Approximate Optimal Power Allocation Scheme for Wireless Ad hoc Networks in Rayleigh Fading Channels

Dan Xu[1], Zhisheng Niu[2], Fangyu Hu[1]
[1]Department of Electronic Engineering and Information Science
University of Science and Technology of China
Hefei 230027, P. R. China, xudan03@mail.ustc.edu.cn, hufy@ustc.edu.cn
[2]State Key Laboratory on Microwave and Digital Communications
Department of Electronic Engineering, Tsinghua University
Beijing 100084, P. R. China, niuzhs@tsinghua.edu.cn

***Abstract*— In this paper, a distributed power allocation scheme for mobile ad hoc networks in Rayleigh fast fading environments is proposed. With fluctuated SINR, outage probability is introduced as a QoS parameter. An optimization model is formulated to minimize total transmit powers with constraint conditions that outage probability of each link should be satisfied. A closed-form near optimal solution to the model is presented based on the exploration of power ratio factor at the boundary of system capacity. The solution is implemented in the networks by a distributed power allocation scheme. In terms of the scheme, each link can determine its power ratio factor independently. The inequity problem of the power allocation algorithms in incremental fashion is avoided. Meanwhile the scheme is more time-saving than those schemes in iterative manner. By performance evaluation, we verify our analysis on the optimization model and validate a better result of our scheme in comparison with the counterpart. That is, a larger system capacity is admissible by our method.**

***Index Terms*— Ad hoc Networks, Power Allocation, Rayleigh Fast Fading, Optimization, Closed-form.**

I. INTRODUCTION

Power allocation or power control is very significant for mobile ad hoc networks. A wise power allocation scheme can prolong the battery life of each node, moreover, quality of service(QoS) of each link is guaranteed and system capacity can be enhanced. The difficulty to design efficient power allocation schemes for ad hoc networks mainly comes from the distributed attribute of the networks. Meanwhile the time-varying nature of wireless links inflicts another great challenge.

Much work [1][2][3][4][5][7][9] has investigated distributed power allocation for ad hoc networks. Among these schemes, there is a so-called incremental fashion such as PCDC[1], PCMA[2] and DRNP[3]. In the incremental method, no optimization objective, e.g., minimizing total transmit powers is available. An existing link in the networks will restrict transmit power of an incoming link in order to guarantee its QoS requirement, i.e., the signal-to-interference-and-noise ratio(SINR) threshold. Thus inequity problem that former links have more privileges than the latters is unavoidable. Moreover, improper power allocation to an existing link will block a new link, which results from the lack of renegotiation of network resource for optimization. Different from the incremental scheme, [4]-[10] propose power allocation schemes, such as F-M, DCPC, DPC/ALP, DPCMAC etc., for ad hoc networks or distributed wireless links in the iterative manner. An optimization model to minimize total transmit powers is formulated. In this kind of scheme, SINR requirement of each link is satisfied simultaneously. Each link steps to the optimal value of transmit power iteratively. The iterative method will inevitably be time-consuming. However, network topology of ad hoc networks varies fast. Thus the optimal convergence point may not be reached.

The schemes discussed above only consider large-scale fading by assuming static channel gains between transmitters and receivers. However, fast fading makes channel gains fluctuate randomly. In these schemes, to resist deep fade resulted from fast fading, each transmitter should increase transmit power level, which is supposed to be energy-consuming.

When wireless channels incur fast fading, receive powers at a node become random variables. Hence receive SINR is also a random variable. The state SINR falls below a required value is regarded as outage. When outage probability is considered as a QoS parameter, power allocation schemes do not need to gather the rapidly changing information of channel gains by fast fading. It should only gather the information of distance dependent path loss periodically. In [11], the authors investigate power allocation scheme for wireless links affected by Rayleigh fast fading. Outage probability is considered. An optimization model to minimize total transmit powers is formulated as a geometric programming problem. However, no distributed power allocation scheme is proposed. Another limitation is that background interference at each receiver is ignored. These drawbacks make it not suitable for ad hoc

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networks. In [12], the authors take the expectation of SINR as a QoS parameter, an iterative scheme similar to F-M is proposed.

To the best of the authors' knowledge, there is no closed-form power allocation under Rayleigh fast fading environments so far. In this paper, an optimization model is presented to minimize the sum of transmit powers. The constraint conditions of the model require outage probability of each link not to exceed the predefined threshold. A near optimal solution to the model is explored in a closed-form format. In terms of the solution, each link can decide its power ratio factor independently according to its QoS requirements. The determination of power ratio factor overcomes the inequity problem in the incremental method. Based on the solution, a distributed power allocation scheme called NOPSF(Near Optimal Power Allocation Scheme for Fast Fading Channels) is proposed. By performance evaluation we verify our discussion and validate a better performance of NOPSF than it counterpart OPSSC(Optimal Power Allocation Scheme for Static Channels). That is, a larger capacity is admissible by our scheme. Here system capacity has a novel definition. Both number of active links and QoS are considered. A wiser power allocation scheme will make larger capacity admissible. Nevertheless system capacity has a boundary beyond which no power allocation scheme is available. Thus we will embed call admission control in our scheme.

The rest of this paper is organized as follows. Section II presents the network model and a novel definition of system capacity. Section III formulates a global optimization model and gives a near optimal solution. This is followed by a distributed power allocation scheme in Section IV. Section V is performance evaluation and discussions. Finally, the conclusions are drawn in Section VI.

