Wireless Power Control

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CS 8292 : Advanced Topics in Convex Optimization and its Applications
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Outline

• Wireless Power Control by LP

• Wireless Power Control by GP

• Extensions to Multiple Antennas by SDP
Wireless Cellular Network

- 3G CDMA2000 EV-DO cellular network link adaptation maximizes uplink/downlink rate using power control
Power-control software blamed for iPhone 3G reception issues

by Tom Kratz

A plausible scenario for the iPhone 3G reception problems has emerged: it's a power thing.

RoughlyDrafted reported Thursday that a source with AT&T blamed "faulty" power-control software inside the iPhone 3G for the dropped calls and poor reception that owners have been experiencing since the device was released in July. In short, the iPhone 3G demands too much power—more than is necessary—from a local cell tower to maintain a connection, and when multiple iPhones try to glom onto the same tower, the problem snowballs.

The iPhone OS 2.0.2 software update was designed to fix this power-control problem, according to RoughlyDrafted's source. However, the source believes that the problems will not go away entirely until all iPhone 3G owners—or quite a few of them—update to the latest firmware.

http://news.cnet.com/830113579_31002802637.html
System Considerations

• How to solve optimally nonconvex power control problems?

• How many ways to characterize optimality?

• How to design distributed power control algorithms with fast convergence and good performance guarantees?

• How fast is fast?

• Can we leverage existing technology?

• What is the industry impact?
System Model

- **Interference channel with single-user decoding:** Treat interference as additive Gaussian noise.

- **Control interference and meet objective using power control.**

![Diagram of a system model with nodes and edges, including transmitters (Tx1 and Tx2), receivers (Rx1 and Rx2), and nodes with labels G_{11}, G_{21}, G_{12}, and G_{22}. The diagram includes noise signals n_1 and n_2.]
Performance Metrics

• **Signal-to-Interference Ratio:**

\[
\text{SIR}_l(p) = \frac{G_{ll} p_l}{\sum_{j \neq l} G_{lj} p_j + n_l}
\]

with \( G_{lj} \) the channel gains from transmitter \( j \) to receiver \( l \) and \( n_l \) the additive white Gaussian noise (AWGN) power at receiver \( l \)

• Attainable data rate (nats per channel use) is a function of SIR, e.g., Shannon capacity formula \( r_l = \log(1 + \text{SIR}_l) \)

• Power constraints \( \mathbf{1}^\top \mathbf{p} \leq \bar{P} \)
Let \( F \) be a nonnegative matrix with entries:

\[
F_{ij} = \begin{cases} 
0, & \text{if } i = j \\
\frac{G_{ij}}{G_{ii}}, & \text{if } i \neq j
\end{cases}
\]

and

\[
v = \left( \frac{n_1}{G_{11}}, \frac{n_2}{G_{22}}, \ldots, \frac{n_L}{G_{LL}} \right)^\top.
\]

Let \( B \) be:

\[
B = F + \left( \frac{1}{\bar{P}} \right) v 1^\top.
\]
Foschini’s Power Minimization

- \[ \text{minimize } \sum_l p_l \]
  \[ \text{subject to } \text{SIR}_l(p) = \frac{G_{ll}p_l}{\sum_{j \neq l} G_{lj}p_j + n_l} \geq \gamma_l \quad \forall l. \]

- Matrix notation:

  \[ \text{minimize } 1^\top p \]
  \[ \text{subject to } (I - \text{diag}(\gamma)F)p \geq \text{diag}(\gamma)v. \]


- IS-95 CDMA Systems, Qualcomm 3G Systems
Foschini’s Power Minimization

• The optimal power vector $p^*$ is:

$$p^* = (I - \text{diag}(\gamma)F)^{-1} \text{diag}(\gamma)v.$$

• Fixed-point algorithm: $p(k + 1) = \text{diag}(\gamma)Fp(k) + \text{diag}(\gamma)v$

• Distributed Power Control (DPC) algorithm (more illuminating form):

$$p_l(k + 1) = \frac{\gamma_l}{\text{SIR}_l(p(k))}p_l(k) \quad \forall l.$$

• Geometric convergence to $(I - \text{diag}(\gamma)F)^{-1}\text{diag}(\gamma)v$ if and only if $\rho(\text{diag}(\gamma)F) < 1$
Dualities

