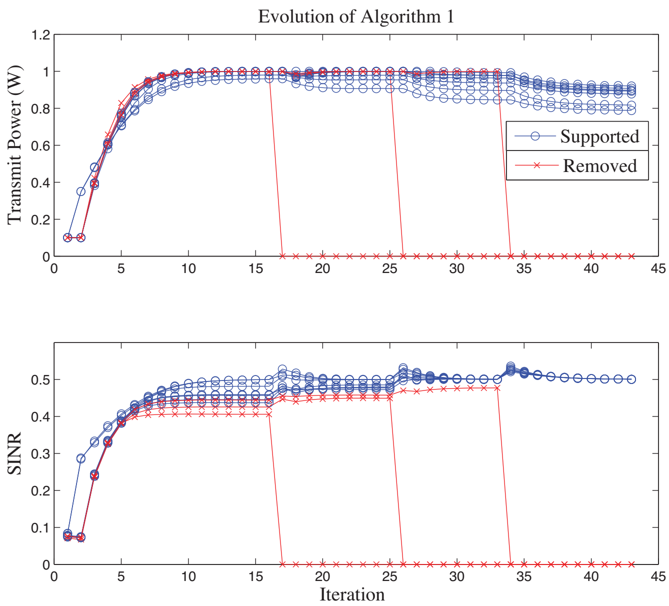

Fig. 7. The evolution of transmit power and SINR for Algorithm 1. The blue lines are the seven supported users. The red lines are the three removed secondary users.

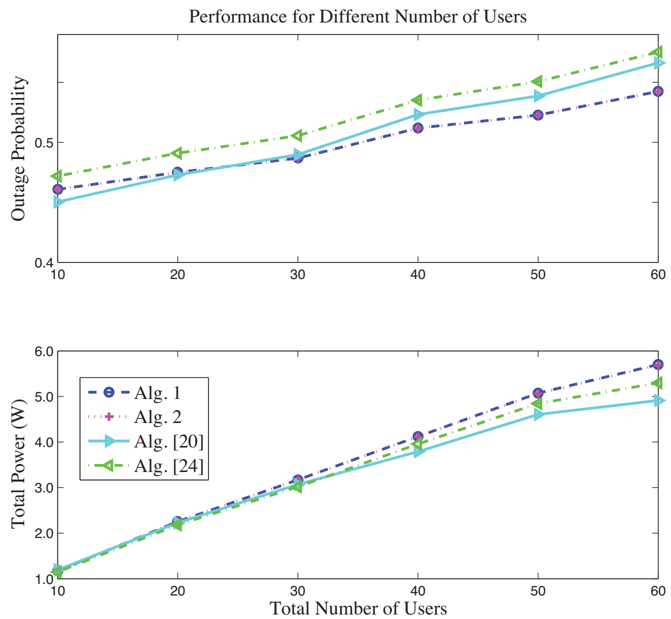

Fig. 8. Average outage probability and average total energy consumption versus the total number of users. The lower bounds of all the SINR thresholds are set to be the same, i.e., $\bar{\gamma}_l = -6$ dB for all l and the weight ω is set to be 0.01.

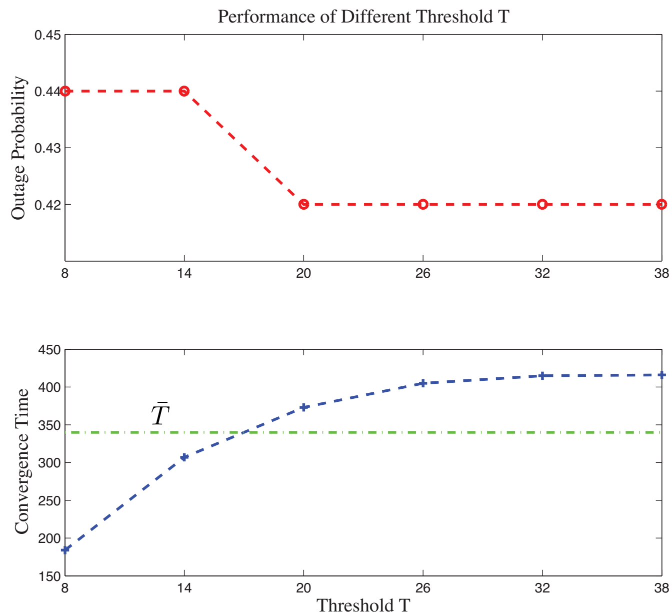

Fig. 9. Average outage probability and average convergence time for different threshold T in a fifty-users example. The lower bounds of all the SINR minimum thresholds are set to be the same, i.e., $\bar{\gamma}_l = -8$ dB for all l.

In comparison, the algorithm in [20] yields the same maximal feasible set as ours by removing the $\{5, 10, 3\}$-th users. As compared to the DPC algorithm, our algorithm increases the system capacity from 20% to 70%.

