Placement of Wavelength Converters for Minimal Wavelength Usage in WDM Networks

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Abstract—**An important goal of the design of WDM (wavelength division multiplexing) networks is to use less wavelengths to serve more communication needs. According to the wavelength conflict rule, we know that the number of wavelengths required in a WDM network is at least equal to the maximal number of channels over a fiber (called maximal link load) in the network. By placing wavelength converters at some nodes in the network, the number of wavelengths needed can be made equal to the maximal link load. In this paper we study the problem of placing the minimal number of converters in a network to achieve that the number of wavelengths in use is equal to the maximal link load. For duplex communication channels, we prove that an optimal solution can be obtained in polynomial-time. For unidirectional communication channels, which was proved to be NP-complete, we develop a set of lemmas which lead to an efficient approximation algorithm whose approximation ratio is two.**

I. Introduction

Wavelength division multiplexing (WDM) [7, 24] divides the bandwidth of an optical fiber into many non-overlapping wavelengths (WDM channels), so that multiple communication channels can operate in parallel on a single optical fiber at different wavelengths.

There are basically two types of architectures of WDM network systems: single-hop systems and multihop systems. In *single-hop* systems [14], each pair of communication nodes has a logical channel configured and the same wavelength is used through out the route of the channel. There is no wavelength conversion in the intermediate nodes in the route of a channel. In *multihop* systems [15], the channel of a pair of nodes can consist of several path segments (called *lightpaths*), each of which may use a different wavelength. Wavelength conversion is needed at a node where the channel uses different wavelengths on the incoming and outgoing fibers. A node which is capable of wavelength conversion must be equipped with a wavelength converter. In a network with sparse wavelength conversion, only a fraction of nodes can be equipped with converters due to the high cost of wavelength converters [20]. It has been anticipated that wavelength converters would remain to be expensive devices in the next few years [3].

Wavelengths are another kind of resource in WDM networks. The number of wavelengths available in a network is always limited due to the complexity of hardware structure of optical switches. In WDM networks, when two channels share a common fiber, they must be assigned with different wavelengths on the shared fiber. It is obvious that the number of wavelengths required in a system should be at least equal to the maximal number of channels over a fiber (called the *maximal link load*), because each channel over the fiber requires a different wavelength. By using wavelength converters, the number of wavelengths required can be made equal to the maximal link load in a system. This feature is called *load-wavelength assignability*. A simple way of achieving this feature is to equip every network node with a converter. In such a system, the number of wavelengths required is equal to the maximal link load of fibers. However, it is too expensive to place a wavelength converter at every network node.

In this paper we consider the problem of the *optimal placement of converters* (OPC) in multihop WDM networks: Given a WDM network, place the minimal number of wavelength converters in the network so that the number of wavelengths required is equal to the maximal link load. For duplex communication channels, we prove that the OPC problem can be solved in polynomial-time; for unidirectional communication channels, which has been proved to be NP-complete [23], we develop a series of lemmas which lead to an efficient approximation algorithm for solving the OPC problem and the approximation ratio is two.

The study of the OPC problem has great implications to network design and applications. Firstly, by achieving load-wavelength assignability, the number of wavelengths needed in a system is made minimal, because the low bound of the number of wavelengths required is equal to the maximal link load. Secondly, with load-wavelength assignability, network applications are free to maximally utilize the network bandwidth without worrying about the availability of wavelengths, so long as the maximal link load is kept less than or equal to the number of wavelengths employed. Thirdly, by using the minimal number of converters to achieve the load-wavelength assignability, it reduces the hardware cost of a network.

II. Technical Preliminaries

A. Network Model

The WDM network under consideration is modelled as a connected graph $G(V, E)$, where V is the vertex-set representing the set of nodes in the network and E is the edge-set representing physical fiber links between nodes. Each link carries two oppositely-directed fibers, for data transmissions in the two directions of the link.

In this paper we consider two types of communication chan-

This work is supported in part by City University of Hong Kong under grant No. 7000778 and National 973 Information Technology and High-Performance Software Program of China.