• GP duality: Application to energy-robustness tradeoff

\[
\begin{align*}
& \text{minimize} & & \sum_l e^{\tilde{p}_l} \\
& \text{subject to} & & \log(\gamma_l/\text{SIR}_l(\tilde{p})) \leq 0 \quad \forall l, \\
& & & \tilde{p}_l \quad \forall l.
\end{align*}
\]

(5)

• LP Duality: Application to Beamforming uplink-downlink duality

• Perron-Frobenius Duality (later on max-min weighted SIR): Application to distributed fast algorithm
Power Minimization with Beamforming

• Base station has $M$ transmit antennas, each mobile user has one receive antenna (Single-cell multiuser MISO channel)

• Let $h_l$ denote the $M$-dimensional channel response from base station to $l$th user, $u_l$ denote the $M$-dimensional transmit beamforming vector, instantaneous transmitted signal denote

$$x_{mt} = \sum_{l=1}^{L} b_l u_l$$

where $b_l$ is the transmitted data signal for the $l$th user

• Received signal at the $l$th user:

$$r_{cv_l} = b_l u_l^\dagger h_l + \sum_{j \neq l} b_j u_j^\dagger h_l + n_l.$$
• Downlink SIR dependent on power $p$ and transmit beamformers $U = [u_1 \ldots u_L]$

$$SIR_l(p, U) = \frac{|h_l^\dagger u_l|^2}{\sum_{j \neq l} |h_l^\dagger u_j|^2 + v_l}.$$
Power Minimization with Beamforming

- Total power minimization:

\[
\begin{align*}
\text{minimize} & \quad \sum_l \|\mathbf{u}_l\|^2 \\
\text{subject to} & \quad \gamma_l (\sum_{j \neq l} |\mathbf{h}_l^\dagger \mathbf{u}_j|^2 + v_l) - |\mathbf{h}_l^\dagger \mathbf{u}_l|^2 \leq 0 \quad \forall l \\
\text{variables:} & \quad \mathbf{u}_l, \quad \forall l.
\end{align*}
\]

- Change-of-variable technique: \( \tilde{\mathbf{U}}_l = \mathbf{u}_l \mathbf{u}_l^\dagger \quad \tilde{\mathbf{H}}_l = \mathbf{h}_l \mathbf{h}_l^\dagger \)

- Equivalent problem in new variables:

\[
\begin{align*}
\text{minimize} & \quad \sum_l \text{Tr}(\tilde{\mathbf{U}}_l) \\
\text{subject to} & \quad \gamma_l (\text{Tr}(\tilde{\mathbf{H}}_l \tilde{\mathbf{U}}_j) + v_l) - \text{Tr}(\tilde{\mathbf{H}}_l \tilde{\mathbf{U}}_l) \leq 0 \quad \forall l, \\
\text{variables:} & \quad \tilde{\mathbf{U}}_l, \quad \forall l.
\end{align*}
\]

- Relax the problem by removing the rank-one constraint leads to semidefinite program (SDP)
• There exists a rank-one solution for $\tilde{U}_l$ for all $l$ of the SDP

• Many other extensions including second order cone program (SOCP) preprocessing

• Stanford CVX, SeDuMi toolbox for SDP

• State of the art of SDP & SOCP solvers for large (but not huge) problems is quite satisfactory


Max-Min Weighted SIR

- **Downlink** case: consider \( \max_{p \geq 0} \min_l \frac{\text{SIR}_l(p)}{\beta_l} \) subject to \( \sum_l p_l \leq \bar{P} \).

- **Uplink** case: consider \( \max_{p \geq 0} \min_l \frac{\text{SIR}_l(p)}{\beta_l} \) subject to \( p_l \leq \bar{P} \ \forall \ l \).

- Can reformulate as a GP and solve using interior point algorithm.

- Can reuse the existing DPC algorithm in any way?

- For distributed solution and its complete analytical solution, see lectures later.
Summary

• A seemingly hard problem can be solved by exploiting the problem structure

• How many possible ways to solve it? And which is the best way?\(^1\)

Reading assignment:


\(^1\)The one that gives you the best mileage!