

# Energy Efficient Broadcast Routing in Static Ad Hoc Wireless Networks

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**Abstract**—In this paper, we discuss energy efficient broadcast in ad hoc wireless networks. The problem of our concern is: Given an ad hoc wireless network, find a broadcast tree such that the energy cost of the broadcast tree is minimized. Each node in the network is assumed to have a fixed level of transmission power. We first prove that the problem is NP-hard and propose three heuristic algorithms, namely, shortest path tree heuristic, greedy heuristic, and node weighted Steiner tree-based heuristic, which are centralized algorithms. The approximation ratio of the node weighted Steiner tree-based heuristic is proven to be  $(1 + 2\ln(n - 1))$ . Extensive simulations have been conducted and the results have demonstrated the efficiency of the proposed algorithms.

**Index Terms**—Ad hoc wireless networks, energy efficient, broadcast routing, heuristic algorithm.

## 1 INTRODUCTION

WIRELESS ad hoc networks have received significant attention in recent years due to their potential applications in battlefield, disaster relief operations, and so on. A wireless ad hoc network consists of a collection of mobile hosts dynamically forming a temporary network without the use of any existing network infrastructure. In such a network, each mobile host can serve as a router. A communication session is achieved either through a single-hop transmission if the communication parties are close enough or through relaying by intermediate nodes otherwise. The selection of relay nodes is a major issue in routing algorithm design. Another important issue in the routing in ad hoc networks is the energy consumption. In ad hoc networks, mobile hosts are powered by batteries and it may be impossible to recharge or replace batteries during a mission. Therefore, the limited battery lifetime imposes a constraint on the network performance. In order to maximize the network lifetime, the traffic should be routed in such a way that the energy consumption is minimized.

In this paper, we address the problem of broadcast routing in ad hoc wireless networks. The nodes in wireless networks communicate via radio signals, which are broadcast in nature. When omnidirectional antennas are used, every transmission by a transmission node can be received by all nodes within its transmission range. Consequently, if multiple nodes are within the immediate communication vicinity of a transmitting node, a single transmission can reach all these receivers.

Broadcast is a communication function that a node, called the source, sends messages to all the other nodes in the networks. Broadcast is an important function in applications

of ad hoc networks, such as in cooperative operations, group discussions, and so on. Broadcast routing is finding a broadcast tree, which is rooted from the source and contains all the nodes in the network. The energy cost of a broadcast is defined as the sum of energy cost of all the nodes that transmit the broadcast message in the broadcast tree.

The problem of our concern is: Given an ad hoc wireless network in which each node has a fixed energy and a broadcast group, find a broadcast tree such that the energy cost of the broadcast tree is minimized. We first prove that the problem is NP-hard and then propose three heuristic algorithms, namely, shortest path tree heuristic, greedy heuristic, and node weighted Steiner tree-based heuristic. The approximation ratio of the node weighted Steiner tree-based heuristic is proven to be  $(1 + 2\ln(n - 1))$ . The three proposed algorithms are centralized, which require the network topology information in prior. Extensive simulations have been conducted and the results have demonstrated the efficiency of the proposed algorithms.

## 2 RELATED WORK

The energy-efficiency problem in wireless network design has received significant attention in the past few years. Some works are on the configuration of a network topology with good (or required) connectivity by using minimal power consumption [1], [2], [3], such as minimizing the maximum power of nodes or minimizing the total power consumption of all nodes. Some other works about energy-efficient broadcast are focused on routing protocols, such as in [4], [5], [6]. These routing protocols are distributed routing algorithms that select routes with less energy cost. There is no theoretical analysis or guarantees over the performance.

Most of the works on energy efficient broadcast/multicast are focused on configuring energy power of each node. That is, given the geometry position of a set of nodes in a plane, find the transmitting power of each node such that the energy cost of the broadcast/multicast tree is minimized [7], [8], [9], [10], [11]. Some fundamental issues associated

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with energy-efficient multicast were discussed in [7] and several multicast schemes were proposed and evaluated. In [8] and [9], some energy-efficient broadcast/multicast algorithms were proposed, namely, the Broadcast Incremental Power (BIP), Multicast Incremental Power (MIP) algorithms, MST (minimum spanning tree), and SPT (shortest-path tree). The proposed algorithms were evaluated through simulations, but little is known about their analytical performances in terms of the approximation ratios. The authors of [10] got the quantitative characterization of performances of these three greedy heuristics. By exploring geometric structures of Euclidean MSTs, they proved that the approximation ratio of MST is between 6 and 12, the approximation ratio of BIP is between  $13/3$  and 12, and the approximation ratio of SPT is at least  $n/2$ , where  $n$  is the number of receiving nodes. In [11], the problem of broadcasting in large ad hoc wireless networks was discussed and a method MLE (Minimum Longest Edge) based on MST was proposed. This algorithm provided a scheme to balance the energy consumption among all nodes.