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TABLE I
THE NUMBER OF WAVELENGTHS REQUIRED FOR CHANNELS IN TREE AND RING NETWORKS.

Networks	Unidirectional channels	Duplex channels
Tree	$\frac{5}{3}L$ wavelengths are sufficient [4] $\frac{5}{4}L$ wavelengths are necessary [4]	$\frac{3}{2}L$ wavelengths are sufficient [16] $\frac{3}{2}L$ wavelengths are necessary [16]
Ring	$(2L-1)$ wavelengths are sufficient [23] $(2L-1)$ wavelength are necessary [23]	

TABLE II
THE NUMBER OF CONVERTERS REQUIRED FOR OPC PROBLEM

Networks	Unidirectional channels	Duplex channels
Star	No converter is needed [23]	One converter is required*
Ring	One converter is required [23]	One converter is required*
Tree	OPC is polynomial-time solvable*	
Mesh	OPC is polynomial-time solvable*	
Cube	OPC is polynomial-time solvable*	
General	OPC has 2-approximation algorithm*	OPC is polynomial-time solvable*

nels: duplex (bidirectional) and unidirectional. In a duplex channel, data can be transmitted in both directions of the channel. The wavelength conflict rule for duplex channels is that channels over the same link must use different wavelengths on the link. In a unidirectional channel, data are transmitted in one direction from the source to the destination. The wavelength conflict rule for unidirectional channels is that channels over the same link and in the same direction must use different wavelengths. That is, two unidirectional channels over the same link but in the opposite directions can use the same wavelength.

We assume that each converter has the *full conversion* capability. That is, a converter can convert an incoming wavelength to any outgoing wavelength at the node.

Our problem is, for a given network, to locate a set of nodes so that the load-wavelength assignability can be achieved if this set of nodes are equipped with wavelength converters. Formally, we define this set as the following:

DEFINITION 1 *A set of nodes S, $S \subseteq V$, is said to achieve load-wavelength assignability if, by equipping each node in S with a wavelength converter, the number of wavelengths required for all channels can be made equal to the maximal link load over fibers in the system.*

B. Related Work

In single-hop systems, minimizing the number of wavelengths can be achieved by using proper routing or wavelength assignment techniques. Since the number of wavelengths needed is at least equal to the maximal link load in a system, an important method to reduce the number of wavelengths is to route channels in a load balancing fashion [1, 23], which aims at minimizing the maximal link load in a system. This problem has been proved to be NP-hard even for some simple network topologies, such as ring [19]. A widely used approach is to formalize the problem using integer linear programming technique and then find approximate solutions [10]. When the routing is done (i.e., the link load is determined), assigning wavelengths properly to channels can also reduce the number wavelengths needed in a system. A lot of research has been done on the study of the minimal number of wavelengths required for a given maximal link load in a system [4, 8, 13, 16]. Due to the complexity of the problem, this kind of study is only limited to the networks of trees or rings. Some best results obtained so far are summarized at Table 1. By "αL wavelength being sufficient" we mean that any set of channels with the maximal link load L can be assigned by using at most αL wavelengths, and by "αL being necessary" we mean that assigning some set of channels with the maximal link load L requires at least αL wavelengths.

In multihop systems, a channel (connection) between two communication nodes may consist of multiple path segments, each of which may use a different wavelength. A converter is required for wavelength conversion at the node where incoming signal and outgoing signal of a channel are in different wavelengths. By introducing wavelength converters into a system, the system blocking probabilities and the wavelengths in use can be significantly reduced. It has been an active research topic on the placement of converters in multihop systems, such that either the system blocking probabilities or the wavelengths used in the system can be minimized. Most of the work focuses on minimizing blocking probabilities or improving system throughput, such as [12, 17, 20, 21, 22, 24]. Some work is on minimizing the wavelengths in use, such as in [11, 23].

