Multiuser OFDM with Adaptive Subcarrier, Bit, and Power Allocation

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- Conclusion
1. Introduction
OFDM

- OFDM: Orthogonal Frequency Division Multiplexing
Advantages

- No intra-cell interference
- Transfer a frequency selective fading channel into some parallel narrow flat fading channels.
- Multiuser diversity
Problems

- How to allocate the subcarriers to different users?
- How to allocate the bits and power to different subcarriers?
  - Power constraint: waterfilling
  - Rate constraint: bit loading
2. Problem Formulation and Solutions
Notations

- $K$ users, $N$ subcarriers
- $R_k$ bits per OFDM symbol for $k$th user
- $c_{k,n}$ bits assigned by $k$th user to $n$th subcarrier
  $c_{k,n} \in \mathbf{D} = \{0, 1, 2, \ldots, M\}$
  if $c_{k,n} \neq 0$, $c_{k,n} = 0$ for all $k \neq k'$
- $f_k(c)$ SNR required for $c$ bits/symbol
- $P_{k,n} = \frac{f_k(c_{k,n})}{\alpha_{k,n}^2}$, $\alpha_{k,n}$: channel gain
Problem Formulation

\[
\begin{align*}
\min & \sum_{n=1}^{N} \sum_{k=1}^{K} \frac{1}{\alpha_{k,n}} f_k(c_{k,n}) \\
\text{s.t.} & \\
& c_{k,n} \in \{0,1,2,\ldots,M\} \\
& R_k = \sum_{n=1}^{N} c_{k,n}, \ k \in \{1,\ldots,K\} \\
& \text{for all } n \in \{1,\ldots,N\}, \text{ if exists } c_{k',n} \neq 0, \text{ then } c_{k,n} = 0, \forall k \neq k'.
\end{align*}
\]

\[
\begin{align*}
\min & \sum_{n=1}^{N} \sum_{k=1}^{K} \frac{\rho_{k,n}}{\alpha_{k,n}^2} f_k(c_{k,n}) \\
\text{s.t.} & \\
& R_k = \sum_{n=1}^{N} \rho_{k,n} c_{k,n}, \ k \in \{1,\ldots,K\} \\
& 1 = \sum_{k=1}^{K} \rho_{k,n}, \ n \in \{1,\ldots,N\} \\
& \rho_{k,n} = \begin{cases} 
1, & \text{if } c_{k,n} \neq 0, \\
0, & \text{if } c_{k,n} = 0.
\end{cases}
\end{align*}
\]
Reformulation and Lagrangian

- Reformulate the problem as a convex optimization problem

\[
\begin{align*}
\min \quad & \sum_{n=1}^{N} \sum_{k=1}^{K} \rho_{k,n} f_k \left( \frac{r_{k,n}}{\rho_{k,n}} \right) \\
\text{s.t.} \quad & R_k = \sum_{n=1}^{N} r_{k,n}, \quad k \in \{1, \ldots, K\} \quad \text{and} \quad 1 = \sum_{k=1}^{K} \rho_{k,n}, \quad n \in \{1, \ldots, N\}
\end{align*}
\]

where \( r_{k,n} = c_{k,n} \rho_{k,n} \).

- \[
L = \sum_{n=1}^{N} \sum_{k=1}^{K} \frac{\rho_{k,n}}{\alpha_{k,n}^2} f_k \left( \frac{r_{k,n}}{\rho_{k,n}} \right) - \sum_{k=1}^{K} \lambda_k \left( \sum_{n=1}^{1} r_{k,n} - R_k \right) - \sum_{n=1}^{N} \beta_n \left( \sum_{k=1}^{K} \rho_{k,n} - 1 \right)
\]
Solution: subcarrier allocation

\[
\lambda_{q,k} = \begin{cases} 
\frac{f_k'(0)}{\alpha_{k,n}^2} & f_k^{-1}(\lambda_k \alpha_{k,n}^2) < 0 \\
\lambda_k & 0 < f_k^{-1}(\lambda_k \alpha_{k,n}^2) < M \\
\frac{f_k'(M)}{\alpha_{k,n}^2} & f_k^{-1}(\lambda_k \alpha_{k,n}^2) > M
\end{cases}
\]

for the \( n \)th subcarrier \( \rho_{k',n}^* = 1, \rho_{k,n}^* = 0 \) for all \( k \neq k' \)

where \( k' = \arg \min_k H_{k,n}(\lambda_{q,k}) \)

\[
H_{k,n}(\lambda) = \frac{1}{\alpha_{k,n}^2} \left[ f_k\left(f_k^{-1}(\lambda \alpha_{k,n}^2)\right) - \lambda \alpha_{k,n}^2 f_k^{-1}(\lambda \alpha_{k,n}^2) \right]
\]
A Simple Case

If we relax the rate constraint to \( R_{\text{total}} = \sum_{k=1}^{K} \sum_{n=1}^{N} r_{k,n} \) then

- \( \lambda_{q,k} \) are the same for different \( k \),

- \( \frac{dH_{k,n}(\lambda)}{d(\alpha_{k,n}^2)} < 0 \)

- \( k' = \arg \max_k \alpha_{k,n}^2 \)

Adaptive solution: allocate the subcarrier to the user who has the largest channel gain.
Solution: bit and power allocation

- **Formulation:**

  \[
  \min_{c_n \in \{0,1,2,...,M\}} \sum_{n=1}^{N} \frac{f(c_n)}{\alpha_n^2}
  \]

  s.t. \( R_{total} = \sum_{n=1}^{N} c_n \)

- **Greedy algorithm:**

  **Initialization:**
  
  For all \( n \), let \( c_n = 0 \), and \( DP_n = [f(1)-f(0)]/a_n^2 \);

  **Bit Assignment Iterations:**
  
  Repeat the following \( R \) times:
  
  \( \tilde{n} = \arg \min_n DP_n; c_{\tilde{n}} = c_{\tilde{n}} + 1; DP_{\tilde{n}} = [f(c_{\tilde{n}} + 1) - f(c_{\tilde{n}})]/a_{\tilde{n}}^2 \);

  End;
3. Simulation Results
Simulation results (1)

![Graph showing simulation results for different modulation techniques. The graph plots RMS delay spread (ns) on the x-axis and average bit SNR (in dB) on the y-axis. The modulation techniques compared are OFDM-OBA, FDMA-OBA, and OFDM-EBA. The OFDM-OBA and OFDM-EBA lines show a slight increase in average bit SNR with increasing RMS delay spread, while the FDMA-OBA line remains relatively constant.](image)
Simulation results (2)

![Graph showing simulation results for OFDM-OBA, FDMA-OBA, and OFDM-EBA with varying number of users.](image)
4. Conclusion
Conclusion

- The total transmit power can be reduced by about 5dB from the case without adaptive subcarrier allocation.
- The total transmit power can be reduced by about 12dB from the case without adaptive bit and power allocation.
- In the frequency selective channel, the adaptive method can exploit the capacity of multiuser OFDM systems.
Thank you!