II. NETWORK MODEL AND DEFINITION

Suppose the ith link in a scenario, the transmit power of the ith link is P_{ti}, the receive power is P_{ri}. Since the link is affected by distance dependent path loss, shadowing loss and fast fading, the relation between P_{ti} and P_{ri} is

$$P_{ri} = F_{ii} G_{ii} P_{ti}, \tag{1}$$

where G_{ii} represents path loss and shadowing loss, which can be deemed as a constant for a relatively long time scale. F_{ii} represents the effects by fast fading. In Rayleigh fast fading channels, receive power is deferring to negative exponential distribution. Then both F_{ii} and P_{ri} are negative exponential random variables. Also there is $E[P_{ri}]$=$G_{ii}P_{ti}$. Assume the kth link is adjacent to the ith link, whose transmitter will bring interference to the receiver of the ith link. The interference is denoted by I_{ki}. Here we assume interference also suffers fast fading. Then I_{ki} is also a random variable deferring to negative exponential distribution. There is $E[I_{ki}]$=$G_{ki}P_{tk}$, where G_{ki} represents path loss form the kth transmitter to the ith receiver. SINR at ith receiver is given by

$$\Gamma_i = \frac{P_{ri}}{\sum_{k \neq i} I_{ki} + I_i^0}, \tag{2}$$

here I_i^0 is a constant which represents background interference at the ith receiver. Γ_i is also a random variable. Let γ_i is the minimum SINR requirement of the ith link, the outage probability of the ith link is

$$O_i = \Pr\left\{ \frac{P_{ri}}{\sum\limits_{k \neq i} I_{ki} + I_i^0} < \gamma_i \right\} = \Pr\left\{ P_{ri} < \gamma_i \sum_{k \neq i} I_{ki} + \gamma_i I_i^0 \right\}. \tag{3}$$

Since P_{ri} and I_{ki} are deferring to negative exponential distribution. It can be deduced that O_i is represented by the following expression:

$$O_i = 1 - (1+\gamma_i) e^{-\frac{I_i^0 \gamma_i}{G_{ii} P_{ti}}} \prod_k \left(\frac{G_{ii} P_{ti}}{G_{ii} P_{ti} + \gamma_i G_{ki} P_{tk}} \right), \tag{4}$$

here subscript k includes i. Outage probability should be smaller than a predefined threshold. The threshold is determined in terms of the traffic type of the ith link.

A. The definition of system capacity

For ad hoc networks with multimedia traffics carried, system capacity does not mean the number of active links only. QoS of each link should be the other factor of system capacity. In the paper, QoS of a link can be represented by (Γ_i, O_i), where Γ_i and O_i are SINR and outage probability respectively. We give the definition of system capacity as follows,

$$S = \{N, ((\Gamma_i, O_i), (i = 1, \ldots, N))\}. \tag{5}$$

Here N is the number of active links. System capacity S has boundary S_{max}, beyond which no any power allocation is feasible, i.e., the capacity can not be sustained. System capacity and its boundary can not be quantitated. However, when there is only one type traffic in the network, the number of links can represent system capacity on the premise of specific QoS requirements. A wiser power allocation scheme makes capacity admissible closer to the boundary. Equivalently, with the same capacity, a better power allocation scheme will provide a lower transmit power for each link.

III. OPTIMIZATION MODEL AND SOLUTION

Suppose there are N active links in the networks, each link is associated with specific QoS requirements. We explore the optimization model as

$$\begin{cases} \min \sum_{i=1}^N P_{ti} \\ s.t. \\ O_i = 1 - (1+\gamma_i) e^{-\frac{I_i^0 \gamma_i}{G_{ii} P_{ti}}} \prod_{k=1}^N \left(\frac{G_{ii} P_{ti}}{G_{ii} P_{ti} + \gamma_i G_{ki} P_{tk}} \right) \leq \delta_i, \\ \qquad (i = 1, \ldots, N). \end{cases} \tag{6}$$

In the optimization model (6), the constraint conditions require outage probability of each link not exceed a threshold. Concerning the model, we give the following theorem.

Theorem 1: If (6) has optimal solution, the solution exactly makes outage probability of each link equal to the threshold.

Proof: See Appendix I.

According to Theorem 1, the optimal solution to optimization model (6) is equivalent to that of the following equation set:

$$e^{-\frac{I_i^0\gamma_i}{G_{ii}P_{ti}}}\prod_{k=1}^{N}\left(\frac{G_{ii}P_{ti}}{G_{ii}P_{ti}+\gamma_i G_{ki}P_{tk}}\right)=\frac{1-\delta_i}{1+\gamma_i}, \quad (i=1,\ldots,N). \tag{7}$$

Equation (7) is a non-linear equation set, the solution is very hard to derive. Thus we should find a near-optimal solution with low computation complexity. Meanwhile, since we discuss power allocation for ad hoc networks, the distributed attribute requires the solution can decide each transmit power as distributed as possible. That is, each link should make full use of its own information, while use parameters of the other links less. A link's own information includes QoS requirements and path loss between the receiver and the other transmitters. These parameters closely correlate to transmit power as we discussed. We should find the intrinsic relation between them.