The solutions developed in [7], [8], [9], [10], [11] are mainly based on geometry features of the nodes in the plane. Some other solutions are based on graph theory (i.e., based on the connectivity among the nodes in the network), such as in [12], [13], [14]. In [12], the minimum-energy broadcast problem was addressed and proven to be NP-hard in general and an  $O(n^{k+2})$  algorithm was proposed for the problem under the assumption that each node is able to reach all the other nodes in the network, where  $n$  is the number of nodes and  $k$  is the number of transmitters. The algorithm is a not polynomial time algorithm when  $k = n$ . The works in [13], [14] assume each node has a limited number of adjustable power levels. In [13], the authors first gave a formal proof of the NP-hardness of the problem for both geometry version and graph version. A heuristic based on MST algorithm was proposed, but no performance ratio was given. In [14], another heuristic algorithm based on the directed Steiner tree method was proposed. The performance ratio of the proposed algorithm is  $n^\varepsilon$ , where  $\varepsilon$  is a constant between 0 and 1. For the special case of the problem where each node has a fixed level of power, a heuristic with performance ratio  $\log^3 n$  was proposed.

The problem that we consider in this paper is similar to works in [13], [14]. We assume each node has a preconfigured transmission power and we aim at, for each broadcast request, finding a broadcast tree that has the minimum energy consumption. This is a more practical issue in real systems because each node would have a transmission power after the network is configured and this power level will not be changed for each broadcast request. We present three algorithms, one of which has performance ratio  $(1 + 2\ln(n-1))$ , which is better than the result  $\log^3 n$  obtained in [14] (the algorithm proposed in [13] does not even have a performance ratio).

### 3 NETWORK MODEL AND PROBLEM SPECIFICATION

The network is modeled by a directed graph  $G = (V, A)$ , where  $V$  represents the set of wireless nodes,  $A$  represents the set of arcs in the network. For each node  $v \in V$ , there is a

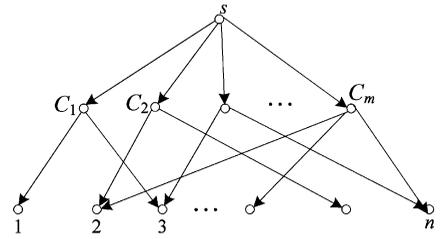


Fig. 1. The auxiliary graph for Theorem 1.

given level of transmission power  $p(v)$ . For any two nodes  $v_1$  and  $v_2$ , if  $v_2$  is in the transmission power range of  $v_1$  (that is  $d^\alpha(v_1, v_2) \leq p(v_1)$ , while  $d(v_1, v_2)$  is the distance between  $v_1$  and  $v_2$ , and  $\alpha$  is a value between 2 and 4), then there is an arc  $(v_1, v_2) \in A$  (i.e., a directed link from node  $v_1$  to node  $v_2$ ).

Suppose  $T$  is a broadcast directed tree sourced from  $s$ . There are two kinds of nodes in  $T$ : the nodes that need to transmit/relay broadcast messages for  $s$  and the nodes that only receive broadcast messages from  $s$ . We assume that a node costs energy only when it does transmissions. Let  $NL(T)$  denote the set of nonleaf nodes of  $T$ . The total energy cost  $C(T)$  of  $T$  can be represented as:

$$C(T) = \sum_{v \in NL(T)} p(v). \quad (1)$$

Our problem is: Given a broadcast request sourced from  $s$  and  $p(v)$  for each node  $v$ , find a broadcast tree rooted from  $s$  such that total energy cost of the tree is minimized. We call this problem the MEB (Minimum Energy Broadcast) problem for short.

We assume the node position is static or changed slowly. Node mobility is not considered in this paper. The ad hoc networks are quite different from the wired networks due to the nature of wireless communication and lack of infrastructure support. They pose many new challenges that are never seen in wired or cellular networks, even the mobility is not addressed.

### 4 ALGORITHMS

In this section, we first prove the MEB problem is NP-hard.

