

We use a straightforward greedy algorithm, which puts as many cookies as possible on each row before going to the next row. The algorithm clearly runs in linear time. We need to show that any optimal solution has the same penalty cost as the solution obtained by this greedy algorithm.

Consider some optimal solution. If this solution is the same as the greedy solution, then we are done. If it is different, then there is some row  $i$  which has enough space left over for the first cookie of the next row. In this case, we move the first cookie of row  $i + 1$  to the end of row  $i$ . This does not change the total penalty cost, since if the width of the cookie moved is  $d$ , then the reduction to the cost of row  $i$  will be  $d+s = d+1$ , for the cookie and the space before it, and the increase of the cost of row  $i+1$  will also be  $d + 1$ , for the cookie and the space after it. (If the moved cookie was the only cookie on line  $i + 1$ , then by moving it to the previous row the total cost is reduced, a contradiction to the assumption that we have an optimal solution.) As long as there are rows with enough extra space, we can keep moving the first cookies of the next rows back without changing the total cost. When there are no longer any such rows, we will have changed our optimal solution into the greedy solution without affecting the total penalty cost. Therefore, the greedy solution is an optimal solution.