Theorem 2: With the constraint conditions of (6) are met equity, increasing each transmit power to infinite by the same scale makes system capacity boundary S_{max} admissible.

Proof: See Appendix II.

Actually increasing transmit power of each link is equal to deceasing background interference at each receive node to zero. Then capacity loss caused by background interference is eliminated, thus system capacity boundary is approached. Here background interference is distributed at each receiver with different levels. Nevertheless, the distinction of background interference is negligible when transmit power of each link reaches infinite. We have mentioned in [13] that in wireless cellular networks, capacity loss caused by background noise at BS can be compensated by increasing each user's power. Actually, the circumstance that each transmit power reaches infinite should be a ideal condition, by which we can investigate the nature of the relations among each transmit power. We further give a theorem as below.

Theorem 3: When transmit power of each link reaches infinite, the ratio among P_{ti} $(i=1,\ldots,N)$ uniquely determine outage probability of each link.

Proof: See Appendix III

Increasing transmit power to infinite let the networks can stay in the state of capacity boundary. At the boundary, maximum number of links or highest QoS is admissible. Meanwhile, outage probability of each link is equal to the threshold. Here the state each transmit power reaches infinite does not means system capacity is at the boundary. Within the boundary, outage probability of each link should be far smaller than the threshold when transmit power reaches infinite. By the above analysis, we first explore power ratio among each transmit power at capacity boundary.

By Theorem 3, transmit power of the ith link can be represented by $P_{ti}=\eta_i\phi$, here ϕ is a power constant factor which is the same of each link. At system capacity boundary, ϕ reaches ∞. η_i is the power ratio factor of the ith transmitter. Then η_i $(i=1,\ldots,N)$ determines O_i $(i=1,\ldots,N)$ completely at capacity boundary. At the boundary S_{max}, (7) can be written as

$$1-(1+\gamma_i)\prod_{k=1}^{N}\left(\frac{G_{ii}\eta_i}{G_{ii}\eta_i+\gamma_i G_{ki}\eta_k}\right)=\delta_i, \quad (i=1,\ldots,N). \tag{8}$$

Equation (8) is equivalent to

$$\ln G_{ii}\eta_i=\frac{1}{N}\ln\frac{1-\delta_i}{1+\gamma_i}+\frac{1}{N}\sum_{k=1}^{N}\ln(G_{ii}\eta_i+\gamma_i G_{ki}\eta_k), \quad (i=1,\ldots,N). \tag{9}$$

Concerning the ith link and the jth one, in terms of (9) , there is

$$\ln\frac{G_{ii}\eta_i}{G_{jj}\eta_j}=\frac{1}{N}\ln\frac{(1-\delta_i)(1+\gamma_j)}{(1-\delta_j)(1+\gamma_i)}+\frac{1}{N}\sum_{k=1}^{N}\ln\frac{(G_{ii}\eta_i+\gamma_i G_{ki}\eta_k)}{(G_{jj}\eta_j+\gamma_j G_{kj}\eta_k)}, \quad (i,j=1,\ldots,N). \tag{10}$$

The second item on the left hand side depends on the power ratio factor of each link. However, we can make an approximation here. Notice the last item of the equation should multiply a factor $1/N$, we can suppose power ratio factor of each link is similar, this will not bring too much influence on the last item, especially when QoS requirements of each link is similar. Then (10) can be written as

$$\ln\frac{G_{ii}\eta_i}{G_{jj}\eta_j}=\frac{1}{N}\ln\frac{(1-\delta_i)(1+\gamma_j)}{(1-\delta_j)(1+\gamma_i)}+\frac{1}{N}\sum_{k=1}^{N}\ln\frac{(G_{ii}+\gamma_i G_{ki})}{(G_{jj}+\gamma_j G_{kj})}, \quad (i,j=1,\ldots,N). \tag{11}$$

By (11), we get power ratio between the ith link and the jth one as follows,

$$\frac{\eta_i}{\eta_j}=\frac{G_{jj}}{G_{ii}}\frac{[(1-\delta_i)(1+\gamma_j)\prod_{k=1}^{N}(G_{ii}+\gamma_i G_{ki})]^{\frac{1}{N}}}{[(1-\delta_j)(1+\gamma_i)\prod_{k=1}^{N}(G_{jj}+\gamma_j G_{kj})]^{\frac{1}{N}}}, \quad (i,j=1,\ldots,N). \tag{12}$$

Then we get power ratio factor of the ith link at system capacity boundary. That is

$$\eta_i=\frac{1}{G_{ii}}\left[\frac{(1-\delta_i)}{(1+\gamma_i)}\prod_{k=1}^{N}(G_{ii}+\gamma_i G_{ki})\right]^{\frac{1}{N}}, \quad (i=1,\ldots,N). \tag{13}$$

In term of expression of (13), each link can determine its power ratio factor by its own information, which includes the link's QoS requirements and path loss between the receiver and other transmitters. However, the independency sacrifices a little capacity loss by the approximation. Another merit of power ratio factor is the inequity problem is avoided. Determination of transmit power has no relation with the sequence of affiliation of each link. By (13), transmit power of the ith link at system capacity boundary is

$$P_{ti}=\frac{\phi}{G_{ii}}\left[\frac{(1-\delta_i)}{(1+\gamma_i)}\prod_{k=1}^{N}(G_{ii}+\gamma_i G_{ki})\right]^{\frac{1}{N}},\phi\to\infty \qquad (i=1,\ldots,N). \tag{14}$$