Our work is to find the minimal number of converters and their placement in a general topology network to achieve load-wavelength assignability. This work is inspired by [23], which discusses the problem in star and ring networks. The important result obtained in [23] is that to achieve load-wavelength assignability, for unidirectional channels, no converter is required in star networks and only one converter is needed in ring networks. The results obtained in this paper (indicated by stars), together with those obtained in [23], are summarized in Table 2.

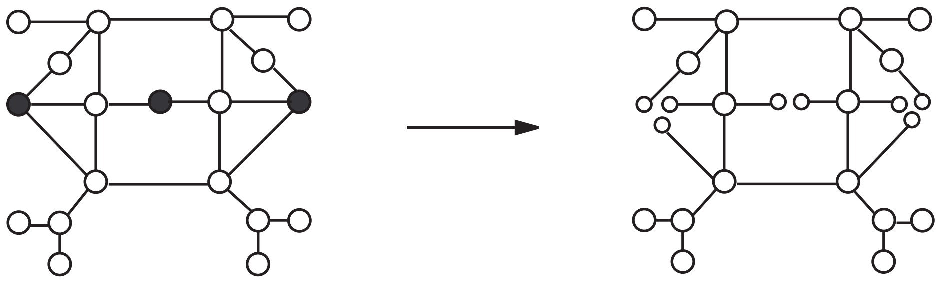

Fig. 1. (a) The original graph $G(V,E)$ and S consists of black nodes. (b) The obtained graph $G_S(V,E)$ from splitting nodes in S.

III. ALGORITHMS

A. The Methodology

The study of load-wavelength assignment in special networks, such as paths, stars, and rings, can be done easier than that in general topology networks. Our approach is to decompose a general topology network into edge-disjoint simple subgraphs, such as paths or spiders. A *spider* is a tree that consists of several paths (called *legs*) with one end of each of these paths incident to a common node (called *body*). Paths and stars are two special cases of spiders.

The decomposition of a graph is done by a *splitting operation* [23] described as the follows. See Fig. 1 for its illustration. Given a graph $G(V,E)$ and a subset $S \subseteq V$, generate a new graph $G_S(V,E)$ by splitting each node $s \in S$ into $\delta(s)$ nodes, where $\delta(s)$ is the degree of s in $G(V,E)$. Each edge (s,t) in $G(V,E)$ becomes edge (s',t) in $G_S(V,E)$, where s' is a newly generated node from splitting s in $G(V,E)$. By studying the obtained graph $G_S(V,E)$, we reduce the OPC problem in general topology networks into the cases of special networks, such as paths and spiders.

We will develop the necessary and sufficient condition of the OPC problem for duplex channels, and prove the problem can be solved efficiently. For unidirectional channels, we transform the OPC problem to a well known vertex cover problem, which is NP-complete [5], and then propose a 2-approximation algorithm to solve the OPC problem.

B. OPC Problem for Duplex Channels

LEMMA 1 *Given graph $G(V,E)$, subset $S \subseteq V$ achieves load-wavelength assignability if and only if every connected component of $G_S(V,E)$ is a path.*

PROOF. "If": Given any set of routed channels with the maximal link load L in $G(V,E)$, consider them in $G_S(V,E)$. These channels that go through nodes in S are broken into several pieces, and each connected component is a path in $G_S(V,E)$. The channels that go through the entire or part of a path can be assigned wavelengths in the following way. See Fig. 2 for its illustration.

Step 1. Order these channels from left to right according to the starting points in the path.

Step 2. Assign each of these channels the wavelength which is free and has the least value. A wavelength becomes free once its assigned channel terminates.

Since the maximal link load is L, it is obvious that Step 2 uses at most L wavelengths to assign wavelengths to channels without causing wavelength conflict. Moreover, the wavelength assignment in each connected component of $G_S(V,E)$ is independently, and there is no wavelength conflict between the channels in different components. This is because each node in S has the full wavelength conversion capability.

