On Error Exponents for Lossless Streaming Compression of Correlated Sources

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***Abstract*—We derive upper and lower bounds for the error exponents of lossless streaming compression of correlated sources under the blockwise and symbolwise settings. For low rates, the upper and lower bounds for blockwise codes coincide. For symbolwise codes, the bounds also coincide under a certain condition on the symbol pairs we wish to decode—namely, that their indices are asymptotically comparable to the blocklength.**

I. Introduction

In this paper, we consider lossless compression of streaming data. We assume there are correlated sources to be compressed. Instead of knowing all the source symbol pairs at the time of compression, the encoder pair has access to only one symbol pair per unit time in the streaming scenario. The decoder in the streaming scenario takes some extra time, called a fixed delay, to either produce an estimate of a symbol pair (symbolwise code) or an estimate of a sequence of symbol pairs (blockwise code). See Fig. 1 for an illustration. At time $j+\Delta$, each encoder has access to $j+\Delta$ symbols and the decoder produces an estimate of the symbol pairs generated until time j. The model can be simplified to lossless streaming compression of single source with (resp. without) side information by removing the second encoder (resp. second source). Our interest in this paper is to explore the error exponent (i.e., the speed of exponential decay of the error probability) for lossless streaming compression of multiple correlated sources (i.e., streaming Slepian-Wolf (SW) coding). We denote the blockwise error exponent as $E^*_{\mathrm{bl}}(R_X,R_Y)$ and the symbolwise error exponent as $E^*_{\mathrm{sy}}(R_X,R_Y,\{j_n\})$ where (R_X,R_Y) is the rate pair of the encoders and $\{j_n\}_{n\ge1}$ is an arbitrary sequence indicating the indices we wish to decode.

Related Work: Draper, Chang and Sahai [1] derived lower bounds for the error exponent of streaming SW coding. Chang and Sahai derived the error exponent for lossless streaming compression without and with side information in [2] and [3] respectively. Palaiyanur studied lossless streaming compression of a source with side information with and without a discrete memoryless channel between the encoder and the docoder in [4]. Finally, Zhang analyzed the error exponent of lossless streaming compression of a single source using variable-length sequential random binning in [5].

II. Main Results

We consider a discrete memoryless correlated source (X,Y) with joint distribution P_{XY} on $\mathcal{X}\times\mathcal{Y}$. Both sources produce one symbol per unit time. The decoder decodes a sequence of symbols or a symbol pair with a fixed delay.

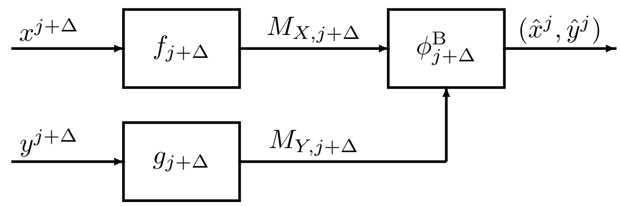

Fig. 1. Lossless compression for correlated streaming sources with sequential encoders and a blockwise decoder at time $j+\Delta$.

Define $\underline{E}_{\mathrm{sw}}(R_X,R_Y)$ and $\overline{E}_{\mathrm{sw}}(R_X,R_Y)$ to be lower and upper bound of the reliability function for SW coding [6]. Denote $\mathcal{R}(P_{XY})$ to be the set of rate pairs such that the above lower and upper bounds coincide and denote $\mathcal{R}'(P_{XY})$ as the set of rate pairs such that $\overline{E}_{\mathrm{sw}}(R_X,R_Y)$ is an strictly increasing function of (R_X,R_Y).

Theorem 1 (Upper Bound)**.** *For any rate pair (R_X,R_Y) such that $((1+\alpha)R_X,(1+\alpha)R_Y)\in\mathcal{R}'(P_{XY})$,*

$$E^*_{\mathrm{bl}}(R_X,R_Y)\le\overline{E}_{\mathrm{sw}}((1+\alpha)R_X,(1+\alpha)R_Y)\quad(1)$$

$$E^*_{\mathrm{sy}}(R_X,R_Y,\{j_n\})\le\overline{E}_{\mathrm{sw}}((1+\alpha)R_X,(1+\alpha)R_Y)\quad(2)$$

Theorem 2 (Lower Bound)**.** *For any rate pair (R_X,R_Y),*

$$E^*_{\mathrm{bl}}(R_X,R_Y)\ge\underline{E}_{\mathrm{sw}}((1+\alpha)R_X,(1+\alpha)R_Y)\quad(3)$$

$$E^*_{\mathrm{sy}}(R_X,R_Y,\{j_n\})\ge\underline{E}_{\mathrm{sw}}((1+\alpha)R_X,(1+\alpha)R_Y)\quad(4)$$

where (4) *holds for all $\{j_n\}$ such that $\lim_{n\to\infty}\frac{j_n}{n}=1$.*

Corollary 3. *For any rate pair (R_X,R_Y) such that $((1+\alpha)R_X,(1+\alpha)R_Y)\in\mathcal{R}(P_{XY})$, we have*

$$\underline{E}_{\mathrm{sw}}((1+\alpha)R_X,(1+\alpha)R_Y)=\overline{E}_{\mathrm{sw}}((1+\alpha)R_X,(1+\alpha)R_Y).\quad(5)$$

For all $\{j_n\}$ such that $\lim_{n\to\infty}\frac{j_n}{n}=1$, we have

$$E^*_{\mathrm{bl}}(R_X,R_Y)=E^*_{\mathrm{sy}}(R_X,R_Y,\{j_n\})\quad(6)$$

References

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