Equation (14) is the solution to optimization model (6) at the capacity boundary S_{max}. We know power level of each transmitter is very limited, the infinite value of power constant factor ϕ can not be achieved. However, within capacity boundary, increasing transmit power of each link to infinite will make outage probability smaller than the threshold. Thus, we can scale ϕ down to the level that the outage probability, i.e., O_i $(i=1,\ldots,N)$, is equal to δ_i exactly. Concerning the ith link, according to constraint condition in (6), there is

$$1-(1+\gamma_i)e^{-\frac{I_i^0\gamma_i}{G_{ii}\eta_i\phi}}\prod_{k=1}^{N}\left(\frac{G_{ii}\eta_i}{G_{ii}\eta_i+\gamma_i G_{ki}\eta_k}\right)\le\delta_i. \tag{15}$$

By deduction, The minimum value of ϕ satisfying (15) is

$$\phi_i=\frac{I_i^0\gamma_i}{G_{ii}\eta_i\ln\left(\frac{1+\gamma_i}{1-\delta_i}\prod_{k=1}^{N}\frac{G_{ii}\eta_i}{G_{ii}\eta_i+\gamma_i G_{ki}\eta_k}\right)}. \tag{16}$$

The maximum one of ϕ_i should be selected as the uniform measure for every link, i.e., $\phi_{max}=\max\{\phi_i\}$ $(i=1,\ldots,N)$. Then we have given a near-optimal closed-form solution for (6), i.e., $P_{ti}=\eta_i\phi_{max}$ $(i=1,\ldots,N)$.

The solution is a near-optimal power allocation scheme for distributed wireless links in fast fading channels, which is called NOPSF. We have mentioned the existed algorithms often regard channel states be constant. When the channels suffer fast fading, each link must enhance transmit power to overcome deep fading. We select a scheme form these methods which can provide the largest capacity as the optimal power allocation scheme for static channels (OPSSC), the OPSSC method can be formulated by the following model.

$$\begin{cases}\min\sum_{i=1}^{N}P_{ti}\\ s.t.\\ \Gamma_i=\frac{G_{ii}P_{ti}}{\Sigma_{k=1,k\neq i}^{N}G_{ki}P_{tk}+I_i^0}\ge\gamma_i,(i=1,\ldots,N).\end{cases} \tag{17}$$

Similar to optimization model (6), the solution will make constraint condition in (17) met equity. Hence OPSSC method is converted to the solution to the following equation set.

$$\frac{G_{ii}P_{ti}}{\Sigma_{k=1,k\neq i}^{N}G_{ki}P_{tk}+I_i^0}=\gamma_i,\qquad (i=1,\ldots,N). \tag{18}$$

Equation (18) is a linear equation set whose solution is easy to derive. Here we do not give the expression of the solution. The OPSSC method make the largest capacity admissible among all the schemes which suppose channel gains are static. However, concerning OPSSC scheme, it must increase each transmit power when the channels suffer fast fading. That is, outage probability requirement of each link should also be satisfied. Hence we will compare our scheme with OPSSC to verify if our scheme can sustain a larger capacity.

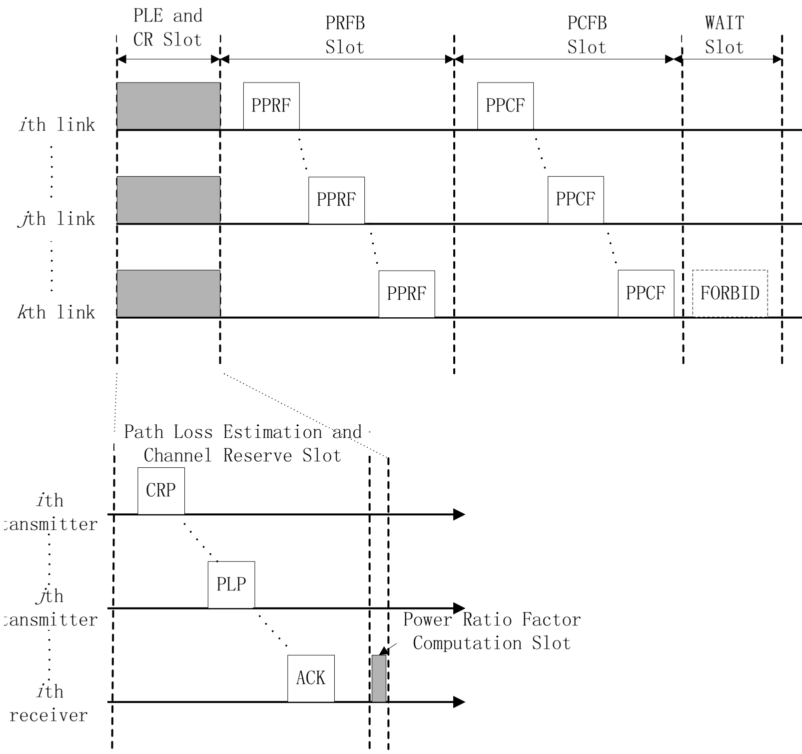

Fig. 1. Time sequence graph for the implementation of NOPSF

IV. A Distributed Power Allocation Scheme

When outage probability is considered, only the information of distance dependent channel gains should be gathered, which varies very slowly. Thus our power allocation scheme can be implemented at relatively long interval. We explore a distributed power allocation algorithm as follows.