**Theorem 1.** *The MEB problem is NP-hard.*

**Proof.** We will show that the set cover problem is polynomial time reducible to the MEB problem.

The set cover problem is defined as the following: Given a set  $I$  of  $n$  elements,  $C = \{C_1, C_2, \dots, C_m\}$ ,  $C_j \subseteq I$ ,  $j \in \{1, 2, \dots, m\}$ , and a positive integer  $K$ . Does  $C$  contain a set cover for  $I$  of size  $K$  or less, i.e., a subset  $J \subseteq \{1, 2, \dots, m\}$  such that  $\bigcup_{j \in J} C_j = I$  and  $|J| \leq K$ ?

We construct a directed graph

$$G = (V, A).$$

$V = \{s\} \cup C \cup I$ , where  $s$  is the source of broadcast. For each  $C_j \in C$ , there is an arc  $(s, C_j)$ . For any  $i \in I$ ,  $C_j \in C$ , if  $C_j$  cover  $i$ , i.e.,  $i \in C_j$ , then there is an arc from  $C_j$  to  $i$  (see Fig. 1). We also assume that  $p(v) = 1$  for any node  $v \in V$ .

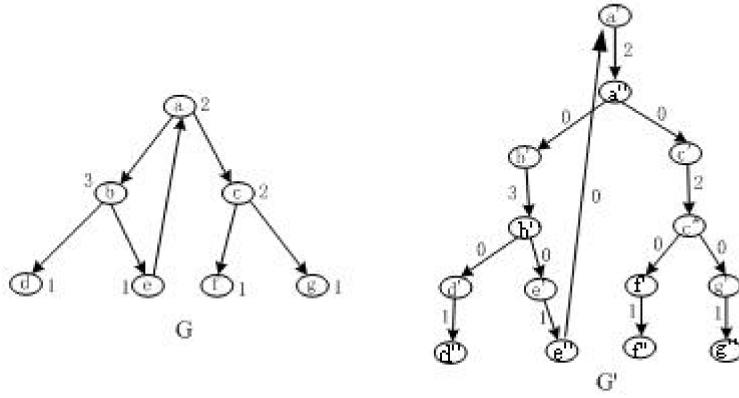


Fig. 2. The transformation  $G$  to  $G'$ .

In the following, we will prove that there is a set cover of size  $k$  if and only if there is a broadcast tree rooted from  $s$  and its energy cost is  $1 + k$ .

“only if”: Suppose  $\{C_j, C_{j_2}, \dots, C_{j_k}\}$  is a set cover of  $I$ . That is,  $\bigcup_{1 \leq t \leq k} C_{j_t} = I$ . We construct a broadcast tree  $T$  in which  $s$  and  $C_{j_1}, C_{j_2}, \dots, C_{j_k}$  transmit the broadcast message. The energy cost  $C(T)$  is equal to  $1 + k$ .

“if”: Suppose  $T$  is a broadcast tree rooted from  $s$ , whose energy cost is  $1 + k$ . Since  $T$  must span  $I$ , so

$$\{C_j | \text{out-degree of } C_j \text{ is at least one in } T\}$$

must cover all nodes in  $I$ , therefore

$$\{C_j | \text{out-degree } C_j \text{ is at least one in } T\}$$

is a set cover of  $I$ . Because

$$C(T) = 1 + |\{C_j | \text{out-degree } C_j \text{ is at least one in } T\}|,$$

and  $C(T) = 1 + k$ , therefore the size of this set cover is  $k$ .

Since the minimum set cover problem is NP-hard [15], the MEB problem is NP-hard.  $\square$

In the next section, we propose three heuristic algorithms. One is based on the directed Steiner tree. The second is a greedy heuristic algorithm. The third is based on the node weighted Steiner tree problem.

#### 4.1 Transforming the MEB to Directed Steiner Tree Problem

In ad hoc networks, the MEB problem is to find a broadcast tree such that the total energy cost of those transmitting nodes is minimized. In our network model, we assign the transmission power of a node as the weight of it. We first transform the network graph  $G$  to a new graph  $G'$  that has weight on arcs.

For each node  $v \in V$ , we split  $v$  into two nodes  $v'$  and  $v''$ , and connect them with a new arc from  $v'$  to  $v''$ . For each arc  $(v', v'')$ , its weight, denoted by  $c((v', v''))$ , is assigned to  $p(v)$ , the weight of  $v$ . For each arc  $(v_1, v_2) \in A$ , let  $(v'_1, v'_2)$  be a new arc in  $G'$  and give it a weight 0. We get a new directed graph  $G'$ , where  $G' = (V' \cup V'', A')$ ,  $V' = \{v' | v \in V\}$ ,  $V'' = \{v'' | v \in V\}$ , and

$$A' = \{(v', v'') | v \in V\} \cup \{(v'_1, v'_2) | (v_1, v_2) \in A\}.$$

Fig. 2 is an example of transforming a network graph  $G$  to  $G'$ . The number at each node in  $G$  is the weight of the node, while the numbers on the arcs in  $G'$  are the weights of the arcs.