Example 3:

It is possible to obtain different maximum feasible sets using different algorithms, as the maximum feasible set may not be unique. Here, we compare the system capacity (equivalently the outage probability) and the energy consumption using different algorithms based on Monte-Carlo (MC) simulations with at least 300 MC runs. For each MC run, transmitter locations are uniformly drawn on a 2 Km $\times$ 2 Km square. For each transmitter location, a receiver location is drawn uniformly in a disc of radius 400 meters, excluding a radius of 10 meters. The primary users (10% of all the users) are randomly selected from all the users and the remaining ones are the secondary users. All the upper bounds of transmit power are fixed as $\bar{p}_l = 1$ W. The channel gains are calculated by $G_{lj} = d_{lj}^{-4}$ where d_{lj} is the Euclidean distance between the jth transmitter and the lth receiver. The receiver noise is set as -60 dBm. In Fig. 8, Alg. 1 is our proposed Algorithm 1 in Section III, Alg. 2 is our proposed Algorithm 2 in Section V, Alg. [20] is the centralized removal algorithm in [20], and Alg. [24] is the algorithm in [24] with the removal metric of a secondary user z satisfying:

$$z = \arg\max_{j \in \mathcal{A}(k)} \sum_{l \neq j} G_{lj} p_j^e + \sum_{l \neq j} G_{jl} p_l^e, \tag{44}$$

where $\mathcal{A}(k)$ is the set of secondary users at the kth iteration and p_l^e is the additional transmission power needed for the lth user to attain its SINR threshold (in contrast to our removal metric based on admission prices).

In terms of finding the maximal possible feasible set, Fig. 8 shows that Algorithm 1 and Algorithm 2 can outperform the algorithm in [20] and the removal heuristic in (44) for admission control when the number of users is large. The algorithm in [20] and the removal heuristic in (44) have a smaller total energy consumption than that obtained by our algorithms due to the fewer secondary users that are admitted. This demonstrates the value of optimizing the admission prices as compared to the metric in (44). Fig. 9 shows that the convergence time becomes longer with a larger T, while the outage probability tends to be smaller. When T is large enough, we observe that Algorithm 1 and 2 always converge. To illustrate the tuning of T, we set $T = 16$ by letting the expected convergence time be $\bar{T} = 340$ and using an empirically outage probability which is $r_o = 0.43$.

VII. Conclusion

We studied the energy-infeasibility problem for energy minimization and secondary user spectrum access control in a cognitive radio network subject to power budget and SINR constraints. We formulated this problem as a vector-cardinality optimization problem and used a sum-of-infeasibilities heuristic to approximately solve the vector-cardinality optimization problem. This led to (admission) price-driven algorithms that jointly optimize the power and admission control. We also studied the tradeoff in energy consumption and admitting as many secondary users as possible. Numerical evaluations showed that our algorithms were computationally fast and converged to equilibrium that was near-optimal in terms of finding the maximal feasible set of secondary users and minimizing the energy consumption. As future work, we will study the globally asymptotically convergence of our algorithms and also extend these price-driven algorithms to the joint spectrum access of both the primary and the secondary users.

Appendix

A. Proof of Lemma 1

From the constraint set of (3), we have:

$$\begin{cases} p_l \le \bar{p}_l \Rightarrow \frac{1}{\bar{p}_l}\mathbf{e}_l^\top \mathbf{p} \le 1, \quad l = 1,\ldots,L_m+L_s, \\ \frac{\bar{\gamma}_l}{\mathsf{SINR}_l(\mathbf{p})} \le 1 + q_l \Rightarrow \mathrm{diag}\left(\frac{\bar{\gamma}}{\mathbf{1}+\mathbf{q}}\right)(\mathbf{F}\mathbf{p}+\mathbf{v}) \le \mathbf{p}, \end{cases}$$
$$\Rightarrow \mathrm{diag}\left(\frac{\bar{\gamma}}{\mathbf{1}+\mathbf{q}}\right)\left(\mathbf{F} + \frac{1}{\bar{p}_l}\mathbf{v}\mathbf{e}_l^\top\right)\mathbf{p} \le \mathbf{p},\ l = 1,\ldots,L_m+L_s,$$

where $\mathbf{e}_l$ denotes the lth unit coordinate vector. Let $\mathbf{H}_l = \mathrm{diag}\left(\frac{\bar{\gamma}}{\mathbf{1}+\mathbf{q}}\right)\left(\mathbf{F} + \frac{1}{\bar{p}_l}\mathbf{v}\mathbf{e}_l^\top\right)$ for all l. Note that $\mathbf{H}_l$ is a nonnegative matrix that is irreducible whenever $\mathbf{F}$ is for all l. Using Theorem 1.6 in [31] (Subinvariance Theorem), we deduce that: Suppose that $\mathbf{H}_l$ is an irreducible nonnegative matrix and there is a vector $\mathbf{p} \ge \mathbf{0}$ with $\mathbf{p} \ne \mathbf{0}$ satisfying $\mathbf{H}_l\mathbf{p} \le \mathbf{p}$ (implying that (3) is feasible), then $\mathbf{p} > \mathbf{0}$ and $\rho(\mathbf{H}_l) \le 1$.