Since in our scheme, an incoming link should also have a power allocation. NOPSF should be implemented with a call admission control scheme, that is, the affiliation of new links should make NOPSF still provide feasible power allocation for each link. The scheme is divided into four stages. First, a receiver estimates distance dependent path loss between the transmitters and itself. Then power ratio factor is computed. Meanwhile, an incoming transmitter should reserve channel with the destination node. Second, power ratio factor of each link should be exchanged. Then power constant factor ϕ is computed independently by each link. Third, power scale factor should be exchanged. In the last stage, the feasibility of power allocation is verified. Figure. 1 is time sequence graph of the procedure to implement NOPSF. We detail the procedure at below.

Step1: In the first stage, i.e., PLE and CR slot, the ith transmitter will send a Channel Reserve Packet(CRP) to the expecting ith receiver if the ith link has not been started. If the ith link has existed in the networks, a Path Loss Estimation Packet(PLP) should be broadcast by its transmitter. Both the packets has the format as follows. The first field records the ID

ID(T)	ID(R)	P_i	P_{ti}	η_i^c	ACTIVE	γ_i	δ_i

of the transmitter itself , the second is the ID of the destined receiver. If ACTIVE field is set TRUE, then this is a CRP packet. Otherwise it is a PLP packet. QoS requirements of the ith link are kept in γ_i and δ_i fields respectively. P_i is the maximum power level of the ith transmitter. The packet is broadcast with this power level. η_i^c represents the current value of power ratio factor if the ith link is active. If it is not active, the field is the latest power ratio factor of the link associated with the ith transmitter. P_{ti} is the current transmit power level of the ith link if it is active. However, the field of P_{ti} is set zero if the ith transmitter has not associated with any link yet.

Step2: Each node receives the CRP or PLP packets will average the receive power to estimate the mean value. Suppose the jth receiver, the average receive power is P_{ij}, then distance dependent path gain is $G_{ij}= \frac{P_{ij}}{P_i}$. Path gain is recorded and updated in each power allocation cycle. When path gains have been estimated, the jth receiver will estimate background interference I_j^0 by the equation $I_j^0{=}P_{rj}^{total} - \Sigma G_{ij}P_{ti}$, here P_{rj}^{total} is the total receive powers of the jth receiver which can be measured. The jth receiver will check if it is the destined receiver of a CRP packet. If it is, also the jth receiver is idle, it will return a ACK packet to the expecting transmitter. If it has already been associated with a session, then a NAK packet will be sent to tell the expiration time of its current link.

Step3: After each receiver estimates distance path loss, power ratio factor of each link is computed by (13). Suppose the ith receiver, if the calculated η_i changes inconsiderably comparing that in CRP or PLP packet from the ith transmitter, that is $|\eta_i - \eta_i^c| < \varepsilon$, where ε is determined in terms of network overhead, then η_i will not be notified to other nodes. This strategy is propitious to reduce unnecessary message exchanges. Otherwise power ratio factor is broadcast via the PPRF packet in the PRFB slot.

Step4: When each receive node gathers power ratio factors of the other links, it will calculate power constant factor ϕ_i according to (16). After that, in the PCFB slot, power constant factor of each link is broadcast via the PPCF packet. A maximum value of ϕ_i, $(i = 1, \ldots, N)$ is selected at each receive node. The ith transmitter will calculate the expectation of P_{ti}^{exp} and send it to the ith transmitter via a NOTIF packet. The format is as follows.

ID(R)	ID(T)	G_{ii}	P_{ti}^{exp}	η_i

Step5: If P_{ti}^{exp} exceeds P_i, meanwhile the ith transmitter has not set up the link with the ith receiver yet, the ith transmitter will delay sending data to the ith receiver and wait until next power allocation cycle. If the ith link is already active, but P_{ti}^{exp} exceeds P_i, the ith transmitter will broadcast a FORBID packet to suspend the probable new link. Thus a WAIT slot is needed to verify if power allocation for each link is feasible.

V. PERFORMANCE EVALUATION

First, numerical analysis is conducted to verify our discussion on the definition of system capacity. Figure. 2 presents the curves to illustrate the relation between system capacity and power allocation. We assume there is only one class of traffic associated with each curve, hence we can use links numbers to represent system capacity. It is observed when the number of links increase, to satisfy QoS requirements of each link, transmit power of each link(Since QoS of each link is identical, transmit power of each link is the same with the assumption network topology is symmetric) must be enhanced. With the same transmit power level, a traffic with higher SINR requirement, or a smaller outage probability threshold will make the number of active links admissible less. It is shown that when each transmit power reaches infinite, the upper limits of the number of active links are differentiated for different classes of traffic. With the increment of transmit power, each curve converges to a different upper limit which is a finite value, we can see the curves become flat with the increment of transmit power. This means system capacity does have boundary. Increasing transmit power of each link can make the boundary sustainable, but the increase in transmit power can not change this boundary.