The MEB problem in  $G$  can be transformed to the following problem in  $G'$ : finding a directed tree  $T$  in  $G'$  that is rooted from  $s'$  and spans all the nodes in  $V'$  such that the sum of the weights of all arcs in  $T$  is minimized.

This is a typical directed Steiner tree problem. The directed Steiner tree problem has been well-studied and several heuristics have been proposed ([16], [17]). Any heuristic of the directed Steiner tree problem can be used to find the solution to our problem. In the simulation, we use the shortest path tree-based heuristic to compute the directed Steiner tree problem on  $G'$  to get a broadcast tree on  $G$ .

#### 4.2 Greedy Heuristic

In this section, we propose a greedy heuristic. The problem can be transformed to a special kind of set cover problem that requires the set cover to form a connected and directed tree rooted at source. The traditional algorithms, such as [18], for the set cover problem cannot be applied directly to our problem.

We introduce two sets: One is cover-set, containing the nodes which transmit/relay messages, the other is covered-set, containing the nodes that are outgoing neighbors of the nodes in the cover-set. This heuristic starts from  $s$ . First,  $s$  is selected to the cover-set and all outgoing neighbors of  $s$  are included into the covered-set. The next, a node in the covered-set, is selected into the cover-set (the selection criteria is explained the next) and its outgoing neighbors are included into the covered-set. This operation is repeated until all nodes in  $V - \{s\}$  are in the covered-set.

In order to choose the nodes into cover-set such that the total energy cost of the broadcast tree is minimized, we introduce the following function:

$$|V_i \cap U| / p(v_i), \quad (2)$$

where  $V_i = \{v_j | (v_i, v_j) \in A\}$  is a set of outgoing neighbors of node  $v_i$  and  $U$  a set of nodes which are not covered so far. Notice that  $v_i \notin V_i$ .

This function represents the number of nodes that a node can cover per energy unit. Each time, a node with the maximum value of this function will be selected into the

cover-set. By doing so, the total energy cost of the broadcast tree can thus be kept small.

The algorithm is formally presented as the following:

**Input**  $G = (V, A)$  and  $s$ .

**Output**  $T$ : a broadcast tree rooted from  $s$ .

$C = \{s\}$ ; //  $C$ : the cover-set

$D = V_s$ ; //  $D$ : the covered-set

**While** ( $D \neq V - \{s\}$ ) **do**

Choose  $v_i \in D - C$  such that

$\max(|V_i \cap (V - (\{s\} \cup D))|/p(v_i))$

$C = C \cup \{v_i\}$ ;

$D = D \cup V_i$

Construct the broadcast tree  $T$  from  $C$ .

**Theorem 2.** Given  $G(V, A)$  and  $s$ , the greedy algorithm can output a broadcast tree in time  $O(n^2)$ .

**Proof.** It is easy to know that the greedy algorithm can output a broadcast tree. In while-loop, there is at most  $n$  loop and, for each loop, finding max value takes  $O(n)$ , then the while-loop can finish in the time of  $O(n^2)$ , and the construction of a broadcast tree in the last line takes the time of  $O(|V|^2) = O(n^2)$ . Therefore, the whole algorithm ends in the time  $O(n^2 + n^2) = O(n^2)$ .  $\square$

### 4.3 A Node-Weighted Steiner Tree-Based Heuristic

In the above greedy heuristic, the algorithm constructs the broadcast tree  $T$  in a top-down fashion, starting from the source node. In contrast, the following heuristic takes a global approach, starting from any node in the network, to construct a broadcast tree that has efficient energy cost.

The basic idea mimics the strategy used in the node weighted Steiner tree problem [19], [20]. In the following, we first introduce some notations.

Let  $s$  be the source node of the broadcast. We first give an ID for each node in  $V$ . For any set  $U$  of nodes which contains  $s$ , let  $H_U$  be the subgraph of  $G$  in which an arc  $e$  of  $G$  is present if and only if the initial node of  $e$  belongs to  $U$ . A strongly connected component of  $H_U$  is said to be an *orphan* if it does not contain the source  $s$  and has no incoming arc. For each orphan component, the node with the smallest ID in this component is referred to as its *head*.

