

Solution for a3 of CS4335

Q1. (a)

Let $d(i)$ be the max number of pairs for the first i numbers in the first i number $a_1 a_2 \dots a_i$.

$$d(i) = \max \{ d(i-1), 1+d(j) \}, \text{ where } j \text{ is the largest integer } < i \text{ and } a_i = a_j.$$

(b) Let $d(i)$ be the max weight of pairs for the first i numbers in the first i number $a_1 a_2 \dots a_i$.

$$d(i) = \max \{ d(i-1), (a_i + a_j) + d(j) \}, \text{ where } j \text{ is the largest integer } < i \text{ and } a_i = a_j.$$

The pseudo code and backtracking are omitted here (20%) Similar to the weighted interval scheduling problem.

Q2.

Let $d(i, j)$ be the cost of the opt solution for the interval $a_i \dots a_j$.

$$d(i, j) = 1 + d(i+1, j-1) \text{ if } a_i = a_j.$$

Otherwise

$$d(i, j) = \max_{i < k < j} \{ d(i+1, j), d(i, j-1), d(i, k) + d(k+1, j) \}$$

Pseudo codes are similar to that of the weighted binary search tree construction problem. (20%)

Q3: Let $d(i, j, k)$ be the max total value the first play can get, where $i+j+k=2q$ for some q .

$$d(i, j, k) = \max \{ a, b, c \}, \text{ where}$$

$$a = a_i + \min \{ d(i-1, j-1, k-1), d(i-1, j, k-2), d(i-1, j-2, k), d(i-2, j-1, k), d(i-2, j, k-1), d(i-3, j, k) \}$$

First play chooses a_i from row 1 and the other two have 6 choices.

$$b = b_j + \min \{ d(i-1, j-1, k-1), d(i, j-1, k-2), d(i-2, j-1, k), d(i, j-2, k-1), d(i-1, j-2, k), d(i, j-3, k) \}$$

First play chooses b_j from row 2 and the other two have 6 choices.

$$c = c_k + \min \{ d(i-1, j-1, k-1), d(i-2, j, k-1), d(i, j-2, k-1), d(i-1, j, k-2), d(i, j-1, k-2), d(i, j, k-3) \}$$

First play chooses c_k from row 3 and the other two have 6 choices.

and all the index on the right hand side must ≥ 0 (if negative, ignore that term.)

Initial value $d(0, 0, 0) = 0$.

Assume that row 1 contains a_1, a_2, \dots, a_n , row 2 contains b_1, b_2, \dots, b_n and row 3 contains c_1, c_2, \dots, c_n . We need to take a_i (b_i and c_i) first before take a_{i-1} . (b_{i-1} and c_{i-1})