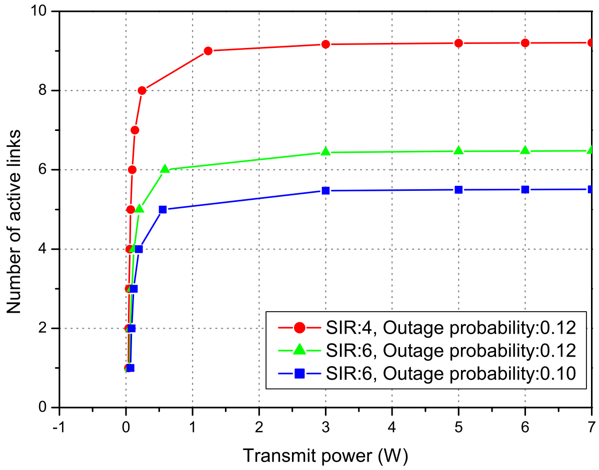

Fig. 2. Admissible number of links with different QoS requirements under NOPSF

The capacity curves affected by background interference are plotted in Figure. 3. A class of traffic with unique QoS requirements denoted by (4, 0.1) is considered, here we can use the number of the active links to represent system capacity . The background interference at each node is set identical. By the figure, with the same transmit power level, a larger background interference will lead to a smaller capacity. However, if there is no power constraint, that is, each transmit power can reach infinite, then the maximum system capacity S_{max} will be the same under different background interference, i.e., the three curves will converge to the same point. This best explains system capacity boundary is admissible when the effects of background interference is eliminated. That is why increasing transmit power of each link can sustain a larger capacity.

By Figure. 4, we compare the performance of the proposed NOPSF with that of OPSSC method. We simulate a series of power allocations for a specific time duration. In this procedure, a link with specific QoS requirements will affiliate

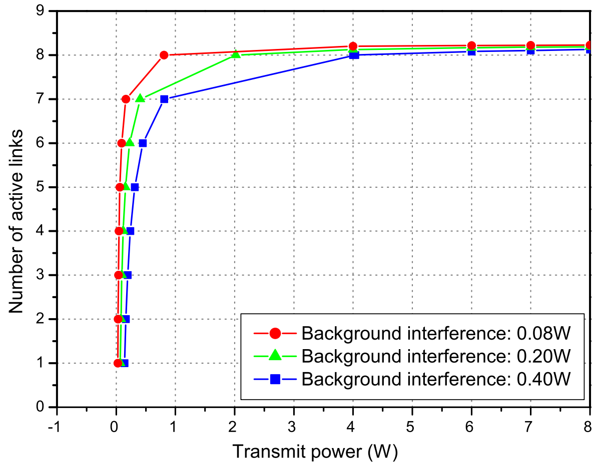

Fig. 3. Capacity curves of a class of traffic affected by different background interference under NOPSF

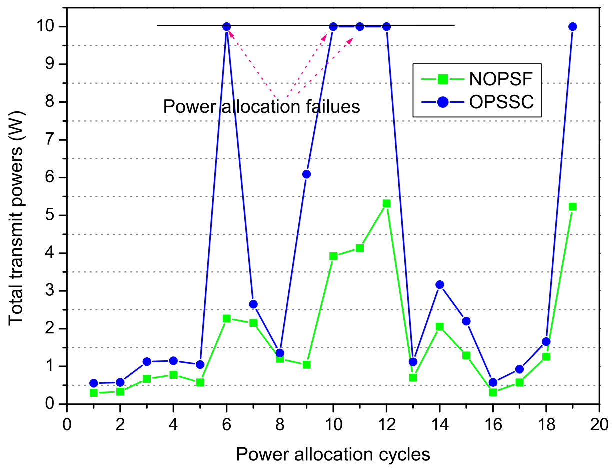

Fig. 4. Comparison between OPSSC and NOPSF in fast fading channels

in the network, or leave the network. QoS requirements of each link is randomly selected from (4, 0.09), (5, 0.1), (5, 0.08), and (6, 0.12). Background interference ranges from 0.1W to 0.15W randomly at each link. Under the same capacity, network topology and background interference, both NOPSF and OPSSC are implemented respectively. The y-axis denotes the sum of transmit powers of all the links, while x-axis is the power allocation cycle. Although the parameter studied is sum of transmit powers, we can also analyze which scheme makes a larger system capacity admissible. That is, under the same capacity, the scheme with higher powers or failed power allocation provides a smaller capacity admissible. It is observed in the figure, in each cycle, the proposed NOPSF always provides a lower sum of transmit powers. While for OPSSC, the sum of transmit powers is higher. Meanwhile, under relatively larger capacity(A large number of links, or strict QoS requirements), OPSSC method fails to provide a feasible power for each link. Here the points of failed power allocations are marked by a top solid line. Under the same circumstances, it is observed that NOPSF still make the capacity admissible by feasible power allocation. Hence NOPSF can sustains a capacity larger than OPSSC.