"Only if": we prove it by contradiction. Assume that one of the connected components in $G_S(V,E)$ is not a path. There are only two possibilities of this component: 1) at least one node in the component has degree greater than two, or 2) the component is a ring. In both cases, it is easy to find an example where three channels require three wavelengths even though the maximal link load is two. This contradicts the load-wavelength assignability. ■

THEOREM 1 *The OPC problem for duplex channel can be solved in time of $O(|E|+|V|)$.*

PROOF According to Lemma 1, the solution to OPC problem for a given $G(V,E)$ is to identify the set S which makes every component of $G_S(V,E)$ a path, and equip every node in S with a converter. The algorithm for identifying such a set S is to find out all the nodes whose degree is greater than two in $G(V,E)$, or to break the ring if $G(V,E)$ is a ring. Therefore, the algorithm can simply search $G(V,E)$. If $G(V,E)$ is a ring, then let S include any node in V; Otherwise let S include all nodes whose degree is greater than two. It is not difficult to see that S, constructed in such a way, is the minimal to make every component of $G_S(V,E)$ a path, and it can be obtained in linear time of $O(|E|+|V|)$. Hence, S is an optimal solution to the OPC problem. ■

C. OPC Problem for Unidirectional Channels

Because wavelength conflict condition for unidirectional channels is different from that for duplex channels, OPC problem becomes more complicated for unidirectional channels. It has been pointed out in [23] that for unidirectional channels there is no need of converter in star networks to achieve load-wavelength assignability. Based on this result, we have the following lemma.

LEMMA 2 *Given graph $G(V,E)$, subset $S \subseteq V$ achieves load-wavelength assignability if and only if each connected component in $G_S(V,E)$ is a spider.*

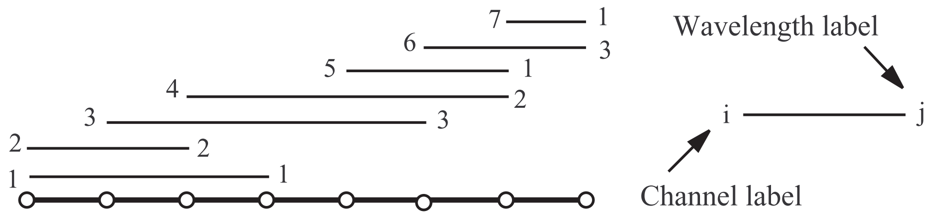

Fig. 2. Wavelength assignment for channels in a path.

PROOF. "If": Given any set of routed channels with the maximal link load L in $G(V,E)$, consider them on $G_S(V,E)$. Each connected component in $G_S(V,E)$ is a spider. See Fig. 3 for the illustration, where the spider body is marked by the black node. The channels on the spider are in either of the two cases: 1) the channels crossing the spider body from one leg to another; or 2) the channels originating from and terminating at the same leg. We consider each of them as the following:

Case 1. Channels crossing from one leg to another. We introduce a bipartite multigraph $B(V_1 \cup V_2, E_{12})$ to represent these channels in the spider, where V_1 and V_2, $V_1 = V_2$, are the set of legs (i.e., each leg of the spider is represented as a node in V_1 and V_2). Each edge $e_{ij} \in E_{12}$ if there is a channel which goes through legs i and j for $i \neq j$. There can be more than one edge between two nodes in $B(V_1 \cup V_2, E_{12})$. For example in Fig. 3(b), there are two edges between node 1 and node 2, as there are two channels going from leg 1 to leg 2. Assigning wavelengths to the channels without causing wavelength conflict becomes the problem of coloring the edges of graph $B(V_1 \cup V_2, E_{12})$ under the condition that two adjacent edges must be in different colors. Since there are at most L channels going from leg i to leg j for $i \neq j$, multigraph $B(V_1 \cup V_2, E_{12})$ has the maximum degree L. Thus, $B(V_1 \cup V_2, E_{12})$ can be colored by using at most L colors (refer to any book on graph theory for details, e.g., [2]).