B. Proof of Theorem 1

Since (14) is a convex optimization problem, we derive its KKT optimality conditions. We introduce nonnegative dual variables $(\boldsymbol{\nu}, \boldsymbol{\lambda}, \boldsymbol{\mu})$ and write the Lagrangian function of (14):

$$L(\tilde{\mathbf{p}}, \mathbf{q}^s, \boldsymbol{\nu}, \boldsymbol{\lambda}, \boldsymbol{\mu}) = \mathbf{1}^\top\mathbf{q} - \sum_{l=1}^{L_m+L_s} \nu_l \log \mathsf{SINR}_l(e^{\tilde{\mathbf{p}}}) - \boldsymbol{\mu}^\top \mathbf{q}^s + \sum_{l=1}^{L_m+L_s} \nu_l \log\bar{\gamma}_l + \sum_{l=1}^{L_m+L_s} \lambda_l(e^{\tilde{p}_l} - \bar{p}_l) - \sum_{l=1}^{L_m+L_s} \nu_l \log(1+q_l). \tag{45}$$

It is easy to obtain the KKT optimality conditions:

$$\begin{cases} \boldsymbol{\nu}^\star \ge 0, \boldsymbol{\lambda}^\star \ge 0, \boldsymbol{\mu}^\star \ge 0, \mathbf{q}^\star \ge 0, \\ \log\bar{\gamma}_l - \log \mathsf{SINR}_l(e^{\tilde{\mathbf{p}}^\star}) - \log(1+q_l^\star) \le 0, \quad \forall l, \\ e^{\tilde{p}_l^\star} - \bar{p}_l \le 0, \quad l = 1,\ldots,L_m+L_s, \\ \nu_l^\star(\log\bar{\gamma}_l - \log\mathsf{SINR}_l(e^{\tilde{\mathbf{p}}^\star}) - \log(1+q_l^\star)) = 0,\ \forall l, \\ \lambda_l^\star(e^{\tilde{p}_l^\star} - \bar{p}_l) = 0, \quad l = 1,\ldots,L_m+L_s, \\ \mu_j^\star q_j^\star = 0, \quad j = L_m+1,\ldots,L_m+L_s, \\ \frac{\partial L}{\partial q_j} = 1 - \mu_j^\star - \frac{\nu_j^\star}{1+q_j^\star} = 0, j = L_m+1,\ldots,L_m+L_s, \\ \frac{\partial L}{\partial \tilde{p}_l} = \lambda_l^\star e^{\tilde{p}_l^\star} - \nu_l^\star + \left(\sum_{i\ne l} \frac{G_{il}\nu_i^\star e^{\tilde{p}_l^\star}}{\sum_{j\ne i} G_{ij}e^{\tilde{p}_j^\star} + n_i}\right) = 0. \end{cases} \tag{46}$$

In particular, from the transformation $p_l^\star = e^{\tilde{p}_l^\star}$ and by defining a new auxiliary variable $x_l^\star = \nu_l^\star / p_l^\star$ for all l, we obtain (15)-(18).

C. Proof of Theorem 2

We prove the stability of Algorithm 1 by the Lyapunov's first method that checks the eigenvalues of the Jacobian of the nonlinear dynamical system given in (20)-(25). Now, they can be concisely expressed as the following nonlinear discrete-time system:

$$\begin{cases} \mathbf{p}(k+1) = \mathrm{diag}\left(\max\{\boldsymbol{\nu}(k), \mathbf{1}\}\right)^{-1}(\bar{\mathbf{F}}\mathbf{p}(k) + \bar{\mathbf{v}}), \\ \mathbf{x}(k+1) = \bar{\mathbf{F}}^\top \mathrm{diag}\left(\max\{\boldsymbol{\nu}(k), \mathbf{1}\}\right)^{-1}\mathbf{x}(k), \end{cases} \tag{47}$$

where $\bar{\mathbf{F}} = \mathrm{diag}(\bar{\gamma})\mathbf{F}, \bar{\mathbf{v}} = \mathrm{diag}(\bar{\gamma})\mathbf{v}$, and $\nu_l(k) = p_l(k)x_l(k)$ for all l. There exists a fixed-point $[\mathbf{p}; \mathbf{x}]$ that satisfies (15) and (19) since (14) has an optimal solution. Let $\mathbf{z}(k) = [\mathbf{p}(k); \mathbf{x}(k)]$ and the vector $\mathbf{p} \circ \mathbf{x}$ denote the Schur product of $\mathbf{p}$ and $\mathbf{x}$. To show the local stability around the fixed-point of (15)-(19), we will study a dynamical system given by:

$$\begin{bmatrix} f_1(\mathbf{p},\mathbf{x}) \\ f_2(\mathbf{p},\mathbf{x}) \end{bmatrix} \le \begin{bmatrix} \mathrm{diag}\left(\mathbf{p}\circ\mathbf{x}\right)^{-1}(\bar{\mathbf{F}}\mathbf{p}+\bar{\mathbf{v}}) \\ \bar{\mathbf{F}}^\top \mathrm{diag}\left(\mathbf{p}\circ\mathbf{x}\right)^{-1}\mathbf{x} \end{bmatrix}. \tag{48}$$

Taking the Jacobian (denoted by $\mathbf{D}$) of the righthand-side (48), we have:

$$\begin{aligned} \mathbf{D} &= \left.\begin{bmatrix} \partial f_1/\partial\mathbf{p}^\top & \partial f_1/\partial\mathbf{x}^\top \\ \partial f_2/\partial\mathbf{p}^\top & \partial f_2/\partial\mathbf{x}^\top \end{bmatrix}\right|_{\mathbf{z}=\mathbf{z}^\star} \\ &= \begin{bmatrix} \mathrm{diag}\left(\mathbf{p}\circ\mathbf{x}\right)^{-1}\bar{\mathbf{F}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} - \mathbf{E}\,\mathrm{diag}\left([\mathbf{p}\circ\mathbf{x}; \mathbf{p}\circ\mathbf{x}]\right)^{-2}, \end{aligned} \tag{49}$$

where:

$$\mathbf{E} = \begin{bmatrix} \mathrm{diag}(\bar{\mathbf{F}}\mathbf{p}+\bar{\mathbf{v}})\,\mathrm{diag}(\mathbf{x}) & \mathrm{diag}(\bar{\mathbf{F}}\mathbf{p}+\bar{\mathbf{v}})\,\mathrm{diag}(\mathbf{p}) \\ \bar{\mathbf{F}}^\top \mathrm{diag}(\mathbf{x}^2) & \mathbf{0} \end{bmatrix}. \tag{50}$$

Let $\hat{\mathbf{D}}$ denote the matrix with entries $|D_{ij}|$. Thus, from (49) and after taking the absolute value on both sides, we have, elementwise:

$$\hat{\mathbf{D}} \le \begin{bmatrix} \mathrm{diag}\left(\mathbf{p}\circ\mathbf{x}\right)^{-1}\bar{\mathbf{F}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} + \mathbf{E}\,\mathrm{diag}\left([\mathbf{p}\circ\mathbf{x}; \mathbf{p}\circ\mathbf{x}]\right)^{-2}. \tag{51}$$

Using Lemma 1 and Corollary 1.5 in [32] which states that $\rho(\mathbf{A}) \le \rho(\mathbf{B})$ for nonnegative matrices $\mathbf{A}$ and $\mathbf{B}$ satisfying $\mathbf{A} \le \mathbf{B}$, we get the inequalities:

$$\begin{aligned} \rho\left(\mathrm{diag}\left(\mathbf{p}\circ\mathbf{x}\right)^{-1}\bar{\mathbf{F}}\right) &\le \rho\left(\mathrm{diag}\left(\frac{\bar{\gamma}}{\mathbf{1}+\mathbf{q}}\right)\mathbf{F}\right) \\ &< \rho\left(\mathrm{diag}\left(\frac{\bar{\gamma}}{\mathbf{1}+\mathbf{q}}\right)\left(\mathbf{F} + \frac{1}{\bar{p}_l}\mathbf{v}\mathbf{e}_l^\top\right)\right) \\ &\le 1. \end{aligned} \tag{52}$$

Therefore, we have $\rho(\mathbf{D}) \le \rho(\hat{\mathbf{D}}) < 1$ once $\mathbf{p}\circ\mathbf{x}$ is sufficiently large enough, i.e., the admission price becomes large. By the Lyapunov's linearization theorem (cf. Chapter 4 in [29]), the nonlinear map in (48) is locally asymptotically stable if the mapping from $\mathbf{z}(k)$ to $\mathbf{z}(k+1)$ has a Jacobian matrix $\mathbf{D}$, where $\rho(\mathbf{D}) < 1$.

The inclusion of T for the inner loop stopping criteria is discussed in the subsection IV-B. Assuming that the initial point for Algorithm 1 is sufficiently close to a local asymptotically fixed point, then Algorithm 1 is guaranteed to converge since (21) is similar to (9) once the constraint set becomes feasible in (3).

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