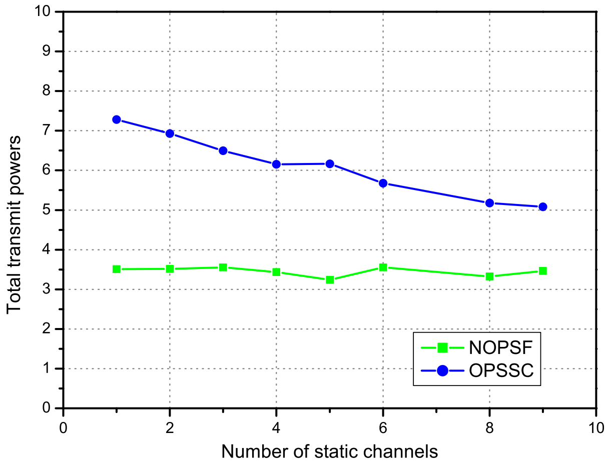

Fig. 5. Comparison between OPSSC and NOPSF in terms of the number of static channels

When the static channels coexist with fast fading channels in the networks. OPSSC method should provide a better performance comparing to the scenario where all the channels suffer fast fading. Under the circumstances, NOPSF may not be so excellent. Then we compare the two schemes in terms of the number of static channels by Figure. 5. Here we study six fixed links under specific network topology. The parameters are listed in Table. I. We set a scenario that static channels coexist with fast fading channels. We Stat. the average sum of transmit powers allocated by NOPSF and OPSSC under the same number of static channels. It is illustrated in the figure, when the static channels increase, the sum of transmit powers of OPSSC method decrease, this is reasonable since OPSSC is optimal for the scenario where there is no fast fading. Concerning NOPSF, there is no obvious decease of sum of transmit powers. However, it is observed NOPSF can still provide a larger capacity, or a low transmit power when static channels coexist with fast fading channels.

Actually the bad performance of OPSSC in fast fading channels comes from the improper determination of power ratio factor of each link. OPSSC method provides a optimal solution for static channels, which means power ratio among each link is optimal in this instance. However, it must enhance transmit power of each link when it suffers fast fading, then the optimal power ratio factors become improper, which will bring capacity loss. The proposed NOPSF first provides a wise power ratio factor for each link in the circumstances of fast fading, although it is not the optimal. Thus a better performance is brought forth by NOPSF.

TABLE I
PARAMETERS OF THE ACTIVE LINKS

Parameter	**Symbol**	**Link**					
		1	2	3	4	5	6
SINR	γ_i	4	5	3	5	4	5
Outage probability	δ_i	0.122	0.134	0.097	0.118	0.135	0.126
Background interference	I_i^0(W)	0.13	0.15	0.16	0.17	0.16	0.13

VI. CONCLUSIONS

In this paper, we propose a distributed power allocation scheme called NOPSF for mobile ad hoc networks in fast fading channels. We fully consider random receive powers and SINR of the link suffering fast fading by introducing outage probability as a QoS parameter. Under the constraint conditions of outage probability requirements, an optimization model is formulated to minimize total transmit powers, or equivalently to maximize system capacity for ad hoc networks. The system capacity is defined as a combination of the link number and QoS of each link. A near optimal solution is presented with low computational complexity, which provides a basis for our distributed power allocation scheme. By performance evaluation, it is proved that NOPSF makes a larger system capacity admissible than the OPSSC scheme.

APPENDIX I
PROOF OF THE THEOREM 1

The outage probability of the ith link is a mono-decreasing function of P_{ti}. While it is a mono-increasing function of P_{tj} $(j=1,\ldots,N, j\neq i)$. Suppose the optimal solution to (6) is P_{ti}^0 $(i=1,\ldots,N)$. If there is a link's outage probability is not equal to the upper limit, without loss of generality, let it be the kth link , i.e., $O_k<\delta_k$. Since O_k is a mono-deceasing function of P_{tk}, there must be a decrement ΔP_{tk}, by which P_{tk}^0 is reduced to $P_{tk}^0{-}\Delta P_{tk}$. Meanwhile the resulted outage probability will increase but inequity $O_k<\delta_k$ is still met due to the continuous attribute of function O_k. The outage probability of other links, i.e., O_i $(i=1,\ldots,N, i\neq k)$ will decease with the increment of P_{tk}^0, that is, the constraint condition in (6) of every link will still be satisfied. So we find a feasible solution, i.e., $(P_{t1}^0,\ldots,P_{tk}^0-\Delta P_{tk},\ldots,P_{tN}^0)$ for (6), which make the constraint condition still met while the sum of the transmit powers is smaller than that of the optimal solution P_{ti}^0 $(i=1,\ldots,N)$. Then the hypothesis of $O_k<\delta_k$ is invalid and all constraints in (6) must be met equity at optimal solution.

APPENDIX II
PROOF OF THE THEOREM 2

By our definition of system capacity, it includes two factors: the number of links and QoS. Capacity boundary may mean the largest number of links, or the highest QoS, thus proof to this theorem should be divided into two parts. First, given a unique QoS requirements, we prove Theorem 2 by the angle of link numbers. After that, we will prove Theorem 2 from the layer of QoS.