Case 2. Channels originating from and terminating at the same leg. Consider each of the legs separately. See Fig. 3(c). Note that each channel belonging to Case 1 is cut into two parts, one in a leg heading the body and the other in another leg leaving the body. We use the same method as described in the proof of Lemma 1 for assigning wavelengths to channels in a path. The channels heading the body of the spider are ordered from left to right by their destinations or the cutting point at the spider body (see Fig. 3(c)), while the channels leaving the spider body are also ordered from left to right by their sources or the spider body. Ties are broken arbitrarily. The channels which have been assigned wavelengths in Case 1 retain the same wavelengths, and the rest of the channels in this leg will be assigned the wavelengths which are free. Thus, all channels (in both Case 1 and Case 2) can be assigned with at most L wavelengths.

For each of those channels that traverse different connected components in $G_S(V,E)$, it can be assigned different wavelengths and this will not cause wavelength conflict, because each node in S has full wavelength conversion capability and this makes the wavelength assignment at each component independent from the others.

"Only if": we prove it by contradiction. Assume that one of the connected components of $G_S(V,E)$ is not a spider. There are only two possibilities of the component. Case 1: There are two nodes having degree greater than two. Case 2: The component is a ring. In both cases, it is not difficult to find a set of channels that yield network load two and require three wavelengths. This contradicts load-wavelength assignability. ∎

Lemma 2 tells us what S consists of. However, it is NP-complete to find the minimal sized S that achieves load-wavelength assignability. The next two lemmas will lead to an approximation algorithm to find the minimal sized S.

LEMMA 3 *If graph $G(V,E)$ has at least one node with degree greater than two, then there exists a minimal sized subset $S \subseteq V$ that achieves load-wavelength assignability and every node in S has degree greater than two in $G(V,E)$.*

PROOF. Let $v \in V$ be the node with degree greater than two, and S be a minimal sized subset of V that achieves load-wavelength assignability. Now assume there is a node $s \in S$ with degree less than or equal to two. Let u be the closest node to s in a path between v and s, and u has degree greater than two (u can be v itself). We consider the following two cases of u:
Case 1. $u \in S$. Let $S' = S \setminus \{s\}$. It is obvious that each connected component of $G_{S'}(V,E)$ is still a spider, because s is a node of degree less than or equal to two.
Case 2. $u \notin S$. Let $S' = S \cup \{u\} \setminus \{s\}$. It is easy to see that each connected component of $G_{S'}(V,E)$ is still a spider, because substituting s with u only makes one component of $G_S(V,E)$ have a longer leg and another split into several paths.

In either cases, S' can achieve load-wavelength assignability because of Lemma 2. In Case 1, S' is a proper subset of S, this contradicts that S has the minimal size. In Case 2, the substitution operation can be repeated until all nodes in S having degree greater than two. The desired subset can thus be obtained. ∎

Lemma 3 allows us to eliminate all nodes whose degree is less than or equal to two in $G(V,E)$ from consideration when searching for the minimal sized S. Even we reduce graph $G(V,E)$ into another graph $G'(V,E)$ by removing from G all the nodes having degree less than or equal to two, it is still NP-hard to find the minimal sized S in G'. Lemma 4 transforms the OPC problem to the well-known vertex cover problem.

DEFINITION 2 *A vertex cover of graph $G(V,E)$ is a set of vertex $C \subseteq V$ such that each edge of G has at least one endpoint in C. The vertex cover problem is to find a vertex cover of the minimal size.*

LEMMA 4 *If every node in $G(V,E)$ has degree greater than two, then $S \subseteq V$ achieves load-wavelength assignability if and only if S is a vertex cover of $G(V,E)$.*