Let  $T$  be an arbitrary arborescence. The cost of  $T$  is defined as the power cost of the root plus the sum of distances to the sinks from root, where the distance along a path is the sum of weights of nodes in this path and does not include the weights of its endpoints. Note that the cost of arborescence is no less than the total energy cost of  $T$ . Fix a set  $U$  of nodes which contains  $s$ . An arborescence is said to be legal (with respect to  $U$ ) if 1) all the sinks are heads and 2) it contains at least two heads if the root is not  $s$  (a root is also counted as a head if it is a head). The quotient cost of a legal arborescence is defined as the ratio of its cost to the number of heads in this arborescence. A *min-quotient legal arborescence* rooted at  $v$  is a legal arborescence rooted at  $v$  with the smallest quotient cost. The quotient cost of a node  $v$  is defined as the quotient cost of a min-quotient legal arborescence rooted at  $v$ . To find the min-quotient legal arborescence rooted at a node  $v$ , 1) compute the shortest (directed) paths in  $G$  from  $v$  to all heads and 2) sort these

paths in the nonincreasing order of the length. Then, the min-quotient legal arborescence must be the union of the first  $i$  shortest paths for some  $i \geq 1$  if  $v$  is the source  $s$  or some  $i \geq 2$  otherwise. Thus, the min-quotient legal arborescence rooted at  $v$  can be computed in polynomial time as is the quotient cost of  $v$ .

Now, we are ready to describe our greedy algorithm. The algorithm maintains a set  $U$  of nodes which will be used for transmission and the set of strongly connected component of  $H_U$ . Initially,  $U$  consists of only the source node  $s$ . Thus,  $H_U$  contains  $n$  components, each component consist of a single node. All components except the one containing the source are orphans. The algorithm uses a greedy strategy iteration. First, it selects a node  $v$  of the smallest quotient cost, next, adds the nonsink nodes of the min-quotient legal arborescence rooted at  $v$  to  $U$  (we denote the nonsink nodes of the min-quotient legal arborescence rooted as  $v$  as  $U(v)$ ), and then updates  $H_U$  and the set of components. This operation is repeated until there is no orphan component. When there is no orphan component, a spanning arborescence is constructed in  $H_U$ , which is used for broadcast routing.

The algorithm is formally presented as the following:

**Input**  $G = (V, A)$  and  $s$ .

**Output**  $T$ : a broadcast tree rooted from  $s$ .

$U = \{s\}$ ; //  $U$ : transmitting node set

$O = \{\{i\} | i \in V - \{s\}\}$ ; //  $O$ : the set of orphan component

**While** ( $|O| \neq 0$ ) **do**

Choose  $v$  with smallest quotient cost;

$U = U \cup U(v)$ ;

Update  $H_U$  and  $O$ ;

Recalculate quotient cost for each node.

Construct the broadcast tree  $T$  from  $U$ .

**Theorem 3.** Given  $G(V, A)$  and  $s$ , the greedy algorithm can output a broadcast tree in time  $O(n^4)$ .

**Proof.** It is easy to know that the greedy algorithm can output a broadcast tree. In while-loop, there is at most  $n$  loop and, for each loop, finding max value takes  $O(n)$ , calculating the quotient cost for all nodes needs  $O(nm^2)$ , then the while-loop can finish in the time of  $O(n^4)$ , and the construction of a broadcast tree in the last line takes the time of  $O(|V|^2) = O(n^2)$ . Therefore, the whole algorithm ends in the time  $O(n^4 + n^2) = O(n^4)$ .  $\square$

Next, we prove that the approximation algorithm of this greedy heuristic is at most  $(2 \ln(n-1) + 1)$ . Let  $opt$  be the minimum total energy cost of the broadcast routing.

**Lemma 1.** The addition of a legal arborescence containing  $h$  heads reduces the number of orphan components by at least  $h/2$ .

**Proof.** After the addition of a legal arborescence containing  $h$  heads, each orphan component containing a head in this arborescence either gets merged into some new component or still survives but is not orphan any more. Furthermore, each new component which does not contain the root cannot be an orphan. If the root of the legal arborescence is not the source node  $s$ , then the reduction in the number of orphan components is at least

$h - 1 \geq h/2$  since  $h \geq 2$ . If  $v$  is the root of the legal arborescence is the source  $s$ , then the component containing  $s$  is not an orphan and, thus, the reduction in the number of orphan components is exactly  $h$ , which is greater than  $h/2$ .  $\square$

**Lemma 2.** *Suppose there are  $l$  orphan components. Then, there is a node  $v$  whose quotient cost is at most  $opt/l$ .*