Given the power allocation P_{ti} $(i=1,\ldots,N)$ for each transmitter, the current active number of links is N. Outage probability of each link is a mono-increasing function of N, i.e., $\frac{\partial O_i}{\partial N}$ $(i=1,\ldots,N)$. Suppose the maximum number of links admissible is N_{max}, meanwhile transmit power of the ith link is P_{ti}, $(i=1,\ldots,N_{max})$. When the maximum number of links N_{max} is given, by Theorem 1, there is

$$O_i = 1-(1+\gamma_i)e^{-\frac{I_i^0\gamma_i}{G_{ii}P_{ti}}}\prod_k\left(\frac{G_{ii}P_{ti}}{G_{ii}P_{ti}+\gamma_i G_{ki}P_{tk}}\right)=\delta_i, \quad (i=1,\ldots,N_{max}). \tag{19}$$

We use the counterevidence method to prove that P_{ti} $(i=1,\ldots,N_{max})$ must tend to be infinite simultaneously to accommodate the maximum number of links. Assuming P_{ti} $(i=1,\ldots,N_{max})$ is finite, then we can set ${P_{ti}}'{=}MP_{ti}$ $(i=1,\ldots,N_{max})$. Now outage probability of link the ith link is

$$O_i = 1-(1+\gamma_i)e^{-\frac{I_i^0\gamma_i}{G_{ii}MP_{ti}}}\prod_k\left(\frac{G_{ii}MP_{ti}}{G_{ii}MP_{ti}+\gamma_i G_{ki}MP_{tk}}\right), \quad (i=1,\ldots,N_{max}). \tag{20}$$

By (19) and (20) we can see scaling P_{ti} $(i=1,\ldots,N_{max})$ up by M times is equivalent to scaling I_i^0 down by M times. Thus outage probability of each link is deceased by this adjustment, i.e.,

$$O_i<\delta_i \qquad (i=1,\ldots,N_{max}). \tag{21}$$

Since O_i $(i=1,\ldots,N)$ is a mono-increasing function of the number of links N, then we can find an increment ΔN, which makes the outage probability of each link increase yet (21) can still be met. Then we find a larger system capacity $N_{max}{+}\Delta N$, with transmit power ${P_{ti}}'$ $(i=1,\ldots,N_{max}+\Delta N)$. Then it violate the fact that N_{max} is the maximum number of links. Hence the assumption that power P_{ti} $(i=1,\ldots,N_{max})$ is finite is invalid.

The above has proved Theorem 2 in the number of links sense. Now we prove Theorem 2 as QoS considered. Suppose there are a group of links among which the ith link has the smallest outage probability O_i. Assuming at system capacity boundary, transmit power of each link is finite. In the QoS sense, system capacity boundary means if some link increase QoS requirement, no power allocation is feasible. That is, O_i can not be decreased. However, we know by above discussions that scaling transmit power of each link up by M times makes

O_i and outage probability of the other links smaller. Then a feasible transmit power allocation can be provided with smaller outage probability of each link. Hence the assumption at capacity boundary, transmit power of each link is finite is invalid. Theorem 2 follows.

APPENDIX III
PROOF OF THE THEOREM 3

Let $P_{ti}=\eta_i\phi$, $(i=1,\ldots,N)$. η_i is power ratio factor of the ith link , ϕ is a constant. Then outage probability O_i can be written as

$$O_i = 1-(1+\gamma_i)e^{-\frac{I_i^0\gamma_i}{G_{ii}\eta_i\phi}}\prod_k\left(\frac{G_{ii}\eta_i}{G_{ii}\eta_i+\gamma_i G_{ki}\eta_k}\right), \quad (i=1,\ldots,N). \tag{22}$$

By (22), We see that increasing ϕ leads to a smaller O_i. When ϕ reaches infinite, $e^{-\frac{I_i^0\gamma_i}{G_{ii}\eta_i\phi}}$ is equal to one. Thus O_i is exclusively determined by η_i $(i=1,\ldots,N)$, and has no relation with ϕ. Theorem 3 follows.

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Dan Xu is a M.S candidate affiliated with Dept. Electronic Engineering and Information Science, University of Science and Technology of China. He also works as a research assistant in Network Theory Lab, Tsinghua University. His research interests include Radio Resource Management and Cross-layer Design for wireless networks.

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Zhisheng Niu graduated from Northern Jiaotong University, Beijing, China, in 1985, and got his M.E. and D.E. degrees from Toyohashi University of Technology, Toyohashi, Japan, in 1989 and 1992, respectively. In 1994, he joined with Tsinghua University, Beijing, China, where he is now a full professor at the Department of Electronic Engineering. He is also an adjunction professor of Northern Jiaotong University and the Institute of Automation of China Academy of Science. From 1992 to 1994, he was with Fujitsu Laboratories Ltd., Kawasaki, Japan. From October 1995 to February 1996, he was a visiting research fellow of the Communications Research Laboratory of the Ministry of Posts and Telecommunications of Japan. From February 1997 to February 1998, he was a visiting senior researcher of Central Research Laboratory, Hitachi Ltd. From January 2001 to February 2001he was a visiting professor of Venture Business Laboratory at Saga University, Japan. He also visited Polytechnic University, NY, in Feb. 2002. He received the PAACS Friendship Award from the Institute of Electronics, Information, and Communication Engineers (IEICE) of Japan in 1991 and the Best Paper Award (1st prize) from the 6th Chinese Youth Conference on Communication Technology in 1999. His current research interests include teletraffic theory, performance evaluation of high-speed broadband integrated networks, and wireless communication networks. Dr. Niu is a senior member of the IEEE and the Chinese Institute of Electronics (CIE) and a member the IEICE. He is now serving as the Chair of Membership Development Committee of Asia-Pacific Board of IEEE Communication Society, the Chair of APCC Steering Committee (ASC), and the Chair of IEEE Communication Society Beijing Chapter.

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Fangyu Hu is currently a professor affiliated with Dept. Electronic Engineering and Information Science, University of Science and Technology of China. His current research interests include wireless networks and communications, wireless resource management.