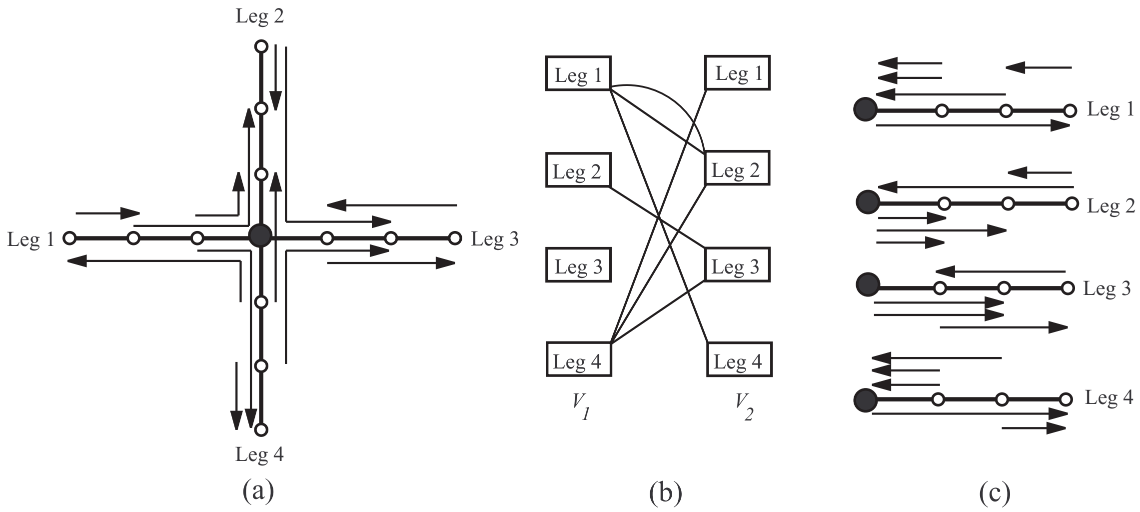

Fig. 3. (a) Wavelength assignment in a spider. (b) Assigning wavelengths to channels crossing the body. (c) Assigning wavelengths to channels within legs.

PROOF. "If": Since S is a vertex cover of $G(V, E)$, every connected component of $G_S(V, E)$ must be a spider (actually a star). From Lemma 2, we have that S achieves load-wavelength assignability.

"Only if": Assume, by contradiction, that there are two nodes u and v in $G(V, E)$ such that edge $(u, v) \in E$ is not incident to any node in S. Then the connected component in $G_S(V, E)$ which contains edge (u, v) is not a spider, because it has two nodes (u and v) with degree greater than two. This contradicts Lemma 2. ■

Lemma 4 tells us that finding the minimal sized subset $S \subseteq V$ to achieve load-wavelength assignability is equivalent in finding the minimal vertex cover of $G(V, E)$. Notice that Lemma 4 requires every node in $G(V, E)$ has degree greater than two. If $G(V, E)$ has degree-one or degree-two nodes, Lemma 4 does not hold any more.

Now we are ready to design an algorithm to find a set S of minimal size that can achieve load-wavelength assignability. Given graph $G(V, E)$, if no node in G has degree greater than two, then G is either a path or a ring. In the case of a path, there is no need of any converter. In the case of a ring network, putting one converter (at any node) in the ring can achieve load-wavelength assignability. Consider the case that G has at least one node with degree greater than two. According to Lemma 3, we do not need to consider degree-one or degree-two nodes. Thus, we first remove every degree-two node in V by substituting two edges incident to the node with one edge linking the two endpoints directly. See Fig. 4 for a simple illustration of the method. Three degree-two nodes (grey nodes in Fig. 4(a)) are removed, resulting in the graph as shown in Fig. 4(b). Then, each degree-one node is removed by contracting it to the node adjacent to it. See Fig. 4(b) for the removal of degree-one nodes in grey, resulting in Fig. 4(c). After these two operations we obtain an induced graph with fewer nodes and edges. See Fig. 4(c). Notice that this induced graph may still have degree-two or degree-one nodes, but they are all contracted nodes and their degrees are greater than two in the *original network graph*. Finally, we employ an algorithm for the vertex cover problem on the induced graph, and obtain a vertex cover C. Set C is able to achieve load-wavelength assignability of the original graph. As the example in Fig. 4(c), a vertex cover of six nodes (in black) is found, which achieves the load-wavelength assignability.

The proposed algorithm is formally presented as below, and its validity is proved in the theorem following the algorithm.