**Proof.** Let OPT be a minimum cost spanning arborescence. The depth of a node is its hop distance from the root in OPT. An arborescence decomposition can be obtained iteratively as follows: At each iterative step, repeatedly prune the sinks which are not heads until all sinks are heads. Choose a node  $v$  of maximum depth such that the subtree rooted at  $v$  contains at least two heads. Such a subtree is a legal arborescence. Delete the subtree rooted at  $v$ . Repeat the iterations until the number of remaining heads is less than two. If there is one head left, the path from the source  $s$  to the remaining head is the last legal arborescence. Let  $A$  denote the legal arborescence with the smallest quotient cost. Since the total cost of all these arborescences is at most  $opt$  and the total number of heads contained in all these arborescences is exactly  $l$ , then the quotient cost of  $A$  is at most  $opt/l$ .  $\square$

**Theorem 4.** *The approximation ratio is at most  $2 \ln(n - 1) + 1$ .*

**Proof.** Suppose that the algorithm runs in  $k$  iterations. Let  $n_0 = n - 1$ , which is the number of initial orphan components. For any  $1 \leq i \leq k$ , let  $T_i$  be the legal arborescence added at iteration  $i$  and let  $n_i$  be the number of orphan components just after iteration  $i$ . Let  $c_i$  be the cost of  $T_i$  and  $h_i$  be the number of heads in  $T_i$ . Then, by Lemma 2,

$$\frac{c_i}{h_i} \leq \frac{opt}{n_{i-1}}.$$

By Lemma 1,

$$n_i \leq n_{i-1} - h_i/2.$$

Combining these two inequalities, we obtain

$$\frac{n_i}{n_{i-1}} \leq 1 - \frac{c_i}{2opt}.$$

Therefore,

$$\frac{n_{k-1}}{n_0} \leq \prod_{i=1}^{k-1} \left(1 - \frac{c_i}{2opt}\right).$$

Taking natural logarithms on both sides and then applying the inequality  $\ln(1 + x) \leq x$ , we obtain

$$\sum_{i=1}^{k-1} c_i \leq 2opt \ln \frac{n_0}{n_{k-1}}.$$

Note that  $n_0 = n - 1$  and  $n_{k-1} \geq 1$ . So, we have

$$\sum_{i=1}^{k-1} c_i \leq 2opt \ln(n - 1).$$

Now, we bound  $c_k$ . By Lemma 2,

$$\frac{c_k}{h_k} \leq \frac{opt}{n_{k-1}}.$$

Since iteration  $k$  is the last iteration,  $h_k = n_{k-1}$ . This implies that  $c_k \leq opt$ .

In summary, we have:

$$\sum_{i=1}^k c_i \leq (2 \ln(n - 1) + 1)opt.$$

Since the total power of all nodes in  $U$  is at most  $\sum_{i=1}^k c_i$ , the theorem follows.  $\square$

## 5 SIMULATION

In simulations, three algorithms, namely, the shortest-path tree based heuristic (SPT-h), Greedy heuristic (Greedy-h), and Node weighted Steiner tree-based heuristic (NST-h) are simulated and compared. We study how the total energy cost is affected by varying three parameters over a wide range: the number of nodes in the network ( $N$ ), the mean value of radius ( $R$ ).

The simulation is conducted in a  $100 \times 100$  2D free-space by randomly allocating  $N$  nodes ( $10 \leq N \leq 100$ ). The radius of transmitter range for each node is generated from a normal distribution with both mean and variance equal to  $R$ . The unit of  $R$  is with respect to the longest distance (i.e., diagonal distance) in the square region. We assume that transmission power ( $P$ ) relates to the node radius ( $r$ ) with function:  $P = r^\alpha$ ,  $\alpha$  is between 2 to 4. Therefore, we can get different topologies by varying these parameters.

We present averages of 100 separate runs for each result shown in the figures. In each run of the simulation, for given  $N$ ,  $R$ , and  $\alpha$ , we randomly place  $N$  nodes in the square, the radius of each node, a source node. Then, we run the three algorithms on this network. Any topology which is not connected is discarded.

Fig. 3 shows the total energy cost versus the number of nodes. From the curves in Fig. 3, we can make following observations:

1. Greedy-h performs the best all the time. The reason is that the NST-h searches for a low cost node at each step, but has no direction to the source. This makes the broadcast tree have more layers than the tree generated by Greedy-h. The Greedy-h and the NST-h both have much better performance than the SPT-h.
2. The results of Greedy-h and NST-h are very close when  $R$  is very small (less than 0.4) or large (more than 0.7). The reason is that, when  $R$  is very small, each node has fewer neighbors to forward the traffic. Thus, all algorithms would compute a similar broadcast tree because there are very few feasible choices. When  $R$  is very large, each node can reach more neighbors and the number of transmitting nodes is less, which makes no much difference in using strategies to choose the transmitting nodes.

Fig. 4 shows the total energy cost versus the mean value of radius ( $R$ ). From the curves in Fig. 4, we observed that

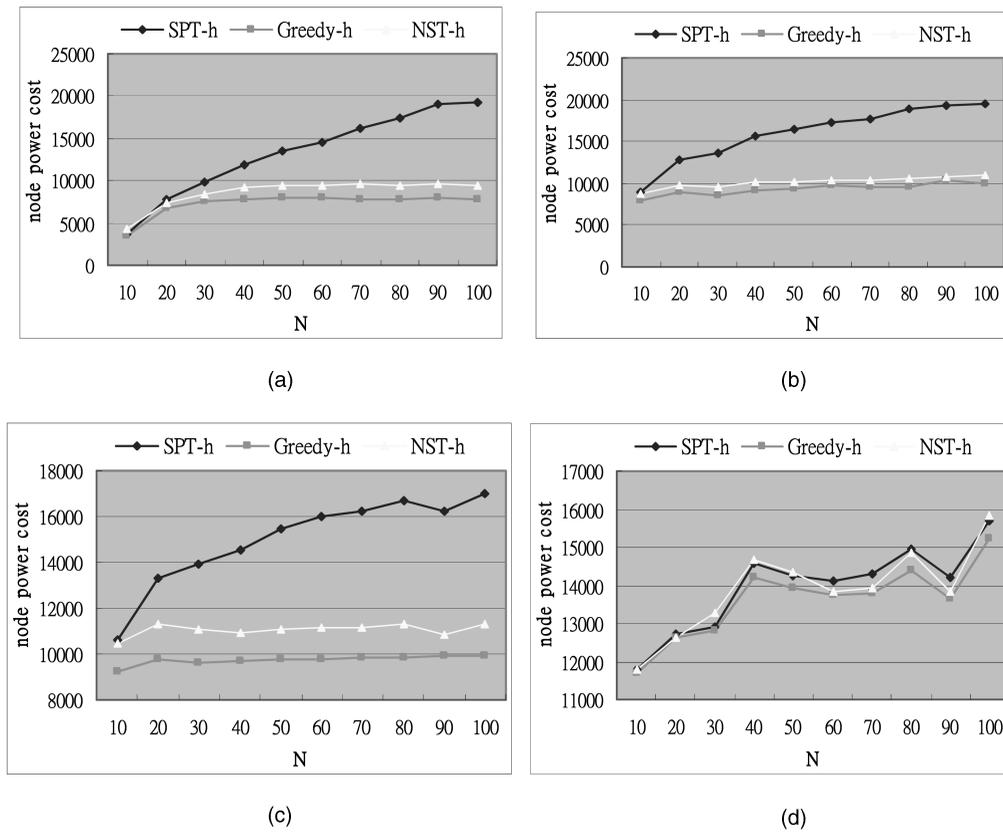


Fig. 3. The total energy cost versus  $N$ . (a)  $\alpha = 2$ ,  $R = 0.2$ , (b)  $\alpha = 2$ ,  $R = 0.4$ , (c)  $\alpha = 2$ ,  $R = 0.5$ , and (d)  $\alpha = 2$ ,  $R = 0.7$ .