Input A graph $G(V, E)$
Output A set $C \subseteq V$ achieving load-wavelength assignability
Step 0 Process simple cases
  if $G(V, E)$ is a ring, *then*
    return C including one node in V.
  if $G(V, E)$ is a path, *then*
    return $C := \emptyset$.
  if $G(V, E)$ is a spider, *then*
    return $C := \{v \in V | v$ has degree greater than two$\}$.
Step 1 Remove degree-two nodes
  $V^2 := \{v \mid v$ is adjacent to exactly two nodes v_1 and $v_2\}$.
  while $V^2 \neq \emptyset$ *do begin*
    choose $v \in V^2$,
    $V := V \setminus \{v\}$.
    $E := E \cup \{(v_1, v_2)\} \setminus \{(v_1, v), (v, v_2)\}$,
    $V^2 := V^2 \setminus \{v\}$.
  end-while
Step 2 Remove degree-one nodes
  $V^1 := \{u \mid u$ is adjacent to exactly one node $u_1\}$.
  while $V^1 \neq \emptyset$ *do begin*
    choose $u \in V^1$,
    $V := V \setminus \{u\}$.
    $E := E \setminus \{(u_1, u)\}$,
    $V^1 := V^1 \setminus \{u\}$.
  end-while
Step 3 Construct a vertex cover
  construct a vertex cover C of the reduced graph $G(V, E)$,
  return C.

In the following analysis of the algorithm, we do not need to

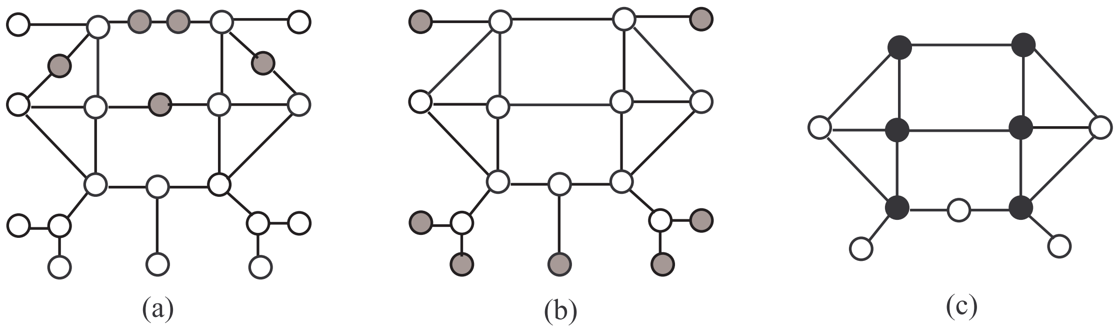

Fig. 4. The proposed algorithm: (a) remove degree-two nodes, (b) remove degree-one nodes, and (c) find a vertex cover of the induced graph.

consider the case that $G(V,E)$ is a spider or a ring, since according to Lemma 2 the algorithm can find an optimal solution in this special case.

THEOREM 2 *The proposed algorithm can produce a set C that achieves load-wavelength assignability for network $G(V,E)$.*

PROOF. Let $G_2(V_2,E_2)$ be the final graph obtained after Step 2. To prove the theorem, it suffices to show that every vertex cover C of $G_2(V_2,E_2)$ achieves the load-wavelength assignability for the original network $G(V,E)$. According to the way that $G_2(V_2,E_2)$ is constructed, it is not difficulty to verify that every connected component of $G_C(V,E)$ is a spider. Thus it follows from Lemma 2 that C achieves load-wavelength assignability for network $G(V,E)$. ■

THEOREM 3 *The proposed algorithm can produce a set C in time of $O(|E|+|V|)$ satisfying $|C| \le 2|S|$, where S is an optimal solution to OPC problem.*

PROOF. It is obvious to see that Step 0-1-2 can be finished in time of $O(|E|+|V|)$. In Step 3, the vertex cover C of $G_2(V_2,E_2)$ can be found in time of $O(|E|+|V|)$ by the following method:

Step 1. construct a maximal matching M of $G_2(V_2,E_2)$ such that any pair of edges in M do not share an endpoint; Moreover, any edge in $E_2 \setminus M$ shares an endpoint with an edge in M.

Step 2. construct C by including both endpoints of each edge in matching M.

Note that every edge in $E_2 \setminus M$ has at least one endpoint matched in M; Otherwise the edge could be added to M to provide a larger matching. This implies that every edge in E_2 has at least one endpoint that is matched and thus C is a vertex cover and can be produced in time of $O(|E|+|V|)$. Hence the proposed algorithm can be finished in time of $O(|E|+|V|)$.

Now we prove $|C| \le 2|S|$. By Lemma 3 we can assume that $S \subseteq V_2$. In fact, we can further assume that S is a vertex cover of $G_2(V_2,E_2)$. If not, there exist two nodes $w_1 \in V_2$ and $w_2 \in V_2$ such that $(w_1,w_2) \in E_2$ with $w_1 \notin S$ and $w_2 \notin S$. According to the rules of the algorithm, w_1 and w_2 has degree greater than two in $G(V,E)$ (they may have degree one or two in $G_2(V_2,E_2)$). Moreover, they are in one connected component of $G_S(V,E)$, because removing degree-one and degree-two nodes from $G(V,E)$ does not destroy its connectivity. This contradicts Lemma 2. To see $|C| \le 2|S|$, consider the edges in maximal matching M. To cover these edges we need at least $|M|$ nodes, since no two of them share a endpoint. This implies that the minimal vertex cover has size at least $|M|$ and thus cover C contains exactly $2|M|$ nodes. Hence $|S| \ge |M| = |C|/2$. ■

As Lemma 4 shows that the OPC problem is equivalent to the vertex cover problem, which is believed unlikely to have approximation algorithms with performance ratio less than two [9], the proposed algorithm for OPC problem is the best possible. Furthermore, the proposed algorithm can find an optimal solution to OPC problem in polynomial-time when graph $G(V,E)$ has some special topologies, such as tree, mesh, torus, and hypercube, because the minimal vertex cover can be computed efficiently in these cases. In the following, we discuss the optimal solutions for the networks with special topologies.

COROLLARY 1 *The proposed algorithm can find an optimal solution to the OPC problem in a tree network in polynomial-time.*

PROOF. When applying the proposed algorithm on a tree, an induced tree is obtained after Step 2. The minimal vertex cover can be found in polynomial-time in Step 3 by the following greedy method:

Step 1. Include all nodes that are adjacent to degree-one node in cover C.

Step 2. If all nodes in the induced tree are covered, then return cover C; else remove all covered nodes from current tree and go to Step 1.

Thus an optimal solution of OPC problem can be found in polynomial-time. ■

COROLLARY 2 *The proposed algorithm can find an optimal solution to the OPC problem in a network of mesh in polynomial-time.*

PROOF. When applying the proposed algorithm on a mesh, an induced mesh is obtained after Step 1 by removing all degree-two nodes. Obviously, an minimal vertex cover can be located in polynomial-time. ■

COROLLARY 3 *The proposed algorithm can find an optimal solution to the OPC problem in a network of torus or hypercube in polynomial-time.*

PROOF. Since all nodes in a torus or a hyper-cube have degree greater than two, Step 1 and Step 2 are skipped. It is easy to see that the minimal vertex cover of a torus is the set that includes every other node in each row and column. The minimal vertex cover of a hyper-cube is such a set that if a node is in the set then its adjacent nodes are not. ■

IV. CONCLUSIONS

We have characterized the optimal solutions to the OPC problem for duplex channels and presented an efficient approximation algorithm for unidirectional channels. The proposed approximation algorithm has the approximation ratio of two, which is believed to be the best result obtained so far [9].

The results obtained in this paper have significant impact to the design of multihop WDM networks. Firstly, they can help understand the relationship between the number of wavelengths required and the placement of converters. Secondly, they can be used to guide the placement of converters at the design of a network. Thirdly, they can help determine the maximal traffic load that the network can support, given the network topology and converter placement.

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