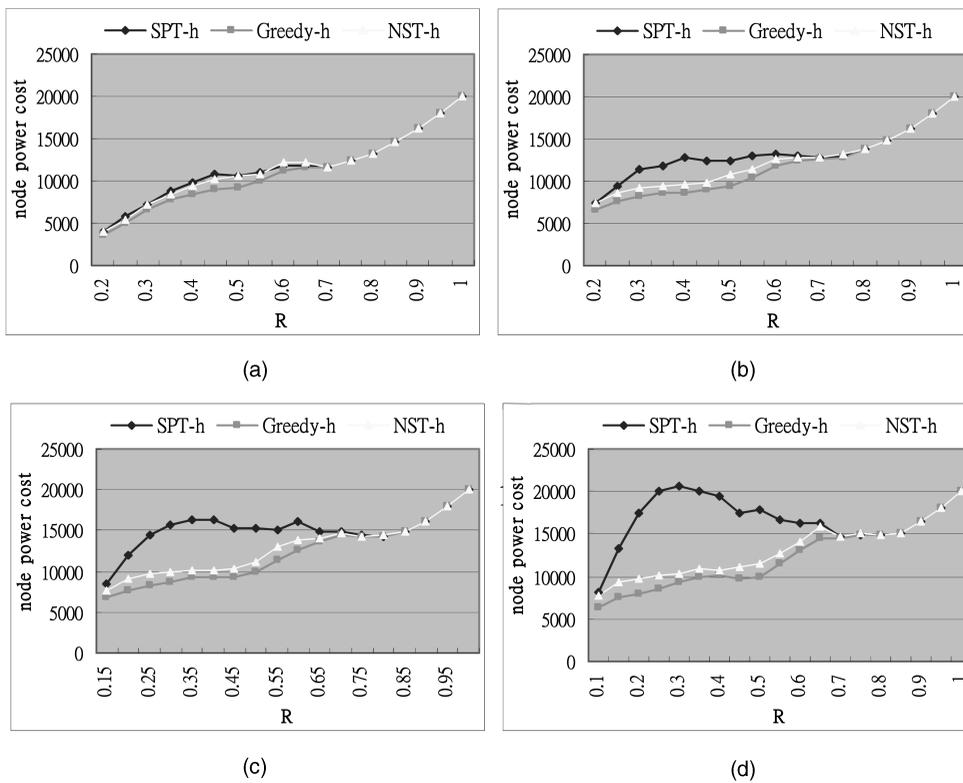


Fig. 4. The total energy cost versus  $R$ . (a)  $\alpha = 2$ ,  $N = 10$ , (b)  $\alpha = 2$ ,  $N = 20$ , (c)  $\alpha = 2$ ,  $N = 40$ , and (d)  $\alpha = 2$ ,  $N = 80$ .

the total energy cost increases as  $R$  increases. When  $R$  is very large, all algorithms are very close because the source

node can almost directly reach all the other nodes without using relaying nodes.

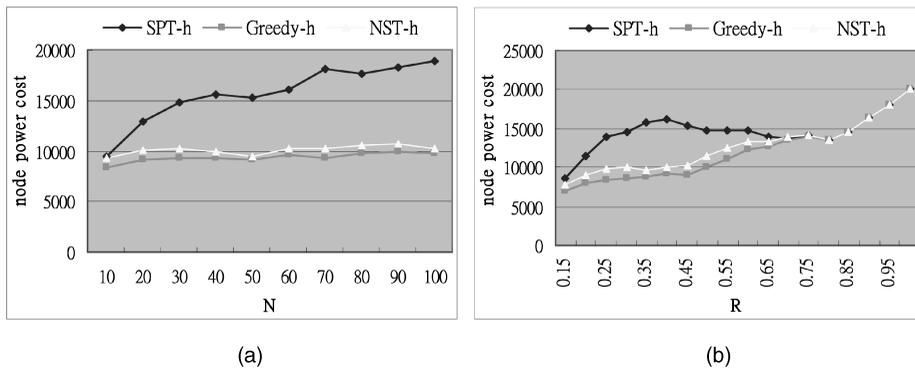


Fig. 5. Uniform radius. (a)  $\alpha = 2$ ,  $R = 0.4$  and (b)  $\alpha = 2$ ,  $N = 40$ .

In the above simulations, nodes have different transmission power (i.e., with different transmission radius). The next group of simulations is for the case where all nodes in the network have the same transmission power.

Fig. 5 shows the cost versus the case where the transmission radius is the same for all nodes. Fig. 5a is the cost versus  $N$  and Fig. 5b is the cost versus  $R$ . The simulation results are quite consistent with the case where nodes have different transmission powers. The difference in the performance of NST-h and Greedy-h becomes very little. Through the results, we can see that our proposed heuristics also perform well in the case of uniform transmission power.

## 6 CONCLUSIONS

We have studied the broadcast routing problem on ad hoc wireless networks. We proposed three algorithms and simulation results have demonstrated that NST-h and Greedy-h perform much better than SPT-h. In particular, Greedy-h outperforms NST-h in most of the time.

In this paper, we did not consider the reception energy cost. In fact, if the reception cost of each node only depends on the hardware, i.e., the reception cost is thus constant irrelevant to the distance from the signal source, the total reception cost of any broadcast tree is a constant regardless of what the tree is. All three proposed algorithms are applicable to this case. If the reception cost is also distance-sensitive, the construction of the broadcast tree will become much more complicated. This will be a subject of our future research. Another future work is the development of distributed algorithms for energy efficient broadcasting which will take node mobility into consideration.

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